DISTURBANCES MONITORING FROM CONTROLLER STATES

Daniel Alazard * **Pierre Apkarian** **

* *SUPAERO, 10 av. Edouard Belin, 31055 Toulouse, France - Email :* alazard@supaero.fr
** *Mathmatiques pour l'Industrie et la Physique, Université Paul Sabatier & ONERA, 2 av. Edouard Belin, 31055 Toulouse, France - Email :* apkarian@cert.fr

Abstract: In this paper, it is proposed to implement a given controller using observer-based structures in order to estimate or to monitor some unmeasured plant states or external disturbances. Such a monitoring can be used to perform in-line or off-line analysis (supervising controller modes, capitalizing flight data to improve disturbance modelling, ...). This observer-based structure must involve a judicious onboard model selected to be representative of the physical phenomenon one want to monitor. This principle is applied to an aircraft longitudinal flight control law to monitor wind disturbances and to estimate the angle-of-attack. *Copyright © 2007 IFAC*

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1. INTRODUCTION

Observer-based controllers are quite interesting for different practical reasons and from the implementation point of view. Probably the key advantage of this structure lies in the fact that the controller states are meaningful variables as estimates of the physical plant states. In (Bender and Fowell 1985) and in the generalization (Alazard and Apkarian 1999), a procedure is proposed to compute the observer-based realization of a n_K-th order given controller for a given (on board) n-th order model of the plant. That's allow to use controller states to estimate plant states (Bender *et al.* 1986). In (Cumer *et al.* 2004), observer-based implementation is used to isolate high level tuning parameters in a complex control law. The general bloc-diagram of the closed-loop involving an observer based controller is shown in Figure 1.

In this paper this structure is used to estimate some plant states but also to monitor some external disturbances. From the implementation point of view, it could be interesting to embed such a monitoring system in the control law to reduce real-time computation. In the same idea, integrated FDI (Fault Detection and Isolation) and control design motivates lots of works (Ding *et al.* 2005, Marcos *et al.* 2005, Aravena *et al.* 2005, Henry and Zolghadri 2005) and could also be an extension of the approach proposed in this paper.

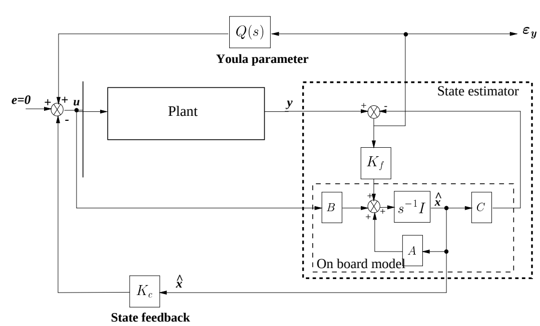

Fig. 1. Observer-based realization of a given controller using YOULA parameterization.

The low-order controller case is more particularly considered here. This case is very interesting from a practical point of view because in many applications (mainly in the field of control design for flexible aerospace vehicles), the order (n_f) of the validation model (including the rigid and the flexible dynamics, the actuator and sensor dynamics,...,) is a very high w.r.t. the controller order ($n_f > n_K$); even if this controller is obtained by an optimal approach on a reduced design model or if the controller is obtained by a frequency-domain classical design. Of course in this case, the separation principle does not hold any more, and the interest of observer-based structure, that is the controller state is an estimate of the plant state, is lost. So the computation of a reduced-order model (called onboard model because it must be integrated in real-time) is required. But, if the n-th order reduced onboard model is representative enough, it is possible to estimate some state variables or to monitor some disturbances taken into account in this onboard model.

Observer-based realization are also interesting to take into account the input reference signal in the control loop when the controller has been design to reject disturbance and to ensure good stability margins, parametric robustness, ... (Alazard 2002). Then, the input reference can be plugged on signal e in Figure 1 to ensure that input reference variations will have no effect on the state estimation error $\varepsilon = x - \hat{x}$ (due to the uncontrollability of state-estimator poles). For instance: $e = K_c x_{ref}$ where x_{ref} is the state reference.

