Unstationnary Control of a Launcher Using Observer-Based Structures

O. Voinot*, **D. Alazard**** and **B. Clément**♮

* ONERA-CERT, Control System Dept., 2 av. Edouard Belin, 31055 Toulouse, FRANCE
** SUPAERO, 10 av. Edouard Belin - BP. 4032, 31055 Toulouse, FRANCE
♮ CNES, Direction des lanceurs, Rond Point de l'Espace, 91023 Evry, FRANCE

Abstract

This paper deals with the design of a gain-scheduled controller for the attitude control of a launcher during atmospheric flight. The design is characterized by classical requirements such as phase/gain margins and flexible mode attenuations as well as time-domain constraints on the response of angle of attack to a worst-case wind profile. Moreover, these requirements must be fulfilled over the full atmospheric flight envelope and must be robust against parametric uncertainties. In order to achieve this goal, we propose a method based on minimal observer-based realizations of arbitrary stabilizing compensators. An original technique to assign the closed-loop dynamics between the state-feedback dynamics and the state-estimation dynamics is presented for the H_∞ compensators case. The structure is used to mix various specifications through the Cross Standard Form (CSF) and to perform a smooth gain scheduling interpolation through an Euler-Newton algorithm of continuation.

Keywords: multi-objective synthesis, robustness, Cross Standard Form, launcher, gain scheduling, observer-based

1 Introduction

This paper presents some techniques based on the observer-based structure to achieve the control of an non-stationary launcher during the atmospheric flight. Some recalls of recent results on the observer-based controller structure are presented in a first time. In [1], a procedure to compute the different parameters (the state-estimation gain, the state-feedback gain and the Youla's parameter) which characterized such a structure is proposed. This procedure requires a generalized non-symmetric Riccati equation to be solved. The Schur decomposition used to solve this Riccati equation involves a combinatory of solutions according to the repartition of the whole closed-loop dynamics between the state-feedback dynamics, the state-estimation dynamics and the Youla parameter dynamics. A systematic choice is proposed in this paper for the particular case of H_∞ controllers. This method is based on the proximity of the closed-loop eigenvalues between the H_2 and H_∞ synthesis and suppose the augmented standard synthesis model (i.e. the standard problem) is available.

The observer-based structure is also exploited to define the Cross Standard Form (CSF) [2, 3, 4] in the discrete-time case. The CSF can be considered as a generalization of the LQ inverse problem to the H_2 and H_∞ inverse problem. It allows to formulate a standard problem from which an initial compensator can be obtained by H_2 or H_∞ synthesis. The CSF is used to mix various synthesis techniques in order to satisfy a multi-objective problem. Indeed, the general idea is to perform a first synthesis to reach some specifications, mainly performance specifications. Then, the CSF is applied to this first solution to initialize a standard problem which will be gradually completed to handle frequency-domain or parametric robustness specifications. This approach is particularly interesting when the designer wants: to take advantage of a initial compensator based on a priori know-how and physical considerations, or to exploit modern optimal control techniques to deal with frequency-domain robustness specifications and trade-offs between various specifications. Others potentialities of this approach, like mixed eigen-structure assignment/H_∞ control or multi-channel control, are proposed in [3]. See also [5] for an alternative approach.

In this paper, we also exploit the observer-based structure for smooth gain-scheduling. We use the technique proposed in [6]. Pellanda *et al.* propose a method to construct adjacent controllers having the same observer structure and preserving a continuous dynamic behavior for each of their elements, independently of the adopted scheduling strategy. In [7, 8], the authors present a method for interpolation of full-order state-space realizations and gains of observer-state feedback controller which ensures closed-loop stability. The observer-based structure is also very interesting from a practical point of view : the compensator state becomes a meaningful estimate of the plant state.

This paper is structured as follows. Section 2 is devoted to the observer-based structure. We present a short recall of the procedure proposed in [1] and an original method for the assignment of the dynamics. The launcher control problem is described Section 3. Section 4 is devoted to the standard form construction and its use to merge together the various design specifications. Finally, Section 5 discuses the non-stationnary results obtained through the proposed methodology.

