On coding for Faster-Than-Nyquist Signaling

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***Abstract*—In this paper, we study the design of sparse graph based codes for Faster-Than-Nyquist (FTN) signaling. Using an asymptotic approach based on EXIT charts, we show that good low-density parity check codes can be designed that perform well under iterative detection and decoding and that have better performance than a FTN scheme using a code optimized for the additive white Gaussian noise channel.**

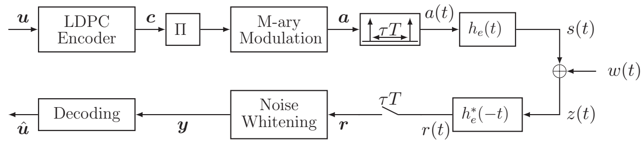

Fig. 1. Overall FTN signaling system

I. Introduction

In his landmark paper [1] Shannon showed that the capacity of band-limited channel can be reached by using sinus cardinal pulses that follow the Nyquist criterion for the absence of InterSymbol Interference (ISI). Later, Mazo introduced Faster Than Nyquist signaling in [2], where he showed that linear modulation can still be performed without loss of error probability, even if symbols are transmitted above the Nyquist rate. The resulting model of the channel proposed in [2] is now known as the ISI channel. Two kind of receivers exist for the ISI channel. Forney proposed the first one in [3], where he showed that Maximum Likelyhood (ML) receiver can be performed by a front-end adapted filter and a transversal noise whitening filter. Later, Ungerboeck proposed a second approach in [4] where he showed that the noise whitening filter could be omitted as long as the noise auto-correlation was taken into account in the ML receiver. Both approaches are also equivalent for symbol or sequence Maximum A Posteriori (MAP) receivers, see [5].

More recently, FTN has known a regain of interest [6]–[8]. In particular, the authors of [7] have shown that Root Raised Cosine (RRC) pulse shaping FTN can provide extra-rate compared with the orthogonal Nyquist transmission, when the RRC pulse has non-null band expansion. This property makes FTN particularly attractive for future telecommunication standards [8].

In this paper, we investigate on the design and the asymptotic performance of sparse graph based codes with FTN signaling. Iterative detection and decoding for FTN signaling is achieved by considering a turbo-equalization scheme based on the discrete equivalent channel with ISI. Based on this model, we perform an asymptotic analysis for turbo-equalized BICM FTN based schemes. From this analysis, we derive an asymptotic optimization of unstructured LDPC codes to investigate on good LDPC code profiles for FTN signaling. By doing so, we aim to study if a *possible* improvement of the overall system efficiency is possible when designing sparse graph based codes matched to FTN signaling in comparison with the use of classical codes optimized for the additive white Gaussian noise (AWGN). Note that in the classical case of mobile communications, frequency selective channels are also varying in time lessening the interest for a code optimization. In the case of FTN signaling over AWGN channel (at least), the discrete linear ISI channel is perfectly known at the receiver and it remains the same during the transmission. Thus, as in the case of partial response channels for magnetic recording, there is a particular interest to investigate on optimized LDPC codes for FTN signaling to derive FTN coding schemes that can perform close to the theoretical limits. For ISI channels, several works have still considered the optimization of LDPC codes. Among them, density evolution is a powerful tool to perform asymptotic analysis and optimization but in the case of ISI channel with LDPC codes, the complexity may become intractable, several orders worse than in the AWGN case [9]. Therefore, we will consider here an approach based on extrinsic information transfer (EXIT) charts. It is a common tool that is used for the asymptotic analysis the convergence of iterative systems [10].

The paper is organized as follows. In Section II, we describe the FTN signaling communication system. In Section III, we derive the asymptotic analysis for FTN signaling scheme. Finally, some optimization and simulation results are given in Section IV. Some conclusions are drawn in Section V.

