Re-balancing problem for assembly lines: new mathematical model and exact solution method

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Abstract

Purpose – The purpose of this study is to develop a new mathematical model and an exact solution method for an assembly line rebalancing problem. When an existing assembly line has to be adapted to a new production context, the line balancing, resources allocation and component management solutions have to be revised. The objective is to minimize the number of modifications to be done in the initial line in order to reduce the time and investment needed to meet new production requirements. The proposed model is evaluated via a computational experiment. The obtained results the efficacy of the proposed method.

Design/methodology/approach – This paper develops a new mathematical model and an exact solution method for an assembly line rebalancing problem with the objective to minimize the number of modifications to be done in the initial line to reduce the time and investments needed to meet new production requirements.

Findings – The computational experiments show the efficacy of the proposed method.

Originality/value – These reconfiguration costs were analysed for different part-feeding policies that can be adopted in an assembly line.

Keywords Programming, Assembly line design

Paper type Research paper

1. Introduction

Actual assembly lines present complex production systems where management decisions at different levels impact times, costs and performances. As a consequence, such optimisation problems as line balancing, resources allocation and component management have to be considered when a new line is designed or an existing line is reconfigured for new products.

Especially, all existing solutions have to be revised when the assembled products change or market fluctuations impose different production volumes and so modifications in the system throughput. In particular, a rebalancing of the existing line leads to a reallocation of the resources used and requires modifications in component management.

This paper develops a new mathematical model and an exact solution method for an assembly line rebalancing problem with the objective to minimize the number of modifications to be done in the initial line to reduce the time and investments needed to meet new production requirements.

This paper is organized as follows. The results of previous research on the studied topic are analysed in Section 2. A formal definition of the assembly line re-balancing problem and an approach for the model linearization are developed in Section 3. An illustrative example is given in Section 4. A computational study is presented in Section 5 and concluding remarks are given in Section 6.

2. Rebalancing versus balancing

The first mathematical formulation of the Simple Assembly Line Balancing Problem was introduced almost 60 years ago by Salveson (1955). During the last decades, this formulation was enriched and intensively studied from various points of view. A detailed analysis of the line balancing problems in different industrial contexts can be found in a recent survey presented by Battaïa and Dolgui (2013).

The most studied version of this problem aims to minimize the number of workstations required for assigning a given set V of tasks under precedence and cycle time constraints. The precedence constraints are given by a directed acyclic graph $G = (V, E)$ over set of tasks V. An edge $(i, j) \in E$ in this graph indicates that task i is an immediate predecessor of task j and, therefore, has to be assigned to a prior or the same work station as task j. Each task $j \in V$ is also characterized by its time, t_j. The sum of task times for the tasks assigned to the same workstation should not exceed a given cycle time denoted by T_0. This problem is known to be NP-hard. Usually, additional assignment constraints exist between tasks, such as incompatibility or exclusion constraints that make the assignment of two tasks to the same work station unfeasible.

Several efficient procedures have been proposed to solve the deterministic version of this problem (Bautista and Pereira, 2009; Pastor and Ferrer, 2009; Sewell and Jacobson, 2012; Morrison *et al.*, 2014). However, due to the dynamic nature of demand and changes in the product characteristics, regular adjustments of the assembly line setup are necessary.

To anticipate the possible changes in the input data, several researchers studied robust formulations of assembly line balancing problems (Xu and Xiao, 2009; Gurevsky *et al.*, 2013b). Other studies were conducted to evaluate the stability of the obtained solutions under variations of task times (Gurevsky *et al.*, 2012, 2013a). These results help to implement robust line balancing solutions and to keep the initial task assignment to some extent without modifications of the line. Nevertheless, at some point, the re-balancing of the line becomes inevitable.

The re-balancing problem is quite different from the original design and balancing problem, as the existing configuration has to be taken into account. As a consequence, the methods and solutions developed for line design and balancing problems cannot be directly used for rebalancing optimisation problems.

Falkenauer (2005) was one of the first to indicate that many assembly lines are not designed from scratch but represent a re-configuration of an existing line. However, contrary to this practical observation, the re-balancing problem has been less studied in the academic literature than the initial balancing problem. To the best of our knowledge, only approximate methods were developed previously to address this problem for assembly lines.

Heuristics and genetic algorithms were used for solving stochastic assembly line rebalancing problem by Gamberini *et al.* (2006, 2009). Three different heuristic methods have been developed by Grangeon *et al.* (2011) for re-balancing of assembly lines in the automotive industry. A COMSOAL-based heuristic for re-balancing of assembly lines that determines a fixed task sequence for a number of different cycle times was proposed by Agpak (2010).