In the next section we recall (from (Alazard and Apkarian 1999)) the procedure to compute the observer-based realization of a given controller for a given model of the plant. In section 3, we consider the case of a longitudinal flight control of a large carrier aircraft. A reduced onboard model (with $n = n_K$) is then built to include a wind model while staying representative of the full order validation model (in this application: $n_f = 111$, $n_K = 12$). Then the observer based realization of the controller is computed on this onboard model and used to feed-forward the input reference signal. The estimation of the wind and of the angle of attack, provided by the state of the observer-based controller, is analyzed by simulations. These simulations take into account the full order model, an input reference profile, measurement noises and various wind profiles to analyze the robustness of the rough wind model included in the onboard model.

2. OBSERVER-BASED REALIZATION OF A GIVEN CONTROLLER

Consider the stabilizable and detectable nth-order model $G(s)$ (m inputs and p outputs) with minimal state-space realization:

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} . \quad (1)$$

Consider also the stabilizing n_Kth order controller $K_0(s)$ with minimal state-space realization:

$$\begin{bmatrix} \dot{x_k} \\ u \end{bmatrix} = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \begin{bmatrix} x_k \\ y \end{bmatrix} . \quad (2)$$

The key idea is to express the compensator state equation as an LUENBERGER observer of the variable $z = Tx$. So, we will denote:

$$x_K = \widehat{z} \quad (3)$$

Then it can be shown (see (Alazard and Apkarian 1999)) that T is the solution a generalized non-symmetric RICCATI equation:

$$[-T \ I] \begin{bmatrix} A + BD_K C & BC_K \\ B_K C & A_K \end{bmatrix} \begin{bmatrix} I \\ T \end{bmatrix} = 0 \quad (4)$$

The characteristic matrix associated with the RICCATI equation (4) is nothing else than the closed-loop dynamic matrix constructed on the state vector $[x^T,\, x_K^T]^T$:

$$A_{cl} = \begin{bmatrix} A + BD_K C & BC_K \\ B_K C & A_K \end{bmatrix} . \quad (5)$$

Such a RICCATI equation can then be solved in $T \in \mathbb{R}^{n_k \times n}$ by standard subspace decomposition techniques, that is:

- compute an invariant subspace associated with a set of n eigenvalues, spec(Γ_n) (spec(A) is the set of eigenvalues of the matrix A), chosen among $n + n_K$ eigenvalues in spec(A_{cl}), that is,

$$\begin{bmatrix} A + BD_K C & BC_k \\ B_K C & A_k \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \Gamma_n , \quad (6)$$

  where $U_1 \in \mathbb{R}^{n \times n}$ and $U_2 \in \mathbb{R}^{n_K \times n}$. Such subspaces are easily computed using Schur decompositions of the matrix A_{cl}.
- compute the solution

$$T = U_2 U_1^{-1} . \quad (7)$$

Then, 3 cases can be encountered:

- full-order controller ($n_K = n$): one can compute a state feedback gain $K_c = -C_K T - D_K C$, a state estimation gain $Kf = T^{-1} B_K - BD_K$ and a static YOULA parameter $Q(s) = D_K$ such that the observer-based structure fitted with the YOULA parameter (depicted in Figure 1) is equivalent to the initial controller form the input-output behavior.
- augmented-order controller ($n_K > n$): then the YOULA parameter becomes a dynamic transfer of order $n - n_K$,

- reduced-order controller ($n_K < n$): then one can built a state-observer based structure if $n_K \geq n - p$ where p stands for the number of measurements of $G(s)$. If $n_K < n - p$, a model reduction is required to built a (partial) state-observer realization.

Note that there is a combinatory set of solutions according to the choice of n auto-conjugate eigenvalues among $n+n_K$ closed-loop eigenvalues. The range of solutions can be reduced according to the following considerations :

- a set of auto-conjugated eigenvalues must be chosen in order to find a real parameterization,
- an uncontrollable (resp. unobservable) eigenvalue in the system must be selected in the state-feedback dynamics (resp. state-estimation dynamics),
- lastly, the state-estimation dynamics (spec($A - K_f C$)) is usually chosen faster than the state-feedback dynamics (spec($A - BK_c$)).

The separation principle of observer based realization allows to state that :

- the closed-loop eigenvalues can be separated into n closed-loop state-feedback poles (spec($A-BK_c$)), n closed-loop state-estimator poles (spec($A - K_f C$)) and the Youla parameter poles (spec(A_Q)),
- the closed-loop state-estimator poles and the Youla parameter poles are uncontrollable by e,
- the closed-loop state-feedback poles and the Youla parameter poles are unobservable from ε_y. The transfer function from e to ε_y always vanishes.