II. System description and notations

We consider the overall communication system presented in Fig. 1. It can be seen as a bit-interleaved coded modulation scheme (BICM) composed of a channel code serially concatenated with a Q-ary linear modulation pulse-shaped using FTN signaling. At the transmitter, a binary message vector $\boldsymbol{u} \in GF(2)^K$ is first encoded into a codeword $\boldsymbol{c} \in GF(2)^N$

using a low-density parity-check (LDPC) error correcting code. A LDPC code C_H is usually defined using a sparse parity check matrix H of size $M \times N$, where N is the codeword length, $M \geq N - K$ the number of parity check equations and K is the information length in bits. The code rate R is defined as $R = K/N \geq 1 - M/N$, with equality if H is full rank. $\boldsymbol{c}$ is a binary vector that belongs to the null space of H if $H\boldsymbol{c}^\top = \mathbf{0}$ where $^\top$ is the transposition operator. Based on H, one can derive an associated bipartite graph, often referred to as Tanner graph [11]. The Tanner graph consists in two sets of nodes: the variable nodes associated with the codeword bits (columns of H) and the check nodes associated with the parity-check constraints (rows of H). An edge joins a variable node (VN) n to a check node (CN) m if $H(m,n) = 1$. Irregular LDPC codes are usually defined with their edge-perspective degree distribution polynomials $\lambda(x) = \sum_{i=1}^{d_v} \lambda_i x^{i-1}$ and $\rho(x) = \sum_{j=2}^{d_c} \rho_j x^{j-1}$ where λ_i (resp. ρ_j) is the proportion of edges in the Tanner graph connected to VNs of degree i (resp. to CNs of degree j) and d_v (reps. d_c) is the maximum VN (resp. CN) degree. The design rate is given by $R = 1 - \sum_{j=2}^{d_c} \frac{\rho_j}{j} / \sum_{i=1}^{d_v} \frac{\lambda_i}{i}$ [12]. Each codeword $\boldsymbol{c}$ is then interleaved, Gray-mapped into N_S Q-ary complex symbols $\boldsymbol{a}$ and finally pulse-shaped using FTN signaling.

The FTN signaling corresponds to sending the complex symbols $\boldsymbol{a}$ at a symbol rate $R_s = \frac{1}{T_s} = \frac{1}{\tau T}$ symbols per second while pulse-shaping the signal using a T-orthogonal pulse $h_e(t)$. The FTN parameter τ belongs to $[0, 1]$ and the case $\tau = 1$ corresponds to Nyquist signaling. In this paper, we consider root-raised cosine (RRC) filters for pulse shaping. Finally, the signal sent through the channel $s(t)$ is expressed as follows

$$s(t) = \sum_{k=0}^{N_S-1} a_k h_e(t - k\tau T) \quad (1)$$

The receiver observes a noisy version of the signal $s(t)$: $z(t) = s(t) + w(t)$ where $w(t)$ is an Additive White Gaussian Noise with Power Spectral Density $S_w(f) = N_0$. Following the structure of the optimal receiver for the Maximum Likelihood (ML) detection as proposed in [3], we consider in this paper a front-end receiver composed by a matched filter $h_r(t) = h_e^*(-t)$, followed by a sampler at rate R_s and a whitening filter. Let $g(t) = \int_{-\infty}^{+\infty} h_e(x) h_e^*(x - t)dx$ be the global filter. The signal $r(t)$ at the output of the matched filter is given by

$$r(t) = \sum_{k=0}^{N_S-1} a_k g(t - k\tau T) + \int_{-\infty}^{+\infty} w(x) h_e^*(x - t)dx. \quad (2)$$

The matched filter sampled outputs $\boldsymbol{r} = (r_0, \ldots, r_{N_S-1})$ are given by

$$\begin{aligned} r_n &= r(n\tau T) \\ &= \sum_{k=0}^{N_S-1} a_k g((n-k)\tau T) + \int_{-\infty}^{+\infty} w(x) h_e^*(x - n\tau T)dx \\ &= \sum_{k=0}^{N_S-1} a_k g_{n-k} + w_n \end{aligned} \quad (3)$$

where $g_n = g(n\tau T)$ and $w_n = \int_{-\infty}^{+\infty} w(x) h_e^*(x - n\tau T)dx$. Consequently, $\boldsymbol{r}$ can be viewed as $\boldsymbol{a}$ filtered by $\boldsymbol{g}$ altered by an additive Gaussian noise $\boldsymbol{w}$ with auto-correlation $R_w[k] = N_0 g_k$.

