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Non-local equilibrium continuum modeling of partially saturated drying porous media: Comparison with pore network simulations

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ABSTRACT

A two equation continuum model is developed to simulate the mass transfer in drying porous media. The main goal is to capture the so-called non-equilibrium effect. To this end, we operate in a regime where the liquid phase is immobile so that non-equilibrium mass exchange between liquid and vapor phase dominates. The formulation of the model relies on an upscaling technique. This notably permits to formulate the non-local equilibrium phase change term on a firmer basis. The upscaling also indicates that there is no reason to consider an enhancement factor in the vapor diffusion model. The macroscopic model parameters are determined from pore network drying simulations. The same simulations are also used as a reference to compare with the predictions of the non-local equilibrium continuum model. The solution of the two equation continuum model proves that this model simulates the non-local equilibrium effect with reasonable accuracy. Also, the simulations indicate that the non-local equilibrium effect is especially significant at the porous medium surface.

1. Introduction

The evaporation from porous media is a key natural and industrial process. The modeling of drying porous media has been a subject of research for decades. Heat and mass transfer during the two stages of drying is more commonly described using the concept of continuum modeling (CM). A presentation of the most popular continuum model in this context can be found in (Whitaker, 1977), where heat and mass transfer during evaporation in porous medium was extensively modeled in the framework of the volume averaging method. This widely used CM combines the equations for the liquid flow and the vapor flow into a single equation under the assumption of local equilibrium (LE) between the vapor and the liquid phases. However, this assumption has been questioned (Bénet and Jouanna, 1982). It was argued that the equilibrium characteristic time \( \tau_{eq} \approx L^2 / D_v \), where \( L \) is a characteristic length of the porous medium and \( D_v \) is the vapor diffusivity, was too long for the local equilibrium condition to be satisfied during drying. This leads to a different class of models where the liquid flow equation and the vapor flow equation are not merged into a single equation and both contain a non-equilibrium phase change term. One can refer, for instance, to (Li et al., 2019) for a presentation of the NLE CM together with the more classical LE CM model. The NLE effect is considered to be especially significant in hygroscopic materials (see, e.g., Bénet and Jouanna 1982; Bénet et al., 2009; Ouedraogo et al., 2013). However, in capillary porous materials, in which pore liquid is spatially allocated by capillary effects and thus adsorption phenomena are negligible, the NLE effect has been treated as a seemingly less important issue. However, the pore network model (PNM) simulations reported in (Attari Moghaddam et al., 2017) clearly indicate that the NLE effect is also
present during the drying of capillary porous media. As a consequence of the NLE effect, it was shown that the saturation profiles computed from the pore network simulations could not be simulated with the classical LE model using a single set of macroscopic parameters (Vorhauer et al., 2010). Also, the drying kinetics could not be simulated over the complete drying process using the classical one equation CM model (Attari Moghaddam et al., 2017). The conclusion was therefore that a NLE CM formulation should be considered not only for hygroscopic materials as suggested by Bénet and co-workers but also for capillary porous media. The present work is a step in this direction.

The methodology used in this work is the same as in (Attari Moghaddam et al., 2017). PNM simulations are used for generating reference data, such as saturation profiles and drying kinetics. Then, the NLE CM is tested against the volume averaged PNM data. Compared to (Attari Moghaddam et al., 2017), the new feature is that a NLE continuum model is considered. Hints on the formulation of the NLE phase change term in this model are obtained using the volume averaging method. Also, as in previous work on the NLE effect, i.e. (Ouedraogo et al., 2013), we focus on the conditions of low initial saturation. In our case, this corresponds to a situation where the liquid phase is formed by a set of disconnected small clusters. This enables us to concentrate on a situation where the vapor transport is a dominant mechanism. Similarly to (Attari Moghaddam et al., 2017), evaporation is controlled by mass transfer and the temperature variation is negligible.

The paper is organized as follows: In Section 2, the formulation of the two equation CM is described in detail. The pore network drying algorithm is explained in Section 3. The method for computation of macroscopic effective parameters is described in Section 4. Results of PNM drying simulations are presented in Section 5, and macroscopic parameters are discussed in Section 6. NLE CM simulations are compared to PNM drying reference data in Section 7, with some further discussion in Section 8. Finally, Section 9 summarizes the key results and outlines possible future research directions.

2. Continuum model formulation

As discussed in several previous works, i.e. (Li et al., 2019; Ouedraogo et al., 2013) and references therein, the NLE continuum model can be expressed as a system of two coupled equations, one for the liquid flow and one for the vapor flow. When the liquid phase is formed by small disconnected clusters, there is no macroscopic flow in the liquid phase and the model can be simplified. Under these circumstances, it can be expressed as

\[
ed (1) \quad \frac{\partial (1 - S)P_v}{\partial t} = \nabla \cdot \left( (1 - S)D_{\text{eff}} \nabla P_v \right) + \frac{RT}{M_v} \dot{m}
\]

for the vapor flow and

\[
ed (2) \quad \frac{\partial S}{\partial t} \approx \dot{m}
\]