3. APPLICATION TO LONGITUDINAL FLIGHT CONTROL

3.1 *Model and initial controller*

The interconnection structure between initial data is depicted in Figure 2. The validation model $P(s)$ is the 111th order longitudinal model of the aircraft, linearized around the pitch axis in a nominal flight configuration. The dynamics of this model can be detailed in the following way:

- 1 rigid mode: short-period mode,
- 28 structural flexible (bending) modes,
- 40 poles (fast and damped w.r.t. the previous modes) representative of aerodynamics lags and actuators dynamics.
- a 13-th order turbulence model on the vertical wind input w.

In addition to the turbulence model, a first order low-pass filter $V(s)$ (see Figure 2) is introduced on w to take into account the frequency response of the wind. The model $P(s)$ have 5 control signals (or 5 control surfaces): the inner rudder, the outer rudder, and 3 pairs of ailerons (inner, middle and outer) which can be used in a symmetrical way, in this study, to increase longitudinal flight control performances. 5 measurements are available for the control: the pitch rate, the cockpit vertical acceleration and 3 vertical accelerations measured on the wings and on the fuselage to observe the first wing bending mode. The first output of P is the angle of attack α which is not measured. α is used here for analysis purpose.

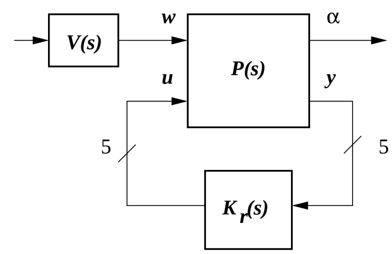

Fig. 2. Interconnection structure of the plant P and initial controller K_r.

The 12-th order controller $K_r(s)$ was designed to accelerate the short-period mode (which is naturally damped enough) and to reduce loads in turbulence. This controller is designed from a 29-th order design model, a multi-objective procedure based on the Cross Standard form and a reduction, which are described in (Alazard *et al.* 2006). The dynamic behavior of the design can be summarized by the root locus presented in Figure 3. This locus presents the open loop dynamics (model dynamics (P) is plotted with black ×, controller dynamics (K_r) is plotted with green ×) and the closed loop dynamics (plotted with black +). The branches from × to + represent pole trajectories when the loop gain varies from 0 to 1 simultaneously on the 5 control inputs.

3.2 *Onboard model computation*

From a a bloc-diagonal realization of the full order model P, the 12-th order onboard model P_o is built in following way:

- the short-period mode and the first three flexible modes are kept (other flexible modes are all truncated),
- the 40-th order subspace associated with the fast and damped poles is balanced and truncated to order 2 ((Alazard 2002)),
- the 13-th order turbulence model is neglected but a 2-nd order Dryden filter is introduced on the wind input to represent roughly this turbulence model and the filter $V(s)$,
- using the output matrix, a change of variable is finally performed in such a way: the first 7

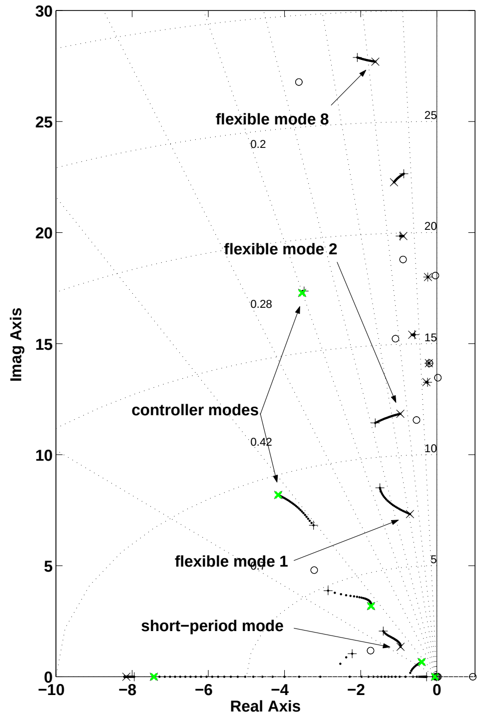

Fig. 3. Root locus of $P(s)$ and $K_r(s)$.