One of the first exact mathematical models and solution approaches were proposed by Makssoud *et al.* (2014) but for the reconfiguration of transfer lines in machining environment. These machining lines are highly automated and allow less flexibility than manual assembly lines where the re-balancing problem is defined differently.

A practical assembly line re-balancing problem in the automotive industry was studied by Altemeier *et al.* (2010). They pointed out that the reconfiguration costs in assembly systems are typically incurred for:

- retraining of workers;
- shifting of tools and storage racks; and
- hanging the delivery of parts.

However, these reconfiguration costs can depend on the part-feeding policies adopted in the assembly line. According to the comparison study of Battini *et al.* (2009), these policies can be classified in three main groups: pallet to work station, trolley to work station and kit to assembly line. In the following, the reconfiguration costs are considered for these policies:

- *Pallet to work station:* As pallets of required components need to be stored at work stations, a modification of the task assignment changes the need in components of each workstation. As a consequence, the supply and the storage of the parts have to be revised according to the new task assignment if the line is re-balanced.
- *Trolley to work station:* In this feeding system, the work stations are supplied directly according to the lists of required components. These lists are used by the warehouse workers to collect the components necessary to complete each activity from the storage area. As a consequence, a modification in the task assignment changes the lists established that have to be adjusted to the new work content at each work station.
- *Kit to assembly line:* This feeding strategy consists in creating a kit of components for every end-product, assembled on the assembly line, each kit contains the products' main components and each kit is associated with one product item. In the warehouse, one kit is prepared for each finished product. This kit passes through the entire line, from the first station to the last, together with the specific end-product. As a consequence, a modification in the task assignment does not have any impact on the established component management system.

The novel concept of *line-integrated supermarkets* was introduced recently (Faccio *et al.*, 2013; Boysen and Emde, 2014). This feeding strategy consists in unifying the advantages of kitting and line stocking. As a result, parts are stored directly at the stations, where kits are prepared by separate logistics workers. As such supermarkets are designed accordingly, the work content of work stations, a modification in the task assignment may cause the re-design of line-integrated supermarkets impacted.

As it can be seen, the most of part-feeding policies are sensible to the re-balancing solutions. As a result, the *minimization of the changes in the initial task assignment* imposed by new production requirements does not only reduce the cost of operators retraining and tool shifting but also the costs incurred by modification of the part-feeding system and procedures.

Taking into account this analysis and the state of the art, in the next section, a mathematical model for an assembly line reconfiguration problem is developed to minimize the number of changes in the initial line.

3. Formal problem definition and mathematical model

In this section, we present a formal definition of the studied problem.

3.1 Problem statement

The following notations need to be introduced:

i, j – indices for tasks;

k – index for workstations;

V – set of tasks to be assigned;

T_j – the processing time of task j, $j \in V$;

E is used for the precedence constraints, it contains all (i and j) such that task i is an immediate predecessor of task j;

A is used for the incompatibility constraints, it contains all pairs $\{i$ and $j\}$ such that tasks i and j cannot be assigned to the same work station;

$M = \{1,2\ [\ldots],\ m\}$ is the set of workstations in the existing line, where m is an upper bound on the number of workstations for the existing line;

$L = \{1,2\ [\ldots],\ l\}$ is the set of workstations in the new line, where l is an upper bound on the number of workstations for the new line;

$Q(j)$ is the interval of workstations in the re-balanced line, where task $j \in V$ can be assigned. It is calculated using the precedence constraints;

C is a relative cost of opening a new work station; and

$x^*_{jk} = 1$ if task $j \in V^* \subset V$ is assigned to workstation k in the initial configuration, 0 otherwise. These constants are used to describe the initial task assignment to work stations. Set V^* contains such tasks j that for $x^*_{jk} = 1$, $k \notin Q(j)$; $x^*_{jk} = 0$ for all $k > m$.

3.1.1 Decision variables

$y_{jk} = 1$ if task $j \in V$ is assigned to workstation k in the re-balanced line, 0 otherwise; $y_{jk} = 0$ for all $k \notin Q(j)$, see constraint 6 in models (1)–(7);

$w_k = 1$ if workstation $k \in L$ is opened in the re-balanced line, 0 otherwise.