for the liquid phase (considered to be immobile). In Eqs. (1) and (2), \(v, \nu, D_{\text{eff}}, S, \rho_l, P_v\) denote the porosity, time, macroscopic vapor diffusion coefficient, liquid saturation, water density and water vapor partial pressure, respectively. \(M_v, R\) and \(T\) represent the molar mass of water, universal gas constant, and temperature; \(\dot{m}\) is the phase change rate between the liquid and vapor phases. The latter is also referred to as the NLE phase change term. Different formulations are proposed for \(\dot{m}\) in the literature. In the works of Bénet and co-workers, i.e. (Ouedraogo et al., 2013) and references therein, \(\dot{m} = \beta \frac{RT}{M_v} \ln \left( \frac{P_{v,eq}}{P_v} \right) \)

where \(P_{v,eq}\) is the equilibrium vapor pressure and \(\beta\) is a phenomenological coefficient which notably depends on the porosity medium microstructure. In (Li et al., 2019), this term was expressed as \(\dot{m} = b \frac{RT}{M_v} \ln \left( \frac{P_{v,eq}}{P_v} \right) \)

where \(b\) is a coefficient and \(S_{\text{res}}\) is the residual water content. The same expression was used in (Pujol et al., 2011) with \(S_{\text{res}} = 0\). In order to clarify the formulation of \(\dot{m}\) a derivation using the volume averaging method (Whitaker, 2013) is presented in Appendix A taking advantage of the mathematical analogy with the problem of diffusion with heterogeneous reaction presented in (Whitaker, 2013). This leads to the expression of the NLE phase change term as

\[
ed (3) \quad \dot{m} \approx \frac{D_{\text{eff}}}{RT} \frac{M_v}{kP_v} \left( P_{v,eq} - P_v \right)
\]

where \(D_{\text{eff}}\) is the specific interfacial area between liquid and vapor phases and \(k\) is a coefficient. In our case \(P_{v,eq}\) is equivalent to \(P_{v,eq}\), \(P_{v,eq}\) is the saturation water vapor partial pressure since adsorption phenomena are not considered in the PNM simulations on the ground that they are negligible in capillary porous media. Another noticeable outcome from the derivation presented in Appendix A is that the effective coefficient \(D_{\text{eff}}\) can be computed as if the liquid was simply acting as a solid obstacle to vapor diffusion. In other terms and contrary to the consideration on vapor diffusion in drying by (Philipp and De Vries, 1957) the derivation does not suggest that an enhancement effect should be considered in the determination of \(D_{\text{eff}}\) as a result of phase change within the representative elementary volume. One can refer, for instance, to (Plumb et al., 1999) for more details on the vapor diffusion enhancement factor.

To simulate the drying problem with the NLE CM (Eqs. (1)-(3)), three macroscopic coefficients must be determined, namely \(D_{\text{eff}}, D_{\text{vap}}\) and \(k\), noting that \(D_{\text{eff}}, D_{\text{vap}}\) are functions of saturation. The method to determine these coefficients is presented in Section 4. Then, initial and boundary conditions must be specified. The pore network is initially saturated with liquid and supposed to be at equilibrium. Thus \(P_{v} = P_{v,eq}\) throughout the computational domain at \(t = 0\). The initial liquid phase saturation is computed by volume averaging the initial liquid water distribution in the pore network. The two equation CM is solved in 1D by discretizing the computational domain by using finite volume method. At the bottom of the domain, a zero flux boundary condition is applied, whereas at the evaporative surface the specific evaporation rate is computed by the following equation:

\[
ed (4) \quad \varepsilon (1 - S) \frac{M_v}{RT} D_{\text{vap}} \frac{P_{v,eq} - P_{v}}{L_{\text{eq}}} = \frac{M_v}{RT} \frac{D_{\text{vap}}}{L_{\text{eq}}} P_{v,eq}
\]

where \(D_{\text{vap}}, L_{\text{eq}}, P_{v,eq}\) and \(P_{v}\) denote the water vapor molecular diffusion coefficient, external boundary layer thickness, unit normal vector directed toward the external boundary layer, vapor pressure of the surface volume element and vapor pressure in the bulk air, respectively. The formulation of the boundary conditions at the evaporative surface is actually not that obvious with the two equation NLE CM model since both liquid and vapor pores can be present at the surface. The question thus arises as to how the external evaporation flux should be split into a contribution from the evaporation of the surface liquid pores and a contribution from surface pores occupied by the gas phase. However, the consideration of an immobile liquid phase, i.e. Eq. (2), simplifies the modeling of the exchange at the evaporative surface since no boundary condition is actually needed in conjunction with Eq. (2). Only boundary conditions for Eq. (1) are needed.

3. Pore network model algorithm

In our pore network model, the pore space is conceptualized by cylindrical tubes referred to as throats. The connectivity between these throats is provided by pore nodes which serve only as com
putational points and do not carry any volume. The radii of the cylindrical throats follow a normal distribution which is based on mean radius and standard deviation. The lateral edges are connected to each other (periodic boundary condition). The network of pore nodes extends outside the pore network domain into the discretized boundary layer. External mass transport occurs through this diffusive boundary layer from the evaporating meniscus throats at ambient conditions. In this work, pore network simulations are carried out in three dimensional space.