After whitening filtering, the equivalent discrete channel model can be written as follows

$$y_n = \sum_{k=0}^{N_S-1} a_k h_{n-k} + w'_n \quad (4)$$

where h_n is the discrete channel impulse response obtained after whitening filtering [3] and w'_n is an additive white Gaussian noise (AWGN) with null mean and variance $\sigma_{w'}^2 = N_0$. Then, based on this channel model, any type of soft-input soft-output (SISO) receiver can be applied.

Based on the preceding equivalent discrete complex baseband channel with inter symbol interference (ISI) model, the joint iterative decoder of the BICM FTN based scheme under consideration is finally reduced to a turbo-equalizer formed by the serial concatenation of a SISO ISI channel detector followed by a SISO LDPC decoder separated by a deinterleaver. Turbo-equalization schemes usually consider full uniform interleaving between the channel code and the modulation symbols [9], [13]. This is sometimes omitted in practical settings when LDPC codes are used owing to a inner LDPC interleaver argument. However, the same studies are implicitly resorting to a uniform interleaving assumption when performing density evolution or EXIT chart analysis for optimization. In this work, we rather consider a partial interleaving approach between the LDPC code and the Q-ary symbols, which leads to a partial deinterleaving between the SISO channel detector and the LDPC decoder. The partial interleaver implies a random interleaving of LDPC codewords bits using different interleaving patterns among variable nodes of the same degree. This is mainly considered for ease of both the analysis and the optimization as discussed in the next section. It can be seen as a multi-edge type approach for edges that link LDPC variable nodes with the channel detector [14]. It can be also thought as a generalization of the approach proposed in [15].

In the following, we consider both trellis based SISO channel detectors such as the bitwise maximum a posteriori (MAP) algorithm (often referred to as the BCJR algorithm) [16] and SISO linear detectors based on a minimum mean square error optimization criterion [17]. We assume bitwise soft outputs for the detector as we are considering a BICM scheme. For LDPC codes, Belief Propagation (BP) decoding [14] is used that performs iterative message passing on the associated Tanner graph assuming independence of the messages after interleaving/deinterleaving. Without loss of generality, we will consider log-likelihood ratio (LLR) based BP decoding.

To perform fair comparisons between schemes using FTN and those at the Nyquist rate ($\tau = 1$), we consider that the signal $s(t)$ occupies a total bandwidth B and has power P_s. Under these considerations, the signal to noise ratio (SNR) is defined as

$$\gamma = \frac{P_s}{N_0 B}. \tag{5}$$

Note that P_s is the power of the signal of interest within its total bandwidth (ie B) and $N_0 B$ is the noise power within the same bandwidth.

The power P_s is obtained from its Power Spectral Density (PSD) $S_s(f)$ as

$$P_s = \int_{-\infty}^{\infty} S_s(f)df = \int_{-B/2}^{B/2} S_s(f)df = \frac{\sigma_x^2 g(0)}{T_s}, \tag{6}$$

where $g(0) = \int_{-\infty}^{\infty} |h_e(t)|^2 dt$. Introducing the energy per signal as the quantity

$$E_s = E(|s(t)|^2) = \sigma_a^2 g(0) = P_s T_s, \tag{7}$$

γ can be expressed as follows

$$\gamma = \frac{E_s}{N_0}\frac{1}{BT_s}. \tag{8}$$

When considering a system with a FTN parameter $\tau = \frac{T_s}{T}$ and a RRC filter with roll-off $\beta = BT - 1$, (8) becomes