2 Observer-based controller structure

2.1 A short recall on the observer-based structure

We recall some recent results on the observer-based controller structure or more generally on compensators involving a state observer (with an estimation gain K_f), a state feedback (with a gain K_c) and a dynamic Youla's parameter $Q(z)$. The structure allows the parametrization of all stabilizing controllers. Alazard and Apkarian [1] propose a procedure to compute the minimal parameterization (K_c, K_f and Q) which characterized this structure, for an arbitrary controller K of order n_K and a system G. This paper just recalls the case of a strictly proper discrete-time system $G(z)$ (n states, m inputs, p outputs):

$$\left[\begin{array}{c} x_{k+1} \\ \hline y_k \end{array}\right] = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \left[\begin{array}{c} x_k \\ \hline u_k \end{array}\right] \quad (1)$$

In discrete-time, one can distinguish 2 observer-based structures: the state predictor structure and the state estimator structure (see [1] for more details). All the results presented in this paper concerns the discrete-time state estimator structure. The Youla parametrization of a controller $K(z)$ built on such a structure can be read:

$$\left\{\begin{array}{ll} \hat{x}_{k+1} & = A\hat{x}_k + Bu_k + AK_f(y_k - C\hat{x}_k) \\ x_{Q_{k+1}} & = A_Q x_{Q_k} + B_Q(y_k - C\hat{x}_k) \\ u_k & = -K_c\hat{x}_k + C_Q x_{Q_k} + (D_Q - K_cK_f)(y_k - C\hat{x}_k) \end{array}\right. \quad (2)$$

where A_Q, B_Q, C_Q and D_Q are 4 matrices of the state representation of $Q(z)$ associated to the state vector x_{Q_k}. $\hat{x}_k$ is an estimate of the state x_k.

Let consider a controller of order $n_K = n$ defined by the following representation:

$$\left[\begin{array}{c} x_{K_{k+1}} \\ \hline u_k \end{array}\right] = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \left[\begin{array}{c} x_{K_k} \\ \hline y_k \end{array}\right] \quad (3)$$

The goal is to compute the state feedback gain K_c, the estimate gain K_f, the static Youla parameter ($Q(z) = D_Q$) and a transformation matrix T such the controller (3) could be described by the state representation (2) when the following change of variable is performed:

$$x_{K_k} = T\hat{x}_k. \quad (4)$$

Then, the following equations can be derived:

$$[-T\ I]\left[\begin{array}{cc} A+BD_KC & BC_K \\ B_KC & A_K \end{array}\right]\left[\begin{array}{c} I \\ T \end{array}\right] = 0 \quad (5)$$

and

$$\begin{array}{rl} AK_f = & T^{-1}B_K - BD_K \\ K_c = & -C_KT - D_KC \\ D_Q = & D_K + K_cK_f. \end{array} \quad (6)$$

The problem is now to solve the Riccati equation (5) and next to compute K_c, K_f and D_Q using (6).

The Hamiltonian matrix associated with the Riccati equation is nothing else than the closed-loop dynamic matrix constructed on the state vector $[x^T, x_K^T]^T$:

$$A_{cl} = \left[\begin{array}{cc} A+BD_KC & BC_K \\ B_KC & A_K \end{array}\right]. \quad (7)$$