In the pore network model, we operate in the special regime where the liquid phase is immobile. In a capillary porous medium, this special regime can be created by considering an initial liquid phase structure which consists of isolated liquid clusters (see Fig. 1). These liquid clusters are minimally sized and are located randomly inside the network. While capillary pumping is considered, we neglect the influence of viscosity in the liquid phase. Moreover, the discontinuity of the liquid phase essentially means that there is no mass transport through the network in liquid phase, eliminating the need to compute liquid pressure field. Hence mass transport essentially occurs only through water vapor diffusion in the gas phase. This means that the volume less pores are used to compute only the gas phase vapor partial pressure through Stefan’s flow based on

$$J_{v,k} = \pi r_k^2 \frac{M_v}{R T_L} \rho_{am} D_{am} \ln \left( \frac{P_{v_j}}{P_{v_j}} \right) \frac{P_{am}}{P_{am}}.$$  \hspace{1cm} (5)

At each pore node, mass balance for vapor flux is applied, enabling us to obtain a system of equations based on linearized version of Eq. (5), which is then numerically solved to obtain the vapor pressure field. This means that we obtain values of partial vapor pressure at each pore node present inside and outside (boundary layer) the network. The discretization of the drying process is associated with the time for emptying of one meniscus throat, during which the vapor flow is assumed to be quasi steady. For the next time step, the vapor pressure field is calculated based on the updated boundary conditions.

As the drying process goes on, the liquid clusters shrink and split into isolated menisci. The calculation of vapor transport is considered independently for each cluster. To this end, it is essential to label the liquid clusters. This is achieved by employing a variant of the Hoshen Kopelman algorithm (Metzger et al., 2006). The drying process goes on until the liquid in the network has been entirely evaporated. We assume that the porous medium is non-hygroscopic; without adsorption a completely unsaturated net work is achieved at the end of the drying process. Furthermore, secondary capillary structures (liquid films and rings) are not considered in the model. In addition to this, due to slow drying (at ambient conditions), the process is assumed to be isothermal, therefore we do not consider the role of viscosity for gas transport and assume a constant pressure at all pore nodes. For the details beyond the present brief discussion of the fundamentals of PNM for drying, the reader may refer to (Prat, 2002; Metzger et al., 2007; Prat, 2011).

4. Method for computation of CM parameters

Macroscopic parameters can be determined from dedicated PNM simulations, e.g. (Ören et al., 1998; Blunt et al., 2001). Most of the literature in this regard pertains to the drainage process, and the focus is on relative and absolute permeabilities and capillary pressure curve. The use of PNM for drying porous media (Nowicki et al., 1992; Attari Moghaddam et al., 2017) has also been done along similar lines with the addition of computation of vapor diffusivity. This requires developing specific computations over a representative elementary volume (REV). In what follows, we proceed somewhat differently and make use of a method inspired from the method used to determine the moisture transport coefficients from drying experimental results in previous works. Typically, in these experiments, transient saturation profiles were determined for a given set of conditions (e.g. Schoeber, 1976; Marchand and Kumaran, 1994; Pel et al., 1996; Gomez et al., 2007). Subsequently these saturation profiles were used to compute moisture transport coefficients. In our case, the saturation profiles are obtained from volume averaging of the appropriate PNM drying simulation data and we also use the volume averaged vapor pressure profiles.

The macroscopic parameters determined from PNM drying simulations are effective vapor diffusivity and specific interfacial area. As mass transport is in the gas phase only (immobile liquid phase), we do not compute liquid diffusivity. The dependency of macroscopic effective vapor diffusivity and specific interfacial area on local saturation is determined using the data obtained from PNM drying simulations. For our PNM, the computation is carried out on two levels: on the scale of pore throats, we compute the liquid saturation and on the scale of volume less pore nodes we compute partial vapor pressure.

The data at the pore scale is transformed into macroscopic scale by dividing the three dimensional pore network into horizontal slices of thickness $dz$ an exemplary 3D pore network is illustrated in Fig. 2. Here one slice is the macroscopic averaging volume over which local saturation and vapor pressure are computed. As shown in Fig. 2, we characterize a slice such that it consists of pore nodes which lie on the side of top bounding plane of the slice, horizontal throats directly connected to these pore nodes and the neighbor vertical throats that are directed towards the bottom. The local saturation of the slice is computed by calculating the ratio of liquid volume in the throats contained in the slice to the total volume of throats in the slice. For the computation of local vapor pressure in the slice, we compute the arithmetic mean of the vapor pressures of the pores contained inside the slice. The characterization of the surface slice is different from other slices due to the fact that in PNM the pore nodes in surface plane do not have any directly connected horizontal throats.

The vapor flux $j_{v,j}$ through all vertical throats in a slice is computed using the vapor pressure of pore nodes through Eq. (5) and

Fig. 1. A 2D representation of 50 x 50 pore network in which the liquid phase (shown in blue) is distributed as isolated clusters. The gas and solid phases are represented in white and grey, respectively. Note that for our discrete simulations 3D pore networks are used. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
Table 1: Structural and physical parameters for PNM simulations.