$$\gamma = \frac{E_s}{N_0}\frac{1}{\tau(1+\beta)}. \tag{9}$$

Let ρ be the spectral efficiency expressed in bits/s/Hz, E_b/N_0 is defined as

$$\frac{E_b}{N_0} = \frac{\gamma}{\rho}. \tag{10}$$

Since $\rho = R_{cm}/((1+\beta)\tau)$ with $R_{cm} = R.\log_2(Q)$, $\frac{E_b}{N_0}$ is given by

$$\frac{E_b}{N_0} = \frac{1}{R_{cm}}\frac{E_s}{N_0} \tag{11}$$

Note that this definition corresponds to the classical definition of $\frac{E_b}{N_0}$.

III. Asymptotic analysis and optimization for LDPC coded FTN signaling

The discrete equivalent baseband channel model is given by (4) when considering a classical Forney-type receiver (ie. matched filtering followed by a whitening filter). The resulting discrete time model is a linear ISI channel with an additive white Gaussian noise. Therefore, the classical use of an AWGN optimized channel coding scheme is not relevant at first sight and it may result in a rate loss (and thus in a decrease of the effective system spectral efficiency). Indeed, in the general case, there are strong evidence that an AWGN optimized code could not approach efficiently the capacity of an ISI channel or at least the channel capacity with uniform inputs [9]. An optimal receiver should consider a channel coding scheme that is designed for a linear ISI channel to fully benefit from all promises of FTN signaling. To this end, we perform an asymptotic analysis for turbo-equalized BICM FTN based schemes. Based on this analysis, we derive an asymptotic optimization of unstructured LDPC codes to investigate on good LDPC code profiles for FTN signaling.

A. Asymptotic analysis of FTN MAP detection

In this section, we briefly perform an EXIT analysis of the FTN bitwise MAP channel detector. Let $T_{ftn}(.)$ denotes the input-output EXIT transfer chart (also referred to as EXIT curve) of the FTN SISO detector, implicitly depending on the channel noise parameter. Analytic expressions of $T_{FTN}(.)$ are usually not available, but they can be evaluated by mean of Monte-Carlo simulations. To evaluate EXIT charts of the channel equalizer, we assume that a priori LLR messages are modelled by consistent and Gaussian-distributed messages with mean m and variance $\sigma^2 = 2m$. It is then possible to compute the associated input mutual information (MI) I_A using a mono-dimensional function of σ noted $J(\sigma)$ [10] with $J(.)$ formally defined as

$$J(\sigma) = 1 - E_x(log_2(1+e^{-x})), x \sim N(\sigma^2/2, \sigma^2),$$

where $E_x(.)$ denotes the expectation with respect to x.

The output mutual information between the send bits and the extrinsic LLRs $I_E(I_A)$ is measured through histograms, as classically done [10] (ie. no Gaussian assumption is made to compute the mutual information at the output). We have finally $I_E = T_{ftn}(I_A)$. The aim of this section is to evaluate the maximum achievable rates for the inner code when curve fitting design of the inner code is considered. This is usually done by conjecturing that the area theorem derived for the binary erasure channel [18] can be applied to more general channels. In particular, when considering BICM schemes over ISI channel, it is usually assumed that the area under the EXIT curves can give a good evaluation of the achievable rate ie $R \leq R^* = \int_0^1 T_{FTN}(I_A)\mathrm{dI_A}$ for the considered channel detector [19].

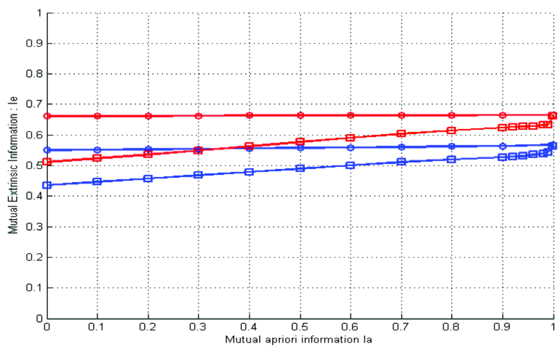

Fig. 2. EXIT charts of a soft linear MMSE based detector from [17] for $\beta = 0.35$ and $(1+\beta)\tau = 1$. Flat EXIT curves with circles correspond to soft demapper EXITs for the AWGN channel.