The objective function (1) aims to minimize at the same time the number of task re-assignments and the number of work stations in the re-balanced line:

$$Minimize \sum_{j \in V^*} \sum_{k \in Q(j)} |x^*_{jk} - y_{jk}| + C \sum_{k \in L} w_k, \qquad (1)$$

Constraint (2) imposes that every task j is assigned to one and only one work station:

$$\sum_{k \in Q(j)} y_{jk} = 1, \forall j \in V \qquad (2)$$

Constraint (3) ensures the precedence relations between the tasks:

$$\sum_{k \in Q(i)} k y_{ik} \leq \sum_{k \in Q(j)} k y_{jk} = 1, \forall (i,j) \in E \qquad (3)$$

Constraint (4) guarantees that the total duration of the tasks assigned to work station k does not exceed the given cycle time:

$$\sum_{j \in V} t_j y_{jk} \leq T_o,\ \forall k \in L \qquad (4)$$

Constraint (5) expresses the impossibility of executing certain tasks at the same workstation:

$$y_{ik} + y_{jk} \leq 1, \forall \{i,j\} \in A, \forall k \in Q(i) \cap Q(j), \qquad (5)$$

Constraint (6) ensures that the variables y_{jk} outside intervals $Q(j)$ are set to 0:

$$y_{jk} = 0 \text{ for all } k \notin Q(j), \qquad (6)$$

As it can be seen, the objective function of the model presented is not linear. In the following, we propose a method to linearize this first model.

3.1.2 Lemma 1

Let $x,y,z \in \{0,1\}$. The logical expression that if $x = 1$ and $y = 1$, then $z = 1$ can be modelled by the following inequality: $x + y \leq z + 1$.

3.1.3 Lemma 2

Let $x,y \in \{0,1\}$. Then the following non-linear expression:

$z: = |\ x - y\ |$ can be linearized using the following inequalities:

$x + y \leq (1 - z) + 1,$

$x + (1 - y) \leq z + 1,$

$(1 - x) + y \leq z + 1,$

$(1 - x) + (1 - y) \leq (1 - z) + 1.$

It is simple to see that $z \in \{0,1\}$. Moreover, only four following cases are possible:

if $x = 1$ and $y = 1$, then $z = 0$,

if $x = 1$ and $y = 0$, then $z = 1$,

if $x = 0$ and $y = 1$, then $z = 1$,

if $x = 0$ and $y = 0$, then $z = 0$.

By applying for these four cases Lemma 1, we obtain the necessary inequalities. To linearize the problem constraints (1)–(6), we introduce a new variable $z_{jk} := |x^*_{jk} - y_{jk}|$, and so by using Lemma 2, we obtain the following model.

The objective function becomes (7):

$$Minimize \sum_{j \in V^*} \sum_{k \in Q(j)} z_{jk} + C \sum_{k \in L} w_k, \qquad (7)$$

The constraints (2)–(6) remain the same, as previously. Additionally, constraints (8)–(11) are introduced:

$$x^*_{jk} + y_{jk} \leq (1 - z_{jk}) + 1, \forall j \in V, \forall k \in Q(j) \qquad (8)$$

$$x^*_{jk} + (1 - y_{jk}) \leq z_{jk} + 1, \forall j \in V, \forall k \in Q(j) \qquad (9)$$

$$(1 - x^*_{jk}) + y \leq z_{jk} + 1, \forall j \in V, \forall k \in Q(j) \qquad (10)$$

$$(1 - x^*_{jk}) + (1 - y_{jk}) \leq (1 - z_{jk}) + 1, \forall j \in V, \forall k \in Q(j) \qquad (11)$$

This linear model can be solved with standard OR solvers as, for example, Cplex or Xpress-MP.

4. Illustrative example

Let us consider the following case study. The initial assembly line is given in Figure 1.

This line has to be rebalanced for a modified product where the following tasks {5, 14, 19, 23, 28 and 29} have been deleted and new tasks {31, 32, 33, 34 and 35} have been introduced. The task times of all tasks are reported in Table I. The line cycle time $T_0 = 15$ s.

Figure 1 Initial line

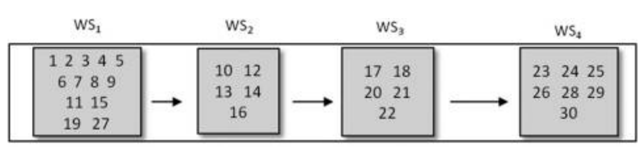

Table I Set of tasks V and their times

Task	Time(s)	Task	Time(s)	Task	Time(s)	Task	Time(s)	Task	Time(s)
1	0.93	8	0.16	15	0.09	22	0.64	31	0.72
2	1.06	9	0.68	16	0.17	24	0.09	32	0.15
3	0.68	10	0.16	17	0.09	25	0.17	33	0.19
4	0.16	11	1	18	0.12	26	0.09	34	0.33
6	0.16	12	0.78	20	1	27	0.12	35	0.97
7	0.68	13	0.64	21	0.78	30	0.91		

The new precedence constraints to be respected are given in Figure 2.

