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An alternative MIP formulation for the Military Flight and Maintenance Planning problem

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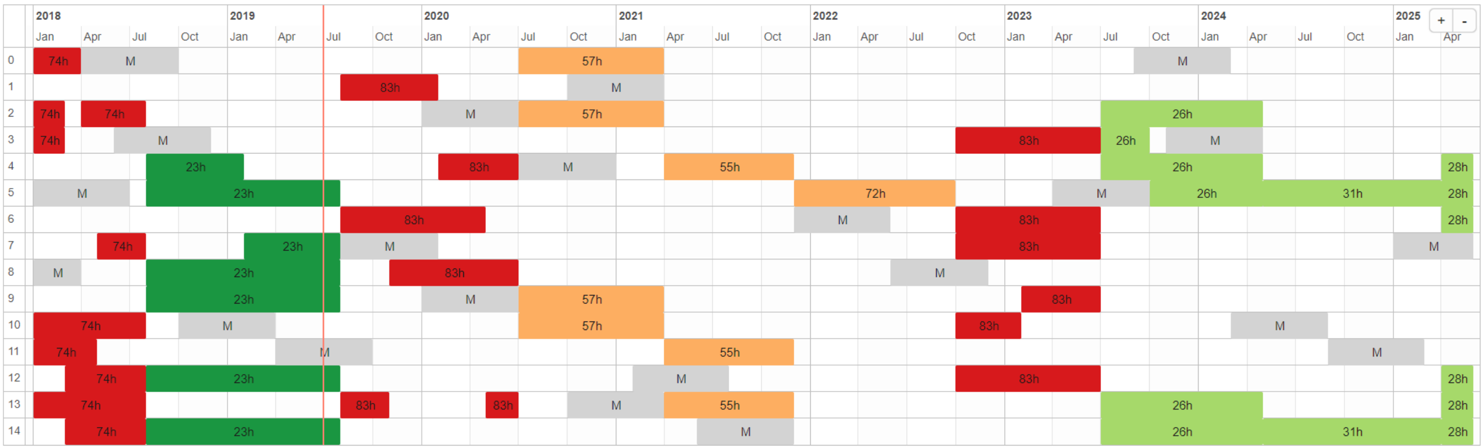
Structure of talk

1. Problem.
2. State of the art.
3. MIP formulations.
4. Machine Learning.
5. Experiments.
6. Conclusions.

Problem (informally)

Assign both **missions** and **maintenance operations** to a fleet of aircraft in order to maximize availability and minimize costs. **Missions** have fixed start and end times and have particular needs in terms of aircraft and flight hours. **Maintenance operations** have specific rules that govern their frequency and capacity.



Problem

- A series of $j \in \mathcal{J}$ pre-scheduled missions are planned along a horizon divided into $t \in \mathcal{T}$ periods.
- Each mission requires a certain **number** R_j of aircraft $i \in \mathcal{I}$ which it employs for H_j **hours** in each period.
- The heterogeneous fleet requires recurrent preventive maintenance operations (**checks**).
- A check takes exactly M^d periods and cannot be interrupted. There is a limited **capacity** for maintenances at each period.
- The objective is to **minimize the total number of checks** while **maximizing the status of the fleet** at the end of the horizon.

Other: soft constraints on the state of the fleet at each period, initial conditions, default consumption, reduce variance of frequency of checks.

State of the art

- FMP: Flight and Maintenance Planning problem.
- In [Cho11], US Army aircraft were assigned daily operations over a year to aircraft in order to minimize the maximum number of maintenances.
- In [Koz08], Greek aircraft had monthly assignments of maintenances and flight hours in order to maximize the availability and final state of squadrons.
- In [VVC15], monthly assignments were done and several objectives were taken into account: availability, serviceability and final state.
- In [SY18], a generalization for different types of maintenances and capacities was done.
- In [Pes+20], the problem was proved NP-Complete.