<table>
<thead>
<tr>
<th>Structural property</th>
<th>Unit</th>
<th>Value</th>
<th>Physical constant</th>
<th>Unit</th>
<th>Value</th>
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<tbody>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Boundary layer discretization</td>
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<td>25 \texttimes} \text{25 \times} \text{4}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean throat radius</td>
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<td>250</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of throat radius</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Throat length</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network porosity</td>
<td></td>
<td>0.594</td>
<td></td>
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</tbody>
</table>

The input parameters that specify the structure and physical conditions for PNM drying simulations are listed in Table 1. To minimize the influence of randomness in throat size distribution and initial liquid phase structure on the macroscopic parameters, we carried out 45 simulations each with different realization of throat size distribution and initial liquid phase structure (location of liquid clusters). All the results shown in this section are averaged over these 45 simulations (unless stated otherwise). As the liquid phase is discontinuous and the size of clusters is very small, the viscosity of liquid phase becomes irrelevant. We operate in the capillary dominated regime as far as the evolution of the liquid phase is concerned. Mass transport through the network is purely controlled by diffusion in the gas phase. To produce the initial liquid phase structure with discontinuous liquid phase, we can have an arbitrary amount of liquid in the network. We choose an arbitrary initial network saturation of 0.26 for the PNM drying simulations and the subsequent CM solution.

5.1. Drying kinetics and saturation profiles

The evolution of overall network saturation $S_{\text{net}}$ with time and the change in normalized total evaporation rate with respect to $S_{\text{net}}$ are shown in Fig. 3. The evaporation rate drops sharply as the surface water evaporates, followed by a consistent decrease in evaporation rate.

- Compared to the classical drying kinetics for capillary porous media, no constant rate period (CRP) is observed. This behavior is in accordance with the initial condition imposed in our simulations for the liquid phase distribution. As illustrated in Fig. 4, the drying situation is characterized by a traveling drying front where the saturation varies abruptly from the initial saturation to zero. The saturation plotted in Fig. 4 and denoted by $S_{\text{sat}}$ corresponds to the saturation determined in each slice of the network (as defined in Sec. 4). Referring to the drying kinetics classical description in three periods (van Brakel, 1980), our case is close to the last period, referred to as the receding front period (RFP) where a dry zone develops into the porous medium from the surface (Pel et al., 1996). As will be discussed later, the dynamics of the drying front, i.e., how the position of the front scales with the elapsed time, is however different in our simulations.

5.2. Non local equilibrium effect

The ratio of $P_v$ to $P_{v,\text{sat}}$ as a function of both local saturation and network saturation characterizing the non local equilibrium is obtained from PNM simulations and illustrated in Fig. 4. Based on continuity of vapor flow, the closer a pore is to the boundary layer, the lower is the vapor pressure of the pore. Hence, when the drying front is on the surface, the vapor pores constituting the surface slice exhibit the lowest vapor pressure of any slice. As the drying front recedes into the network, the NLE effect decreases because the distance from the boundary layer increases. As shown in Fig. 5, the NLE effect increases abruptly, i.e., the ratio of $P_v$ to $P_{v,\text{sat}}$ decreases significantly, for very low saturations ($S_{\text{loc}} < 0.03$ in Fig. 5); this corresponds to the drying front where the saturation varies sharply over a small distance (see the saturation profiles in Fig. 4).
Fig. 3. Variation of network saturation $S_{net}$ with drying time (left) and normalized evaporation rate with respect to $S_{net}$ (right).

Fig. 4. Saturation profiles (left) and normalized vapor pressure profiles (right) obtained from two-equation CM (blue dashed lines) and PNM (black solid lines). CM results are plotted for times that correspond to $S_{net}$ of 95, 90, 80, 60, 40, 20 and 10% of $S_{net.ini}$ for PNM simulation results. The outer surface lies at $z/H = 1$ in both figures. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 5. Averaged value of NLE indicator as a function of local saturation ($S_{loc}$) for different intervals of network saturation ($S_{net}$).

Fig. 6. Ratio of $D_{eff}$ to $D_{eff^*}$ with respect to averaged local saturation. The black solid line represents the fitted profile through the discrete data points shown by blue symbols. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
6. Macroscopic parameters from PNM simulations

6.1. Computation of water vapor diffusivity

Fig. 6 shows the variation of the ratio of \( D_{\text{eff}} \) to \( D_{\text{eff}}' \) with respect to local saturation obtained from PNM drying simulations. \( D_{\text{eff}}' \) is \( D_{\text{eff}} (S_{\text{loc}} = 0) \), i.e. the effective vapor diffusivity for a completely dry network. The local saturation here is computed as an arithmetic mean of saturation of the two successive local averaging volumes between which the vapor transfer occurs. As expected the value of \( D_{\text{eff}} \) is highest when the vapor flux faces no resistance from the presence of liquid phase, i.e. when \( D_{\text{eff}}' \) is equal to \( D_{\text{eff}}' \).