Fig. 2 gives some EXIT curves for a $Q = 8$ phase shift keying (8-PSK) modulation with Gray mapping and different signal to noise ratios. The simulation parameters are given in the label of the figure. The point $(0, I_e(0))$ represents the achievable rate when only soft detection and decoding is performed without iterative decoding. It is sometimes referred to as BCJR once bound [9] or more simply the BICM capacity for the ISI channel. The point $(1, I_e(1))$ represents the achievable rate when perfect ISI cancellation is assumed (perfect a priori), often reported to as matched filter bound. If for orthogonal signaling, this bound can be easily understood, it should be more carefully analyzed in our context. In the case of FTN signaling, it can be interpreted as the achievable rate for an equivalent AWGN channel with perfect Nyquist signaling with a total bandwidth B with rate R_s (classical orthogonal signaling with a null roll-off) and when using BICM with iterative decoding (BICM-ID). As a reference, we have plotted the EXIT curves of the corresponding soft demapper for the same corresponding SNRs. Based on these curves, we can compute by integration an evaluation of the maximum achievable rates for BICM-ID FTN signaling.

B. Asymptotic analysis of LDPC based FTN BICM-ID

We now present an EXIT analysis of a LDPC based FTN BICM-ID scheme. In this paper, we assume the following scheduling: a global iteration ℓ is composed of one FTN SISO detector update followed by one BP iteration (one data-pass plus one check-pass update) for the LDPC code. We further assume partial interleavers, each one is associated with the VNs set of the same degree. Under a Gaussian approximation [20], we can give the different update equations for the FTN BICM-ID scheme as follows.

The combined EXIT function $I^{\ell}_{v\to c}$ for the VNs and the FTN SISO detector at the ℓ^{th} iteration by:

$$I^{\ell}_{v\to c} = \sum_{i=1}^{d_v} \lambda_i I^{\ell}_{v\to c}(i) \tag{12}$$

with

$$I^{\ell}_{v\to c}(i) = J\left(\sqrt{(i-1)[J^{-1}(I^{\ell-1}_{c\to v})]^2 + [J^{-1}(I^{\ell}_{eq\to v}(i))]^2}\right)$$

$$I^{\ell}_{eq\to v}(i) = T_{FTN}\left(J(\sqrt{i}J^{-1}(I^{\ell-1}_{c\to v}))\right)$$

where:

- $I^{\ell}_{v\to c}(i)$ is the average MI associated with LLR messages passed from a VN of degree i to CNs,
- $I^{\ell-1}_{c\to v}$ the average MI associated with LLR messages from CNs to VNs,
- $I^{\ell}_{eq\to v}(i)$ is the average MI for degree-i VN associated with LLR messages from the FTN SISO MAP (equalizer) to the LDPC decoder.

For a degree-j CN, the MI $I^{\ell-1}_{c\to v}$ associated with extrinsic LLRs passed from CN to VN at iteration $\ell - 1$ and relative coded bits is known, under reciprocal channel approximation [15], as:

$$I^{\ell-1}_{c\to v} = 1 - \sum_{j=2}^{d_c} \rho_j J(\sqrt{j-1}J^{-1}(1 - I^{\ell-1}_{v\to c})) \tag{13}$$

Combining the preceding equations, we have finally the following recursion:

$$I^{\ell}_{v\to c} = F\left(\lambda(x), \rho(x), T_{FTN}(.), I^{(\ell-1)}_{v\to c}\right)$$