Previous formulation

- a_{jti} : =1 if mission $j \in J$ in period $t \in \mathcal{T}_j$ is realized with aircraft $i \in \mathcal{I}_j$, 0 otherwise.
- m_{it} : =1 if aircraft $i \in I$ starts a check in period $t \in \mathcal{T}$, 0 otherwise.
- u_{it} : flown time (continuous) by aircraft $i \in I$ during period $t \in \mathcal{T}$.

$$u_{it} \geq \sum_{j \in \mathcal{J}_t \cap \mathcal{O}_i} a_{jti} H_j \qquad t = 1, \ldots, \mathcal{T}, i \in \mathcal{I}$$

$$u_{it} \geq U^{min}(1 - \sum_{t' \in \mathcal{T}_t^s} m_{it'}) \qquad t = 1, \ldots, \mathcal{T}, i \in \mathcal{I}$$

$$rft_{it} \leq rft_{i(t-1)} - u_{it} + H^M m_{it} \qquad t = 1, \ldots, \mathcal{T}, i \in \mathcal{I}$$

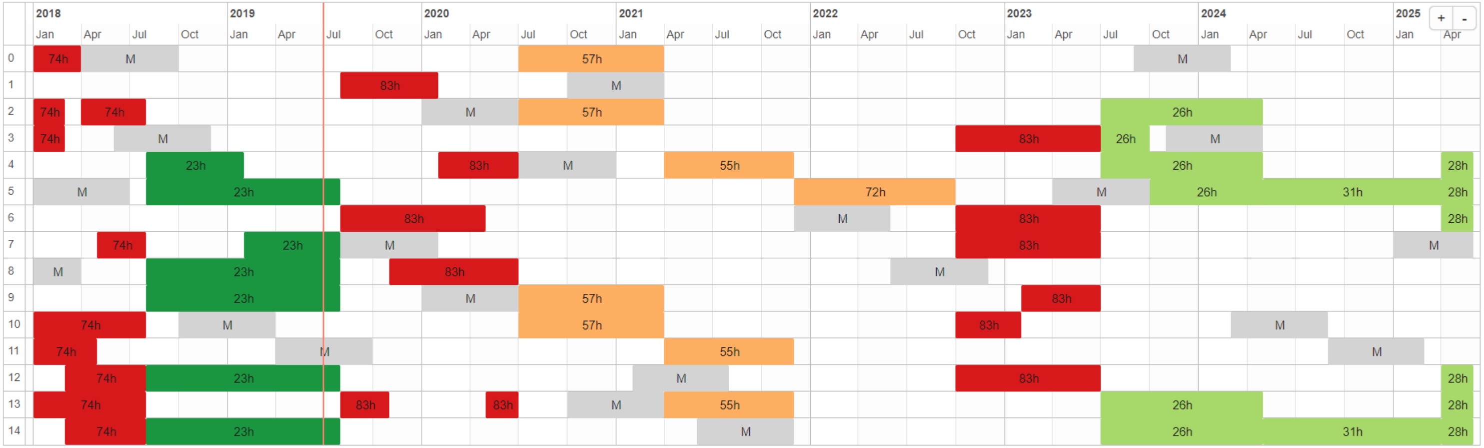
$$rft_{it} \in [0, H^M] \qquad t \in \mathcal{T}, i \in \mathcal{I}$$

New formulation

- $a_{ijtt'}$: =1 if aircraft i starts an assignment to mission j at the beginning of period t and finishes at the end of period t', zero otherwise.
- m_{ip} : =1 if aircraft i uses check pattern p, zero otherwise.
  - each pattern p has a single feasible combination of check starts for an aircraft during the whole planning (usually only 1-2 checks per aircraft).