We observe an initial increase in \( D_{\text{eff}} \) with decreasing \( S_{\text{loc}} \) followed by a intermediate period of approximately constant value and then a sharp increase in \( D_{\text{eff}} \) until zero local saturation has been reached. The plateau for the intermediate local saturation range has generally not been observed in previous works, i.e. (Pel et al., 1996), in which the vapor diffusivity decreases continually with increasing saturation. The difference is that here we start from a different liquid phase distribution where the liquid phase is distributed in small isolated clusters. The results shown in Fig. 6 indicate that the tortuosity of the gas phase does not vary significantly over the intermediate range of saturations in our case.

6.2. Computation of specific interfacial area

Using PNM drying simulations, we compute the specific interfacial area \( a_{\text{pg}} \) i.e. the sum of interfacial area of the interfacial throats contained within a local averaging slice divided by the macroscopic volume of a local averaging slice. Initially each local slice occupies a moderate value of \( a_{\text{pg}} \) of around 150 m\(^2\)/m\(^3\) as illustrated in Fig. 7. As the drying process begins, the uniformly distributed liquid clusters in the evaporating slices split into isolated meniscus throats. As a consequence of the small size of the liquid clusters, the total interfacial area of the resulting isolated menisci is greater than that of the parent cluster. This phenomenon is the cause of the initial increase in \( a_{\text{pg}} \) until a peak is reached at around \( S_{\text{loc}} = 0.16 \). Upon further drying, the rate of local production of interfacial area is overcome by the rate of removal of liquid throats, resulting in a consistent decrease of \( a_{\text{pg}} \) with local saturation.

7. CM simulation results compared with PNM results

The macroscopic effective vapor diffusivity \( D_{\text{eff}} \) and specific interfacial area \( a_{\text{pg}} \) (indicated by fitted functions illustrated in Figs. 6 and 7, respectively) obtained from PNM drying simulations are used to solve the two equation CM (Eqs. (1) and (2)). The only missing parameter is the mass exchange coefficient \( k \). The latter is used as a fitting parameter. Fig. 4 shows the saturation profiles and normalized vapor pressure profiles obtained from the solution of two equation CM at seven distinct times compared with corresponding PNM simulation results for the \( k \) value of 0.05 m/s. The comparison of saturation and vapor pressure profiles indicates that the two equation CM reproduces the profiles with good accuracy.

Fig. 8 shows the evaporation rate obtained by the NLE CM compared with the evaporation rate obtained from corresponding PNM simulations results. The initial saturation assigned to the discretized finite volume elements of the CM is 0.26 and the corresponding initial vapor partial pressure is given as \( P_{v,\text{sat}} \) for all slices. The assigned initial condition for \( P_{v} \) is the reason for the higher initial value of evaporation rate of CM simulation compared to the corresponding initial value of PNM simulation (as can be seen in Fig. 8). However, the CM adjusts the value of \( P_{v} \) of surface volume element (and the surrounding elements) very quickly (in the first second of drying time), therefore this discrepancy in the initial evaporation rate does not affect the solution.

Additionally, we can analyze the ability of two equation CM to reproduce the NLE effect. This is illustrated in Fig. 9. In doing so, we illustrate the comparison of NLE effect reproduced by two equation CM simulation and the corresponding PNM simulation results for varying ranges of network saturation. Based on the convergence of saturation profiles, vapor pressure profiles, total evaporation rate and the NLE effect, it can be said that the two equation NLE CM produces good results.

8. Discussion

8.1. NLE CM simulation results compared with LE front model

Since the drying front is quite sharp, the particular situation considered in this work is classically analyzed using a traveling front model under the assumption of local equilibrium, \( P_{v} = P_{v,\text{sat}} \) at the front, i.e. regardless of the degree of local saturation within

---

Fig. 7. Specific interfacial area \( a_{\text{pg}} \) as a function of local saturation \( S_{\text{loc}} \). The data points are averaged over finite local saturation intervals. The black solid line is a fitted function of \( a_{\text{pg}} \) out of the discrete data points (blue symbols). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 8. Comparison of evaporation rates obtained from PNM drying simulations and that predicted by two-equation CM.
the front region. Under the classical quasi steady approximation for the water vapor transport, the mass balance at the front is expressed as

$$j_v = \frac{M_v}{RT} \frac{D_{ef}}{z_f} P_{v,Sw} \frac{P_{v,net}}{z_f} \epsilon \rho_1 S_{net,ini} \frac{dz_f}{dt},$$

(7)

where $z_f$ denotes the position of the front (distance from the surface) and $S_{net,ini}$ the initial network saturation. Combining Eq. (7) with the right hand side of Eq. (4), i.e. $j_v = \frac{M_v}{RT} \frac{D_{ef}}{z_f} P_{v,Sw} \frac{P_{v,net}}{z_f} \epsilon \rho_1 S_{net,ini} \frac{dz_f}{dt}$, leads to an ordinary differential equation which can be solved analytically. This leads to a solution where the front position typically scales with the square root of time, i.e. $z_f \propto \sqrt{t}$. In what follows, the LE front model is referred to as the LE CM.

Fig. 10 shows the drying front position predicted by LE CM compared with that obtained from PNM drying simulations and the two equation NLE CM. It can be seen that the NLE CM reproduces the drying front position very well. On the other hand, the drying front position predicted by the LE CM is shifted by a certain amount in relation to that obtained from PNM drying simulations.