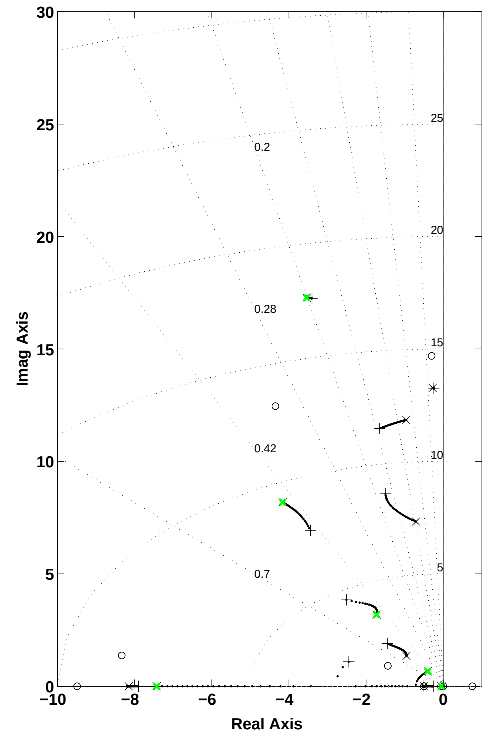

Fig. 4. Root locus of $P_o(s)$ and $K_r(s)$.

states of the onboard model P_o correspond to the 6 outputs (angle-of-attack, pitch rate, 4 vertical accelerations) and the wind w (that is: the output of the DRYDEN filter).

The root locus obtained with K_r and the onboard model P_o is depicted in Figure 4: the evolution of low-frequency modes is quite representative of the behavior observed with the full order model (Figure 3). Then, one can expect that low-frequency signals (like α and perhaps w) could be estimated by states of the observer-based realization of K_r involving this onboard model. Let us note (A_o, B_o, C_o, D_o) the state space realization of the onboard model.

3.3 *Observer-based realization*

The state feedback gain K_c (5×12), the state estimator gain K_f (12×5) and static YOULA parameter (5×5) are computed using the procedure described in section 2 according to the eigenvalue selection depicted in Figure 5: the 12 eigenvalues, among the 24 closed-loop eigenvalues, chosen to solve the RICCATI equation (4) are in fact the 12 eigenvalues which are located on the branches starting from the 12 open-loop eigenvalues of the onboard model $P_o(s)$ (black $\times$), in the root locus depicted in Figure 4. Such a choice can be easily systematized by an simple procedure. Note that in order to have good estimation performance, the 12 fastest close-loop eigenvalues could be selected for the state estimation dynamics but, as a counterpart, the estimation will be very sensitive to measurement noises. After some trials, the choice proposed in Figure 5 seem to be a good trade-off.

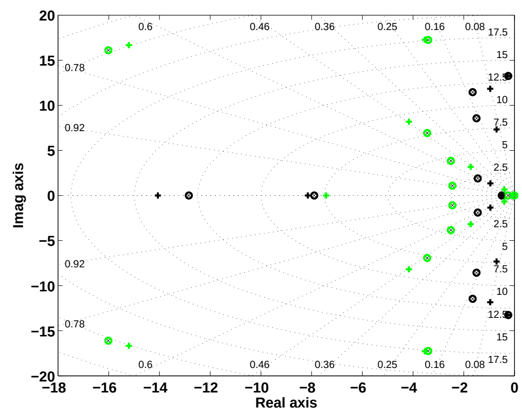

Fig. 5. Eigenvalue selection to solve equation (4): open-loop poles of P_o are marked with black $+$; open loop poles of K_r are marked with green $+$; 24 closed-loop poles are marked with o: the green ones are affected to state-estimation dynamics ($\mathrm{spec}(A_o - K_f C_o)$) and the black ones are affected to state feedback dynamics ($\mathrm{spec}(A_o - B_o K_c)$).

3.4 *Simulation results*

Once the controller K_r is realized using an observer-based structure, it is implemented according to the simulation schema depicted in Figure 6. Of course, this simulation involves the full-order validation model $P(s)$. The physical meaning of the observer-based realization allows the input reference on the vertical load factor N_z to be plugged into a comparator on the estimation of N_z, that is the third state variables of the estimator. The input reference used in simulation is 2 successive and opposite square signals filtered by a low pass feed-forward controller $H(s)$. The responses of the angle-of-attack α and its estimate α_e, when disturbances (wind and noises) are set to 0, are plotted in Figure 7.