$$\sum_{(j,t,t') \in \mathcal{JTT}_{ic}} a_{ijtt'} H'_{jtt'} + U'_{tc} \leq H^M + M(1 - m_{ip})$$

$$i \in \mathcal{I}, p \in \mathcal{P}, c \in \mathcal{C}_p$$



Formulation

$$\text{Max} \sum_{i \in \mathcal{I}, p \in \mathcal{P}} m_{ip} \times W_p$$

$$\sum_{i \in \mathcal{I}, p \in \mathcal{P}_t} m_{ip} \leq C^{max} \qquad t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{I}_j, (t_1, t_2) \in \mathcal{T}_{jt}} a_{ijt_1t_2} \geq R_j \qquad j \in \mathcal{J}, t \in \mathcal{T}\mathcal{J}_j$$

$$\sum_{p \in \mathcal{P}_t} m_{ip} + \sum_{j \in \mathcal{J}_t \cap \mathcal{J}_i} \sum_{(t_1, t_2) \in \mathcal{T}_{jt}} a_{ijt_1t_2} \leq 1 \qquad t \in \mathcal{T}, i \in \mathcal{I}$$

$$\sum_{(j,t,t') \in \mathcal{J}\mathcal{T}\mathcal{T}_{ic}} a_{ijtt'} H'_{jtt'} + U'_{tc} \leq H^M + M(1 - m_{ip})$$

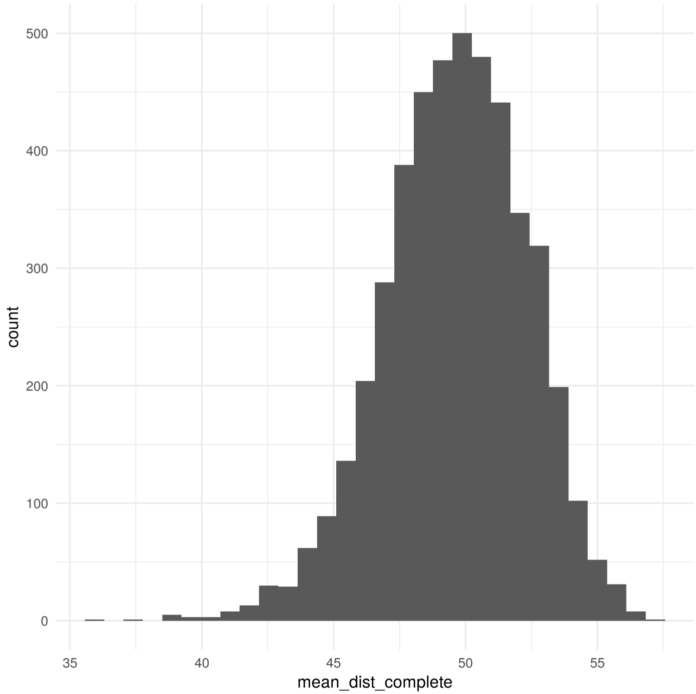
$$i \in \mathcal{I}, p \in \mathcal{P}, c \in \mathcal{C}_p$$

Is this better?

1. It uses 3 times the number of constraints and 3 times the number of variables.
   - variables: 11000 => 28000.
   - constraints: 13000 => 48000.
2. It is still better. Better lineal relaxation, better performance.

Distance between maintenances

- The distance between maintenance has a maximum of E^M periods.
- Depending on the instance, the optimal distance can be shorter.
- This distance conditions the total number of patterns to create.



Forecasting + Optimization

We want to:

1. Train a statistical model to predict the mean distance between maintenances for any given instance.
2. Use this information to limit all possible combinations of patterns to generate.

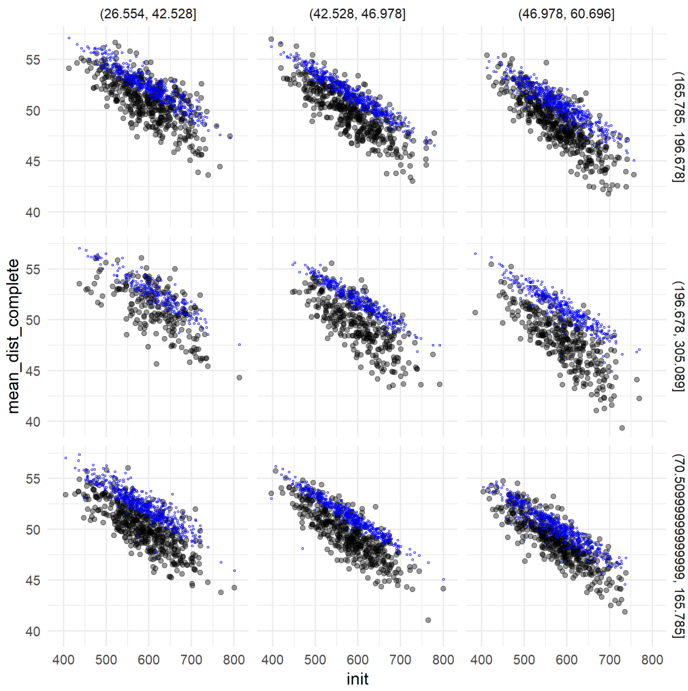
Benefits:

1. **Performance**: a smaller model is easier to solve.
2. **User feedback**: direct feedback about the solution without needing to solve any model.
3. **More stable solutions**: Every aircraft flies an amount that is closest to the mean of the fleet.

The better we're able to predict the optimal distance between maintenances for the whole fleet, the less optimality we will lose

Prediction model

- **Technique**: *Quantile regressions* to estimate upper and lower bounds.
- **Training**: 5000 small instances.
- **Input features**:
    - mean flight demand per period,
    - total remaining flight hours at start (init),
    - variance of flight demand,
    - demand of special missions,
    - number of period where flight demand is cut in two.
- **Output features**: mean distance between maintenances.



Experiments

- Number of instances: medium (1000), large (1000) and very large (1000).
- Time limit at 3600 seconds.
- We seeded instance generation for better comparison.
- CPLEX running 1 thread.

Largest instances have 60 aircraft, 90 periods, ~30 missions (4 active missions at any given time).

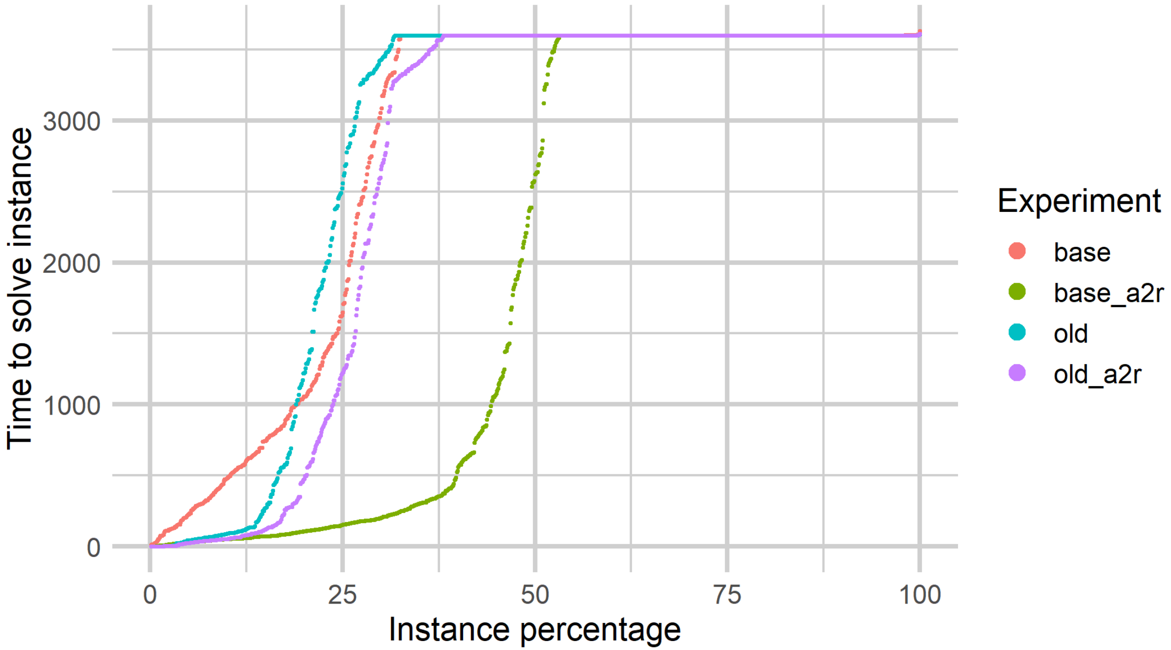
1. Create forecasting model based in 5000 small instances.
2. Use forecasting model to predict bounds on distance between maintenances: $\hat{\mu}^{lb}_{t'-t}$, $\hat{\mu}^{ub}_{t'-t}$
   .
3. Implement the pseudo-cut:

$$
\begin{aligned}
m_{ip} = 0 \qquad & p_{t'} - p_t < \hat{\mu}^{lb}_{t'-t} - tol \\
m_{ip} = 0 \qquad & p_{t'} - p_t > \hat{\mu}^{ub}_{t'-t} + tol
\end{aligned}
$$

1. Recycling.

How good is it (performance)

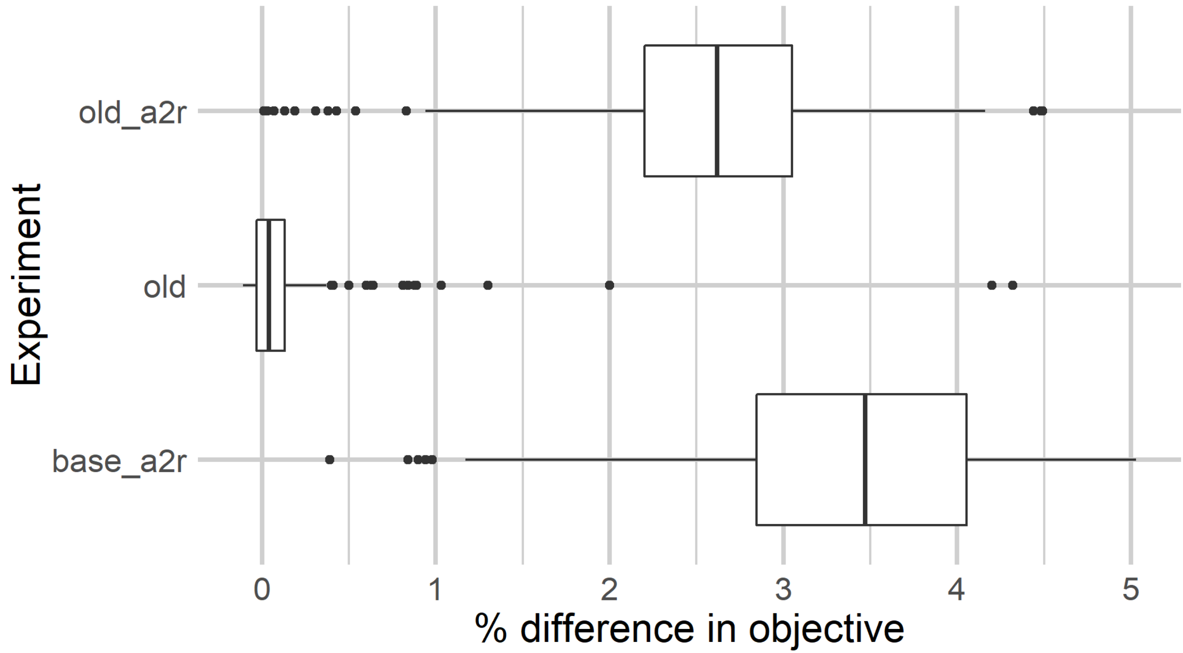
Faster solutions, more solutions.



How good is it (optimality)

For instances were an optimal solution was found (optimum degradation):

- 95% of instances had less than 4% gap with real optimal.



Further steps

- **Better predictions** with better features, or predicting several characteristics of optimal solutions.
- **Predict a distribution** and sample patterns from the distribution instead of predicting patterns.
- **Warm-start Column Generation** with a selected subset of potentially good patterns.
- **Automatize prediction** so it can be easily integrated in other problems.

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