The corresponding comparison of evaporation rate however reveals that the evaporation rates from PNM, LE CM and NLE CM do not differ significantly in the course of the process. Though it should be noted that in NLE CM, the initial evaporation rate drops instantaneously to the correct value corresponding to the initial evaporation rate of the PNM results. On the other hand, in the LE CM the initially overestimated drying rate affects the overall drying time as well as the drying front position as can be seen in the comparison of drying front position with respect to time (Fig. 10). Consistently with the NLE curves in Figs. 5 and 9, the impact of the NLE effect is only important at the beginning of the drying process when the drying front is at the surface or very close to it. Nevertheless, the impact of the error at the beginning associated with the LE CM affects the position of the drying front as illustrated in Fig. 10.

8.2. On the specific interfacial area

A key parameter introduced in the two equation NLE CM is the specific interfacial area $a_{g1}$. It is a measure of liquid or gas phase tortuosity in a two phase flow situation and is considered very important especially in the modeling of drainage process inside a porous medium. The general shape of the $a_{g1}$ curve as illustrated in Fig. 7 is consistent with several previous results from the literature. For example in (Joekar Niasar et al., 2008; Joekar Niasar et al., 2010) we see a similar non monotonous polynomial relationship between saturation and specific interfacial area. For comparison with experimental results we can refer to (Culligan et al., 2006) where drainage experiments are performed on glass beads that have $a_{g1}$ values in similar ranges as the ones shown Fig. 7, i.e. from 0 to 0.23 mm$^{-1}$. More recently, the work done by (Wang et al., 2019) has also qualitatively validated the specific interfacial area function.

While operating in the limiting regime of immobile liquid phase, we could work with different degrees of initial network saturation $S_{net,ini}$. In order to illustrate the effect of degree of initial network saturation on $a_{g1}$, we arbitrarily selected 4 different values of initial network saturation and performed 15 simulations for each value of $S_{net,ini}$. Each with a different realization of throat size distribution and location of liquid clusters. In Fig. 11 the averaged $a_{g1}$ obtained from the various realizations for each case of $S_{net,ini}$ is
Considering the interfacial area is a vital step to circumvent the
Then, it can be observed that our expression of the phase change
Fig. 11 can be related to some considerations about a
phase flow modeling where the specific interfacial area is modeled
as another illustration of the fact that the saturation alone is not
limiting regime of immobile liquid phase, the total interfacial area
illustrated. For all $S_{\text{net,ini}}$, we see a qualitatively similar trend, i.e. an
initial increase in $a_g$ followed by a consistent decrease with
decreasing local saturation. When the results of different $S_{\text{net,ini}}$
are compared quantitatively, it is observed that overall the $a_g$
values decrease with decrease in $S_{\text{net,ini}}$ for their respective local satu-
ration values. This is owed to the fact that while operating in the
limiting regime of immobile liquid phase, the total interfacial area
per unit volume is higher when there are more liquid throats per
unit volume.

This dependency of $a_g$ on the initial saturation illustrated in
Fig. 11 can be related to some considerations about $a_g$ in two
phase flow modeling where the specific interfacial area is modeled
by a dedicated conservation equation, e.g. (Hassanizadeh and Gray,
1990; Hassanizadeh and Gray, 1993). As shown in (Joekar Niasar
et al., 2008) the classical capillary pressure saturation relationship
is incomplete without the consideration of specific interfacial area.
Considering the interfacial area is a vital step to circumvent the
hysteresis in capillary pressure and relative permeability and sat-
uration relationships. Here, the results shown in Fig. 11 can be seen
as another illustration of the fact that the saturation alone is not
sufficient to derive the macroscopic parameters, since the parametrization of the NLE phase change term in the NLE CM, i.e.
Eq. (3), in fact depends on the initial network saturation $S_{\text{net,ini}}$
through the dependency of $a_g$ on $S_{\text{net,ini}}$ illustrated in Fig. 11.

Also, it can be noted from Fig. 11, that $a_g$ varies roughly linearly
with local saturation for sufficiently low local saturation values.
Then, it can be observed that our expression of the phase change
term, i.e. Eq. (3), is consistent with the expression used in (Li
et al., 2019; Pujol et al., 2011), namely
\[ \ln \left( \frac{b}{s} s_{\text{net}} \right) = \frac{E}{M_r} (P_{\text{vp}} - P_V) \]
in this range of saturation, which is the most critical for the NLE effect according to Figs. 5 and 7.

8.3. On the receding front period

The saturation evolution depicted in Fig. 4 resembles the evolu-
tion in the receding front period (RFP) reported in several previous
works where the profiles were determined experimentally, e.g. (Pel
et al., 1996, 2002). However, there is an important difference. In
the experimental results presented in (Pel et al., 1996, 2002), the
position of the front scales linearly with time, see also
(Lockington et al., 2003), whereas in our case the front position
scales with the square root of time (Fig. 10). The difference is
due to the impact of the viscous effects, which are completely
neglected in our simulations since the liquid phase is disconnected.
In the experiments, the liquid is still connected, i.e. percolating
between the front and the sample bottom, and the receding of the
front results from the balance between the evaporation rate and
the liquid flow at the front. In other words, the situation con-
sidered in our simulations rather corresponds to the special case
where viscous effects in the liquid can be completely neglected
compared to capillary effects. In this limiting case, the liquid phase
is disrupted during the receding front period as shown for instance in
(Le Bray and Prat, 1999).