The wind w is generated by a white noise with a PSD (Power Spectral Density) W filtered by $V(s)$. White measurement noises are taken into account with PSD V_q for the pitch rate measurement and V_{N_z} for the 4 vertical acceleration measurements.

Nominal numerical values:

$$W = 1\,m^2/s, \quad V(s) = \frac{1}{s+1},$$

$$V_q = 0.05\,rd^2/s \quad \text{and} \quad V_{N_z} = 1\,m^2/s^3\ .$$

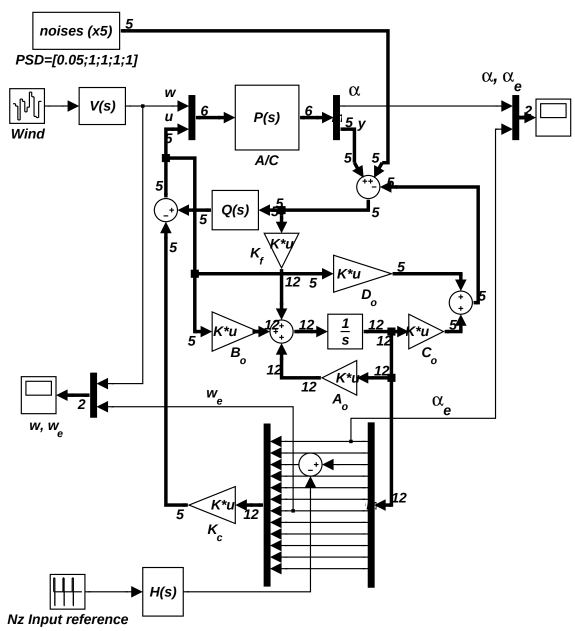

Fig. 6. Simulation schema.

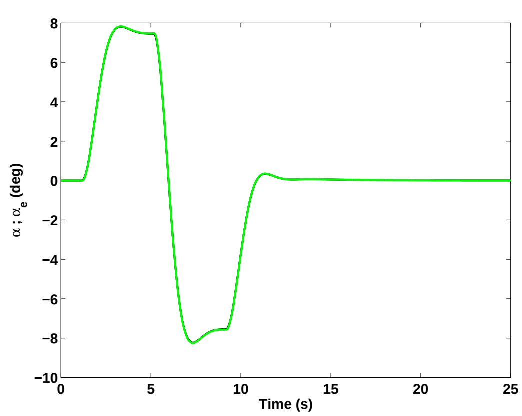

Fig. 7. Black: $\alpha(t)$; green: $\alpha_e(t)$ (without disturbances: $W = V_q = V_{N_z} = 0$).

In presence of disturbances (wind and noise) one can notice in Figure 8, that α_e (the first state of the observer-based controller) stays a very good estimate of α. Such an estimate of the angle-of-attack might be very useful to monitor the state of the A/C and its distance to flight domain boundaries; without the use of any dedicated angle-of-attack sensor. In Figure 9, one can also notice that w_e, that is the 7-th state of the observer-based controller is quite representative of the wind $w(t)$. The wind estimation error $w(t) - w_e(t)$ is insensitive to the input reference profile due to the quasi-uncontrollability of the state-estimation dynamics which works pretty well although the onboard model P_o is consequently reduced w.r.t. the full model $P(s)$.

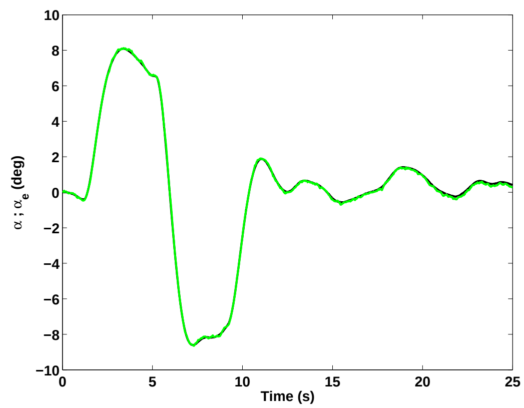

Fig. 8. Black: $\alpha(t)$; green: $\alpha_e(t)$ (with nominal disturbances).