The Riccati equation (5) can then be solved in $T \in \mathbb{R}^{n_k \times n}$ by standard subspace decomposition techniques, that is compute an invariant subspace associated with a set of n eigenvalues, spec(Λ_n) (spec(A) is the set of eigenvalues of the matrix A), chosen among $2n$ eigenvalues in spec(A_{cl}), that is,

$$\left[\begin{array}{cc} A+BD_KC & BC_k \\ B_KC & A_k \end{array}\right]\left[\begin{array}{c} U_1 \\ U_2 \end{array}\right] = \left[\begin{array}{c} U_1 \\ U_2 \end{array}\right]\Lambda_n, \quad (8)$$

where $U_1 \in \mathbb{R}^{n \times n}$ and $U_2 \in \mathbb{R}^{n_K \times n}$. Such subspaces are easily computed using Schur decompositions of the matrix A_{cl}. And, finally compute the solution

$$T = U_2U_1^{-1}. \quad (9)$$

2.2 A new way to choose invariant sub-spaces

Some rules exist to choose the invariant spaces in the resolution of the Riccati equation (5) ensuring the existence and the regularity of T (see [1]). Pellanda [6] proposed a technique based on the relative and absolute controllability and observability, on the modal participation factors and on the decay rate of the eigenvalues to assign them to the state-feedback dynamics, the observer dynamics and the Youla parameter dynamics. In this section, we proposed a solution for the particular case of H_∞ controllers based on the dynamics of the H_2 controller and the continuity of the closed loop dynamics w.r.t. the H_∞ performance index γ from the H_2 synthesis to the optimal H_∞ synthesis. This method assumes that the augmented synthesis model $P(s)$ is available:

$$\begin{bmatrix} \dot{x} \\ \hline z \\ \hline y \end{bmatrix} = \left[\begin{array}{c|c|c} A & B_1 & B \\ \hline C_1 & D_{11} & D_{12} \\ \hline C & D_{21} & 0 \end{array}\right] \begin{bmatrix} x \\ \hline w \\ \hline u \end{bmatrix}. \quad (10)$$

For the problem (10), the H_2 compensator is a pure observer-based compensator and the $2n$ closed-loop eigenvalues verify the separation principle between the state-feedback dynamics and the state-estimation dynamics. This H_2 compensator is also the solution of the H_∞ problem when the wanted performance index γ tends towards infinity. The assumed continuity of the closed-loop dynamics w.r.t. γ suggests to us the following algorithm to compute the equivalent observer-based compensator of an H_∞ controller:

Algorithm 2.1 *Computing the observer-based realization of the H_∞ compensator associated with the standard problem $P(s)$ (equation (10)):*

Step 1 : Computation of initial controllers and initialization :

- *compute the H_∞ controller $K_\infty(s)$ and the corresponding H_∞ performance index: $\gamma_{opt} = \|F_l(P(s), K_\infty(s))\|_\infty$* [1],
- *compute the closed-loop matrix $A_{cl_{opt}}$ for the H_∞ controller (equation (7)),*
- *compute the H_2 controller $K_2(s)$ on the same standard problem $P(s)$, the corresponding H_∞ performance index $\gamma_2 = \|F_l(P(s), K_2(s))\|_\infty$ and identify the state-observer eigenvalues set (Λ_o^-) and the state-feedback eigenvalues set (Λ_f^-) among the $2n$ closed-loop eigenvalues,*
- *initialization: $\gamma^+ = \gamma_{opt}$, $\gamma^- = \gamma_2$ and $A_{cl} = A_{cl_{opt}}$.*

Step 2 : First assignment :

- *assign to Λ_f^+ the uncontrollable eigenvalues of the pair (A, B) (also eigenvalues of A_{cl}, $\forall \gamma$) in order to guarantee U_1 is not singular,*
- *assign to Λ_o^+ the unobservable eigenvalues of the pair (A, C) (also eigenvalues of A_{cl}, $\forall \gamma$) in order to guarantee U_2 is not singular.*

Step 3 : Determination of the dynamics :

- *if it is possible: search 2 sets, among $spec(A_{cl})$, which are nearest (in the least squares sense) to Λ_o^- and Λ_f^- and assign them to the corresponding dynamics sets Λ_o^+ and Λ_f^+. The 2 sets of eigenvalues must be auto-conjugated; if $\gamma^+ = \gamma_{opt}$, go to step 5.*
- *else : choose $\gamma^+ = (\gamma^+ + \gamma^-)/2$ and solve the H_∞ sub-optimal problem :*
  $\min_{K_\infty} \|F_l(P(s), K_\infty(s))\|_\infty \leq \gamma^+$;
  compute the corresponding closed-loop dynamic matrix A_{cl} and go to step 3.