9. Conclusions

In this study, a non local equilibrium (NLE) continuum model
(CM) of drying was studied by comparison with pore network sim-
ulations for the case where the liquid phase is initially distributed
in the form of small disconnected clusters. The CM was derived
with the help of the volume averaging method. This led to the
introduction of the specific interfacial area in the formulation of
the NLE phase change term. Also, the upscaling suggests that there
is no particular reason for introducing an enhancement factor in
the vapor diffusion model.

Pore network model (PNM) simulations indicated that an NLE
effect should be expected not only for hygroscopic materials, as
suggested in several previous works (Bénet and Jouanna, 1982;
Bénet et al., 2009; Ouedraogo et al., 2013), but also in the case of
capillary porous materials. The study also indicates that the NLE
effect is particularly marked at the surface. This should be taken
into account in the modeling of the coupling between the internal
transfer, i.e. inside the porous medium, and the external transfer,
i.e. within the external boundary layer. In other words, the study
suggests that a significant improvement in the modeling of the
drying process by means of continuum models can be expected
from the consideration of the NLE effect in the porous medium
evaporative surface region.

The NLE CM led to a good agreement with the PNM drying sim-
ulations in both the saturation profiles and the drying kinetics.
However, for the considered situation, the NLE CM leads to only
slightly better results than the simpler LE front model. Neverthe-
less, the NLE CM is much more conceptually consistent with the
drying PNM simulations which clearly indicate an NLE effect.
The next step will be to test the NLE CM over a full drying, that is when
the porous medium is fully saturated initially. This will imply to
consider the so called constant rate period (CRP) and, the falling
rate period (FRP) (van Brakel, 1980) where a significant NLE effect
is expected, notably at the evaporative surface. We then expect
that the NLE CM will be not only conceptually better but also quan-
titatively better than the LE CM.

Like other macroscopic parameters, such as the capillary pres-
sure curve or the relative permeabilities, the parameters of the dry-
ing model can be determined from PNM simulations. However, it
should be noted that we have determined these parameters from
the PNM simulations of the drying process in a manner somewhat
similar to the determination of moisture diffusivity from drying
experiments, i.e. (Pel et al., 1996). Within the framework of PNM,
it is more classical to determine the parameters using dedicated
algorithms. We mean here, for instance, a specific algorithm to
determine the permeability curve or the capillary pressure curve,
see, e.g., (Blunt et al., 2001) and references therein. In this respect,
it would be interesting to develop a specific algorithm for deter-
mining the mass transfer coefficient $k$ of the NLE phase change
term, see Eq. (3), so as to study the impact of microstructure on this

Fig. 11. Specific interfacial area $a_g$ as a function of local saturation $S_{\text{loc}}$ simulated
with varying degree of $S_{\text{net,ini}}$. Each of the four functions presented here is the result
of averaging of 15 simulations, each with different realizations of the randomly
generated throat size distribution and the liquid structure.

\[ \ln \left( \frac{b}{s} s_{\text{net}} \right) = \frac{E}{M_r} (P_{\text{vp}} - P_V) \]}
coefficient. In the framework of the volume averaging method, this means developing the closure problem enabling one to compute $k$.

**CRediT authorship contribution statement**

Faeez Ahmad: Conceptualization, Software, Visualization, Writing original draft. Marouane Talbi: Conceptualization, Software. Marc Prat: Conceptualization, Supervision, Writing review & editing. Evangelos Tsotsas: Supervision, Funding acquisition. Abdolreza Kharaghani: Conceptualization, Supervision, Writing review & editing, Funding acquisition.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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**Appendix A**

In this appendix we present the details of derivation of the two equation CM comprising of transport equations for liquid and vapor phases. Starting point is the pore scale description of vapor transport in the domain occupied by the gas phase in the porous medium

$$\frac{\partial P_v}{\partial t} \nabla \cdot (D_v \nabla P_v), \quad (A1)$$

where $\rho_v$ and $D_v$ denote the vapor density and vapor diffusion coefficient, respectively. The boundary condition at the solid gas inter face can be expressed as

$$D_v \nabla \rho_v \cdot \mathbf{n}_{sg} = 0, \quad (A2)$$

where $\mathbf{n}_{sg}$ is the unit vector in the direction perpendicular to the solid gas interface. At the liquid gas interface ($\mathbf{Agl}$), we make use of the Hertz Knudsen Schrage (HKS) equation (Schrage, 1953) and express the boundary condition as

$$D_v \nabla \rho_v \cdot \mathbf{n}_{lg} = \frac{2\sigma}{\gamma} \sqrt{\frac{M_v}{2\pi k_B T_l}} \frac{P_{v,sat}}{P_v}. \quad (A3)$$