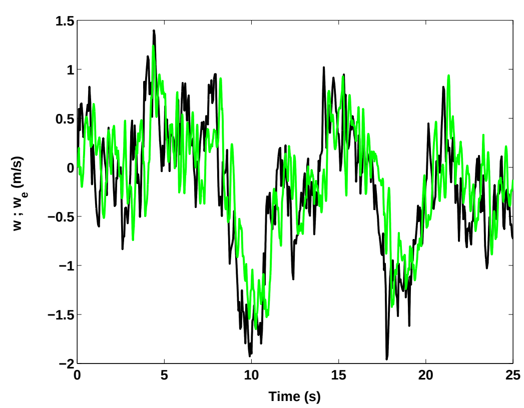

Fig. 9. Black: $w(t)$; green: $w_e(t)$ (with nominal disturbances).

To analyze the robustness of the wind model taken into-account in the onboard model P_o, some variations are made on the generator filter $V(s)$ used in simulation:

- $V(s) = 1/(s + 0.2)$ that is: a low-frequency wind but with a higher magnitude than the nominal wind,
- $V(s) = 1/(s + 5)$ that is: a high-frequency wind but with a lower magnitude than the nominal wind.

In both cases (see Figures 10 and 11), w_e stays quite representative of the wind $w(t)$. One can also notice a 0.3 second delay in the response of

w_e w.r.t. $w(t)$. Such a delay could be critical to used w_e in the low-level control loop, but it can be used in a higher level loop to detect severe wind conditions and to switch the control to a new mode. Note also that this delay can be removed for off-line analysis aiming to improve turbulence knowledge and modelling.

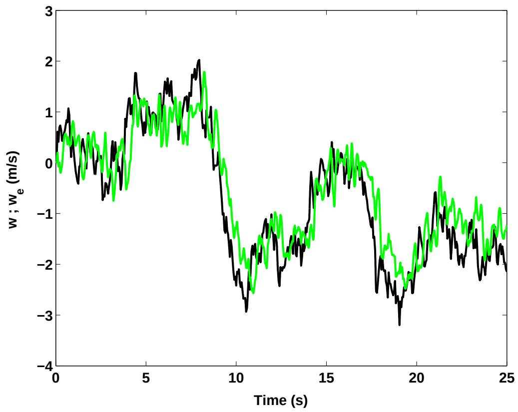

Fig. 10. Black: $w(t)$; green: $w_e(t)$ (with $V(s) = 1/(s+0.2)$).

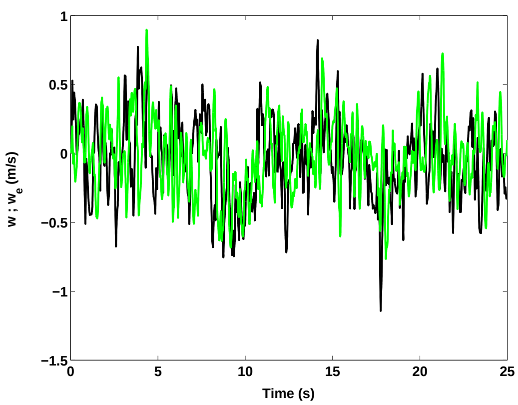

Fig. 11. Black: $w(t)$; green: $w_e(t)$ (with $V(s) = 1/(s+5)$).

4. CONCLUSIONS

Observer-based structure is used to implement a reduced-order controller. The meaningful state of the observer-based controller allows to plug judiciously the input reference in the control law (when the initial controller was designed to meet pure disturbance rejection specifications) and to estimate some plant states or external disturbances taken into account in the onboard model. The reduction of the full-order model to get the onboard model is the key step of this procedure and is based on a practical know-how. Further works are required to define systematically this onboard model.

This approach was applied on a high order aircraft model (order > 100) and low order longitudinal control law (order 12) to monitor the angle-of-attack and the wind. First results are very promising. Various wind profiles were well estimated even the onboard model in based on a very rough and frozen wind model. For a complete validation of this approach, next work will be focused on the parametric robustness analysis of the onboard model. The interest of this approach to design fault detection filters embedded in the controller will be also studied.

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