Step 4 : Tracking the dynamics :

- *let $\Lambda_o^- = \Lambda_o^+$, $\Lambda_f^- = \Lambda_f^+$,*
- *let $\gamma^- = \gamma^+$, $\gamma^+ = \gamma_{opt}$,*
- *let $A_{cl} = A_{cl_{opt}}$ and go to step 3.*

Step 5 : Computing K_c, K_f and D_q :

- *compute T following (9).*
- *compute K_c, K_f and D_q following (6).*

Remark : for the *Output Estimation* (OE) and the *Disturbance Feed-forward* (DF) problems (see [9] for more details), we can also denote that the central controller [10] is a pure observer-based controller with only the state-observer dynamics, respectively the state-feedback dynamics, depending on γ (the H_∞ performance index). Then the determination of the 2 eigenvalue sets (step 3) is obvious.

Some recent results [6, 8, 7] show also the interest to use observer-based realization to assure smooth transition between interpolated controllers. So from the gain scheduling point of view, it could be also interesting to propagate a particular choice from an operating point to another. Pellanda *et al.* [6] proposed a method to construct a family of controllers having the same observer structure and preserving a similar dynamic behavior for each of its member, independently of the adopted scheduling strategy.

2.3 The Cross Standard Form (CSF)

The CSF, previously detailed in [2] for continuous-time systems, is defined in this section for discrete-time systems. The CSF is based on the augmented observer-based structure defined by (2).

Proposition 2.2 *The CSF (Cross Standard Form), $P_p(z)$, associated with the compensator defined by (2), such that :*

$$F_l(P_p(z), K(z)) = 0 \quad (11)$$

[1] $F_l(P(s), K_\infty(s))$ is the lower Linear Fractional Transformation of P and K.

reads :

$$P_p(z) := \left[\begin{array}{cc|c|c} A & 0 & AK_f & B \\ 0 & A_Q & B_Q & 0 \\ \hline K_c & -C_Q & -D_Q + K_cK_f & I_m \\ \hline C & 0 & I_p & D \end{array}\right]. \quad (12)$$

Proof : See [4] for the discrete time or [2] for the continuous time version.

Pratical use : This result can be considered as a generalization, for H_2 and H_∞ criteria and for dynamic LQG output feedbacks, of the solution to the *LQ inverse* problem, extensively discussed in the Sixties and Seventies and which consisted in finding the LQ cost whose minimization restores a given state feedback. This CSF used as such is not of interest since it is necessary to know gains K_c and K_f and the Youla parameter $Q(z)$ to set up the problem $P_p(z)$ and to finally find the initial augmented observer-based compensator. On the other hand, from an arbitrary compensator satisfying some time-domain specifications, one can compute an observer-based realization (i.e. K_c, K_f and $Q(z)$) of this compensator using the technique in [1]. The CSF is then immediately useful to initialize a standard setup which will be completed by dynamic weightings to take into account frequency-domain specifications.

3 Launcher Control Problem

This application considers the launcher inner control loop. The problem is the same as presented in [11, 12, 2].

The discrete-time validation model considered in this paper (that is the full-order model $G_f(z)$) is characterized by the rigid dynamics , the dynamics of thrusters , the sensors and the first 5 bending modes. The launcher is aerodynamically unstable. The rigid model strongly depends on 2 uncertain dynamic parameters A_6 (aerodynamic efficiency) and K_1 (thruster efficiency). The characteristics of bending modes are uncertain. The parameters are konwn as time functions.

