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Unilateral behaviour of microcracks and thermal conduction properties: a homogenization approach

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Damaging effects of microcracks on the elastic properties of brittle materials (rocks, concrete, ceramics) have been extensively studied through experiments and modelling approaches. In the latter case, the homogenization (up-scaling) technique appears as an effective tool to provide the overall properties from the microstructural features of the material [1, 2]. Especially for microcracks, main difficulties arise from: (i) the anisotropy induced by the oriented nature of defects, (ii) their ability to be open or closed according to tension or compression loading and to influence differently the overall response of the material. Such unilateral behaviour is typical of contact problems related to that kind of defects. Regarding the thermal properties of microcracked media, very few investigations on the effects have been done in the literature, even on the experimental point of view [3]. Some micro-macro modelling works have been proposed to derive effective properties of a given Representative Volume Element (RVE) but consider only the open state of microcracks [4]. This work aims to explore a homogenization-based approach to derive thermal properties accounting for unilateral effect. We also intend to perform numerical simulations through a finite-element modelling in order to validate these theoretical results.

Considering stationary thermal conduction, the macroscopic temperature gradient \( G \) (respectively macroscopic heat flux \( Q \)) corresponds to the average of its microscopic quantity \( g \) (resp. \( q \)) over the RVE, i.e. \( G = \langle g \rangle; Q = \langle q \rangle \). Let us consider a 3D RVE composed of a matrix weakened by randomly distributed parallel microcracks. Such a heterogeneous material shows classical matrix-inclusion topology allowing the use of averaging techniques. We can either impose uniform \( G \) or uniform \( Q \) on the outer boundary \( \delta \Omega \) of the RVE. Taking this into account the microscopic and macroscopic properties can be linearly linked through the equation \( g = A \cdot G \) and \( q = B \cdot Q \), where \( A \) (resp. \( B \)) is the second order temperature gradient localization (resp. heat flux concentration) tensor.

The derivation of the closed-form expression for the thermal properties starts with the linear thermal problem. According to the Fourier law,

\[
q = -\lambda \cdot g \quad \text{and} \quad g = -\rho \cdot q
\]

where \( \lambda \) and \( \rho \) are the second order thermal conductivity and resistivity tensors respectively. Therefore, the overall behaviour of the RVE can be given similar to Eq. 1 as:

\[
Q = -\lambda_{\text{hom}} \cdot G \quad \text{and} \quad G = -\rho_{\text{hom}} \cdot Q
\]

Taking all simplifications into account, the effective properties can be given as

\[
\lambda_{\text{hom}} = \lambda_m + f_c (\lambda_c - \lambda_m) \cdot \langle A \rangle_c \quad \text{and} \quad \rho_{\text{hom}} = \rho_m + f_c (\rho_c - \rho_m) \cdot \langle B \rangle_c
\]

where \( \lambda_m \) (resp. \( \lambda_c \)) and \( \rho_m \) (resp. \( \rho_c \)) are the conductivity and resistivity tensors of the matrix (resp. cracks), \( f_c \) is the volume fraction and \( \langle \cdot \rangle_c \) is the mean value over the cracks phase. Also the following relations are respected: \( \lambda_m = (\rho_m)^{-1}, \lambda_c = (\rho_c)^{-1} \).
Based on the works of Eshelby on the single-inhomogeneity problem [5], we develop an Eshelby-like tensor for thermoelasticity. Like the Eshelby tensor in elasticity, the Eshelby-like tensor here depends on the shape and orientation of the crack. The cracks are modelled as a flat oblate ellipsoids with crack density $d$, aspect ratio $\omega$ and unit normal $\mathbf{n}$. We assume both the matrix and cracks are isotropic and their properties can be given by $\lambda_i = \lambda_i \mathbf{I}$ and $\rho_i = \rho_i \mathbf{I}$ with $i = \{m, c\}$ (where $\lambda_i$ and $\rho_i$ are scalars). The unilateral effect is taken into account via the $\lambda_c$ and $\rho_c$. For open cracks, there is no heat transfer so, $\lambda_c = 0$ and $\rho_c = \infty$. Inspired by the works of Deudé et al. [6], the closed cracks (when the crack lips contact) are considered as a fictitious isotropic material such that $\lambda_c \neq 0$ and $\rho_c \neq \infty$. The tensors $A$ and $B$ are found using dilute and Mori-Tanaka schemes.

![Graphs](image)

(a) $\lambda(\mathbf{v})$ and (b) $\rho(\mathbf{v})$ normalized by their initial values for a material weakened by a single array of parallel microcracks of unit normal $\mathbf{n}$ (crack density $d=0.1$).

As an illustration, Fig. 1 shows the effective properties in the direction of unit vector $\mathbf{v}$ derived by means of up-scaling approach. For open cracks, the damage influence is seen mostly in the direction orthogonal to the cracks. Also, the results obtained from dilute and MT schemes are not the same for conductivity, but same for resistivity. For closed cracks, there is a complete deactivation of damage. The simulations we have run so far support these conclusions.

References