Using ideal gas law, we can express Eq. (A1) in terms of $P_v$ as

$$\frac{\partial P_v}{\partial t} \nabla \cdot (D_v \nabla P_v), \quad (A4)$$

Similarly expressing Eqs. (A2) and (A4) in terms of $P_v$

$$D_v \nabla P_v \cdot \mathbf{n}_{sg} = 0, \quad (A5)$$

$$D_v \nabla P_v \cdot \mathbf{n}_{lg} = k(P_{v,sat} - P_v). \quad (A6)$$

where

$$k = \frac{2\sigma}{\gamma} \sqrt{\frac{TR^2}{2\pi k_B M_v}}. \quad (A8)$$

Here with the intention to re formulate the vapor transport equations at the pore scale, we define a new variable $P'_v$ as

$$P'_v = P_{v,sat} - P_v. \quad (A9)$$

Using Eq. (A7) we re formulate the problem of vapor transport at the pore scale in terms of $P'_v$ and express Eq. (A5) as

$$\frac{\partial P'_v}{\partial t} \nabla \cdot (D_v \nabla P'_v). \quad (A10)$$

Similarly, we can express Eqs. (A6) and (A7) as

$$D_v \nabla P'_v \cdot \mathbf{n}_{sg} = 0, \quad (A11)$$

$$D_v \nabla P'_v \cdot \mathbf{n}_{lg} = kP'_v \quad (A12)$$

Invoking the advantage of the analogy between the problem of diffusion and heterogeneous reaction presented in (Whitaker, 2013) and using the volume averaging method (Whitaker, 2013), we upscale the problem leading to the volume transport macroscopic equation

$$\epsilon(1 - S) \frac{\partial P'_v}{\partial t} \nabla \cdot (\epsilon(1 - S)D_{eff} \nabla P'_v) + \nabla \cdot (\mathbf{u}P'_v) + a_{v}kP'_v. \quad (A13)$$

where we use the same notation for the volume averaged partial pressure as in the pore scale problem. In Eq. (A13), $a_{v}$ is the liquid gas interfacial area per unit volume

$$a_{v} = \frac{A_{agl}}{V}. \quad (A14)$$

It is argued in (Whitaker, 2013) that the term $\nabla \cdot (\mathbf{u}P'_v)$ in Eq. (A13) is negligible. Thus, Eq. (A13) can be expressed as

$$\epsilon(1 - S) \frac{\partial P'_v}{\partial t} \nabla \cdot (\epsilon(1 - S)D_{eff} \nabla P'_v) + a_{v}k(P_{v,sat} - P_v). \quad (A15)$$

It is important to note that the effective diffusivity $D_{eff}$ is computed assuming zero flux on the liquid gas interface. Also, note that the HKS theory was introduced for convenience in order to start from a formulation analogous to the one considered in (Whitaker, 2013). It is to be noted that the HKS theory is not included in the PNm drying algorithm.

For the characterization of liquid phase, we begin with the pore scale expression

$$\frac{\partial \rho_l}{\partial t} \nabla \cdot (\rho_l \mathbf{v}). \quad (A16)$$

Applying the volume averaging method leads to

$$\epsilon \rho_l \frac{\partial S}{\partial t} \nabla \cdot (\rho_l \mathbf{v}) + \frac{1}{V} \int_{A_{hg}} \rho_l(\mathbf{v} \cdot \mathbf{n}_{hg}) dA, \quad (A17)$$

where $\mathbf{v}$ is the liquid phase mass average velocity and $\mathbf{w}$ is the velocity of liquid gas interface. At a meniscus, we have

$$\rho_l(\mathbf{v} \cdot \mathbf{n}_{lg}) \approx \rho_s(\mathbf{v} \cdot \mathbf{n}_{lg}), \quad (A18)$$

where $v_s$ is the velocity of the water vapor and

$$\rho_s(\mathbf{v}_v \cdot \mathbf{w}) \cdot \mathbf{n}_{lg} \approx \frac{M_v}{RT} D_{v} \nabla P_{v} \cdot \mathbf{n}_{lg}. \quad (A19)$$

Assuming isolated liquid clusters and considering Eq. (A7), the convective liquid flow can be neglected, enabling us to express Eq. (A17) as

$$\epsilon \rho_l \frac{\partial S}{\partial t} \approx \frac{A_{agl} M_v}{V RT} (P_{v,sat} - P_v). \quad (A20)$$
Taking into account Eq. (A14), we express Eq. (A20) as
\[
\frac{\partial}{\partial t}\frac{\partial S}{\partial t} \approx \frac{R}{\rho} \left( P_v - P_{\text{sat}}(P_v) \right). \tag{21}
\]

The final form of the liquid and vapor phase equations of the two equation CM for the limiting case of immobile liquid phase can thus be expressed as
\[
\frac{\partial}{\partial t}\left( \frac{1}{S} \frac{\partial S}{\partial t} \right) + \nabla \cdot \left( \frac{1}{S} \frac{\partial S}{\partial t} \mathbf{v} \right) + \frac{R}{\rho} \left( P_v - P_{\text{sat}}(P_v) \right), \tag{22}
\]
\[
\frac{\partial}{\partial t}\frac{\partial P_v}{\partial t} \approx \frac{M}{RT} \left( P_v - P_{\text{sat}}(P_v) \right). \tag{23}
\]

References