The available measurements are the attitude angle (ψ) and the velocity ($\dot{\psi}$). The control signal is the thruster deflection angle β. Launcher control objectives for the whole atmospheric flight phase are as follows : performance with respect to disturbances (wind). The angle of attack peak, in response to the typical wind profile $w(t)$ (depicted in dashed plot in Figure 2), must stay into a narrow band ($\pm\, i_{max}$) ; one sampling period of delay margin ; closed-loop stability with sufficient stability margins. This involves constraints on the rigid mode but also on the flexible modes. In fact, the first flexible mode is "naturally" phase controlled (collocation between sensors and actuator) while the other flexible mode must be gain controlled (roll-off). So, the peaks associated with the flexible mode (except for the first) on the NICHOLS plot of the loop gain ($L(s) = K(s)G(s)$) must stay below a specified level X_{dB} for any parametric configurations (see Figure 3 as an example). From the synthesis point of view, the flexible modes are not taken into account in the synthesis model. But a roll-off behavior with a cut-off frequency between the first and the second flexible modes must be specify in the synthesis.

All the objectives must be achieved for all configurations in the uncertain parameter space (22 uncertain parameters), particularly in some identified worst cases. In this paper, the robustness analysis is limited to these worst cases as the experience shown they are quite representative of the robustness problem.

4 Stationary launcher control design

To solve the stationary design problem (at each flight instant) a specific design set-up has been developed on the basis of the Cross Standard Form (CSF). This approach proceeds in 2 steps : the first one aims to satisfy time-domain specification (angle of attack constraint) and the second one is a H_∞ synthesis based on the CSF allowing the frequency-domain specifications (roll-off, stability margins) to be met.

The models used for the synthesis are discrete models including a zero-order hold. The computation of the first step of the synthesis is directly derived from the continuous time synthesis [2]. It consists of an LQG/LTR compensator defined by a state feedback gain K_d^a and a state estimator gain G_d^a. The model (defined by the 4 state space matrices A_d^a, $B_{2_d}^a$, C_2^a and D_{22}) associated with this LQG design is the discrete-time rigid model including a rough first order wind model.

In the second step, to satisfy all frequency domain requirements, an H_∞ synthesis is performed on the standard problem depicted in Figure 1 :

Between inputs w and u and outputs z_2 and y of this standard problem, we recognize the CSF presented in section 2.3 which will inflect the solution towards the previous pure performance compensator (LQG/LTR de-

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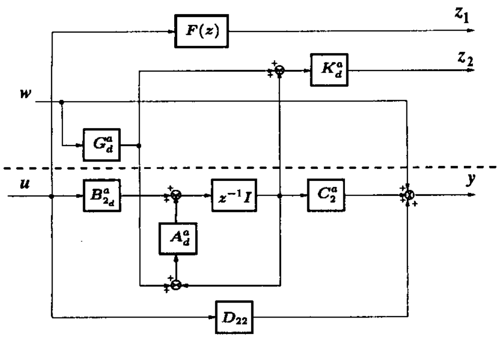

FIGURE 1: $P_f(z)$: setup for the final H_∞ synthesis.

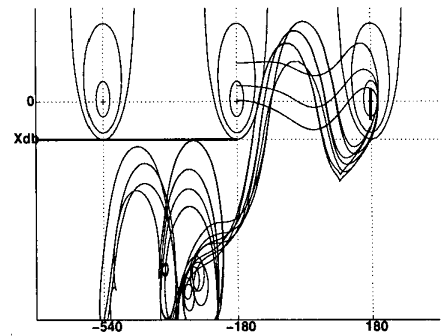

FIGURE 3: $K_2(z)G_f(z)$: NICHOLS plot for worst cases.

sign), and the output z_1 is introduced to specify the second order roll-off behavior with a filter $F(z)$. Then, the H_∞ synthesis provides a 6th-order compensator. Analysis results are displayed in Figures 2 and 3. In Figure 2, we can see that the performance specifications are met. In the Nichols plot (Figure 3), stability margins are good enough for all worst cases and the roll-off behavior is quite satisfactory. A more complete μ-analysis on stationnary launcher control problem is presented in [13].