Exclusion constraints are as follows: {{1, 4}, {1, 17}, {1, 20}, {2, 11}, {3, 24}, {3, 7}, {4, 15}, {6, 24}, {8, 21}, {9, 22}, {10, 15}, {11, 31}, {12, 13}, {12, 20}, {13, 28}, {15, 17}, {16, 17}, {22, 26}, {30, 33}, {31, 32}, {33, 35}.

The optimal solution was obtained in 0.35 second and consists to reassign the following tasks: {2, 4, 10, 12, 21}.

5. Computational results

The proposed method was evaluated on three datasets that consist of 41 test problems obtained in the following way from original 25-tasks line balancing problems. To create the instances of Dataset 1, 25 per cent of tasks were changed (five tasks were deleted and five were added). Dataset 2 contains problem instances where 50 per cent of tasks were modified: five were deleted and eight were added. Finally, Dataset 3 includes problem instances with 75 per cent of modified tasks: 5 tasks deleted and 15 added, i.e. these instances are 34-tasks line re-balancing problems.

Experiments were carried out on PC Intel(R), 2.20 GHz, with 8 Go RAM. The model was coded in C++ with ILOG CPLEX 12.4. The computational results are presented in Table II, where the number of reassigned tasks is reported for each instance for three datasets. The solution time was less than 2 seconds for all instances.

The obtained results show that the method proposed can solve to optimality up to 34-task problems in a reasonable time. So it can be applied for real-life problem step by step after a decomposition of a problem into small sub-problems with approximately 35 tasks. In this case, the solutions obtained for each sub-problem will be optimal. The sole error could be from decomposition. Several decomposition approaches developed for balancing machining lines (Dolgui *et al.*, 2006; Guschinskaya *et al.*, 2008, 2011; Guschinskaya

Figure 2 The new precedence diagram

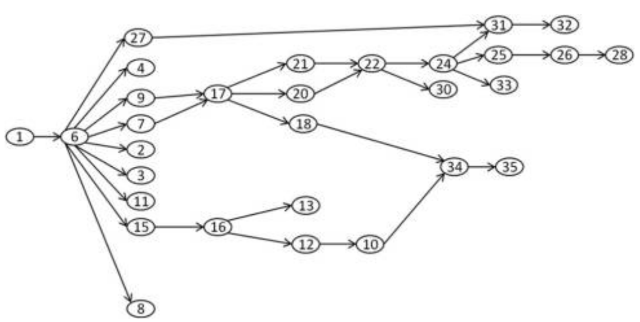

Table II Number of reassigned tasks for Datasets 1-3

Instance	Dataset 1	Dataset 2	Dataset 3
1	22	23	23
2	22	27	23
3	8	14	16
4	12	13	19
5	13	21	27
6	10	12	18
7	10	11	15
8	14	14	20
9	16	18	24
10	9	12	17
11	24	24	30
12	13	14	20
13	11	14	19
14	22	22	27
15	4	6	12
16	14	14	20
17	6	5	5
18	11	15	20
19	12	12	18
20	5	5	11
21	23	23	29
22	21	20	26
23	19	20	26
24	4	4	9
25	13	12	18
26	10	11	17
27	12	13	18
28	4	10	16
29	17	21	27
30	13	13	18
31	10	10	16
32	21	20	26
33	13	15	21
34	13	16	22
35	24	24	29
36	11	15	21
37	14	15	21
38	12	15	21
39	17	18	23
40	10	10	16
41	12	13	19
42	15	17	22

and Dolgui, 2009) can be adapted to reduce the error of decomposition using different advanced techniques. Another possible way of using this model consists in stopping the solver before proving the optimality, just when a given calculation time is expired. In this case, we will also obtain approximate solutions, but usually the quality of this approximation is very good and solutions obtained are near to optimum.

6. Conclusion

The reconfiguration of assembly lines is an optimisation problem of paramount importance in industry. The line modifications caused by rebalancing are costly and concern also the part-feeding system. To reduce these costs as well as the cost of operators re-training and tool shifting, it is necessary to minimize the changes in the initial task assignment imposed by new production requirements.

Taking into account this analysis and the state of the art, a mathematical model for the assembly line reconfiguration is developed to minimize the number of changes in the initial line and an industrial case study is presented.

The case study showed that the model proposed can be successfully applied. The experimentation revealed that the model is capable of solving problems with up to 34 tasks in the precedence diagram within a very short computational time. However, the model size would be too large to obtain the optimal solution of large-scale problems. In this case, decomposition techniques can be used or the model solving can be stopped to obtain approximate solutions.

Moreover, the proposed model may also be used as a validation tool for performance evaluation of heuristic procedures.

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