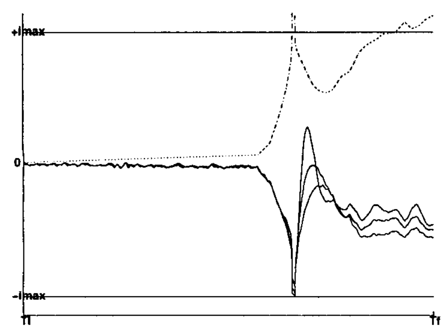

FIGURE 2: angle of attack $i(t)$ (solid) and wind profile $w(t)$ (dashed).

5 Unstationary launcher control

The previous stationary design has been applied for various instants t^i along the flight envelope (10 instants).

Figure 4 depicts the distribution of the closed-loop eigenvalues found by the method proposed in section 2.2 to compute the observer-based realization of the central H_∞ controller at a particular flight instant. On can notice that the state-observer dynamics of the observer-based realization of the H_∞ controller and the state-observer dynamics of the H_2 controller are the same. This property is the direct consequence of the standard problem presented Figure 1 which is a pure DF problem. Therefore, the choice of the particular observer-based realization is systematic at each flight instant and one can assume a correct continuation of these realizations if transitions between models are smooth and if the set of operating points is appropriately chosen. Then, a linear interpolation seems enough to ensure local closed-loop stability for each intermediate value.

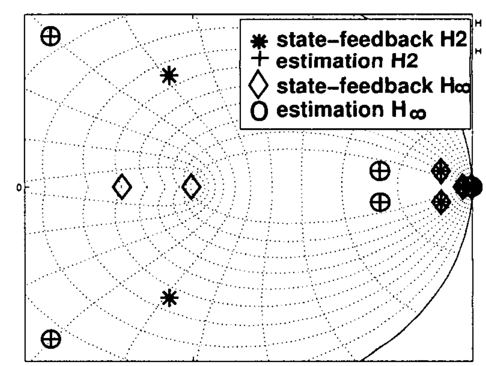

FIGURE 4: Closed-loop eigenvalue map for H_∞ and H_2 syntheses.

Moreover, the state of the interpolated controller $K_e(z,t)$ from observer-based realizations has a physical signification and allows to estimate efficiently the plant states during the flight. Assuming that the linear plant model is available in real-time, the storage of two static gains is only required to update the controller at each sampling instant.

Following these remarks, let us focus on the interpolation of observer-based realizations of various H_∞ controllers. Figure 5 depicted the evolution of the singular value of $K_e(z,t)$ as a function of time t, and one can denote that the evolution of the compensator is very smooth.

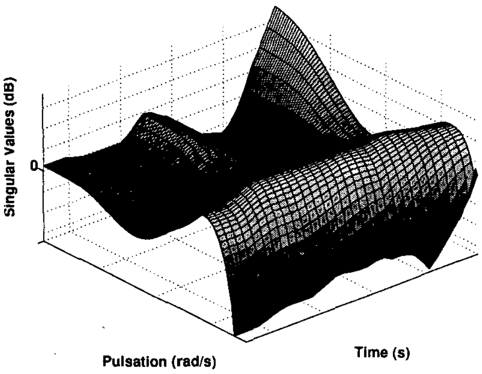

FIGURE 5: Singular value of the compensator $K_e(z,t)$ w.r.t time

6 Conclusion

A complete methodology based on the observer-based realization of H_∞ compensators has been proposed for the attitude control design of a civil launcher. The interest of the CSF to build a standard problem embedding various specifications has been highlighted. The CSF leads to a very specific synthesis setup in which an a priori know-how can be taken into account.

On the non-stationary problem, we have also shown that the observer-based structure can be very interesting to obtain a smooth gain-scheduling. The computation of observer-based realization of each stationary H_∞ compensator is straightforward using the the closed-loop distribution of the associated H_2 synthesis. The observer-based realization of compensators is also very interesting from the real time implementation point of view : as the compensator becomes an estimate of the plant state, such a representation can be recommended to implement failure diagnosis algorithms or to initialize correctly the compensator states during mode switches.

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