The obtained recursion is a linear function with respect to optimization parameters $\lambda_i, i = 1 \dots d_v$ for a given $\rho(x)$ and a given channel noise parameter. With concentrated $\rho(x)$ [20], rate maximization design is equivalent to the following linear programming optimization problem : $\max_{\{\lambda_i\}} \sum_i \lambda_i/i$ with the constraints

$[C0]$ Mixture : $\sum_i \lambda_i = 1$

$[C1]$ Convergence: $F(\lambda(x), \rho(x), T_{FTN}(.), x) > x$

$[C2]$ Stability: $\lambda_2 < e^{-M/4} \sum_{j=2}^{d_c} \rho_j (j-1), M = J^{-1}(T_{FTN}(1)).$

The last inequality for the stability condition can be deduced from a fixed point analysis and has been first conjectured in [21]. This linear programming optimization is then efficiently solved by classical linear programming using discretization of the convergence constraints for $x \in [0,1]$. It can be shown that this linear programming approach is in fact equivalent to a curve fitting approach that searches for optimal combination of combined EXIT functions of VNs and channel detector to match the CNs EXIT curve [15].

IV. RESULTS

In this section, we provide some optimization and simulation results. Fist, in Fig. 3, we provide as an illustration the achievable spectral efficiency for $\beta = 0.35$ and $(\beta + 1)\tau = 1$ for the case of a binary phase shift keying (BPSK) with optimal bit wise MAP detection. As we can see, the achievable spectral efficiency for the FTN scheme (estimated using the area theorem, ie. considering BICM with iterative decoding with perfectly matched inner code) is less than perfect Nyquist signaling with roll-off $\beta = 0$ for a total bandwidth B (with rate R_s). However, when comparing with Nyquist rate signaling with a roll-off $\beta = 0.35$ with the same total bandwidth, we can see that a significant gain can be achieved when using FTN signaling with iterative decoding. When comparing the "BCJR once" achievable spectral efficiency to the FTN one, we can also deduce that all the promises of the FTN scheme come at the price of higher decoding complexity since FTN signaling using iterative decoding should be considered. From the curves, we can also see that allowing a higher maximum degree variable node enables to operate closer to the maximum achievable spectral efficiency. The same conclusion can be drawn for other kinds of modulations and SISO detectors.

In Fig. 4, we also report the bit error rate (BER) performance with $\beta = 0.35$ and $(1+\beta)\tau = 1$ for both a BPSK with $N == 25000$ and a 8-PSK modulation with $N = 24000$. BP decoding is assumed with maximum 250 iterations. For the BPSK case, bitwise SISO MAP is considered while linear SISO MMSE based detector are considered for the 8-PSK case. The performance are compared with an AWGN optimized code [12] with iterative decoding. We can see that for the BPSK case, the AWGN code performance is quite close to the optimized FTN scheme. For the 8-PSK case, the improvement is more significant.

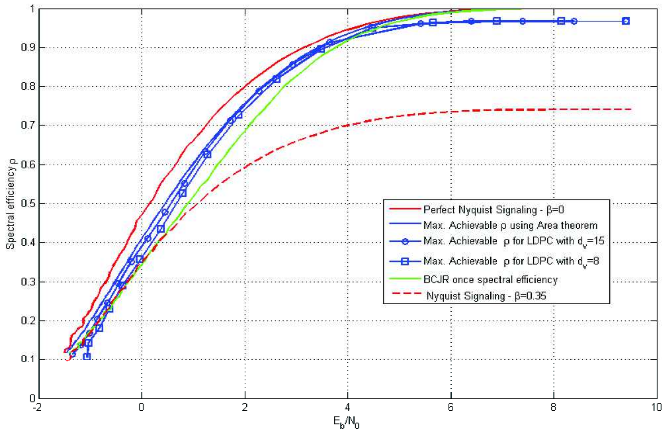

Fig. 3. Achievable Spectral efficiency for BPSK based FTN schemes for different configurations

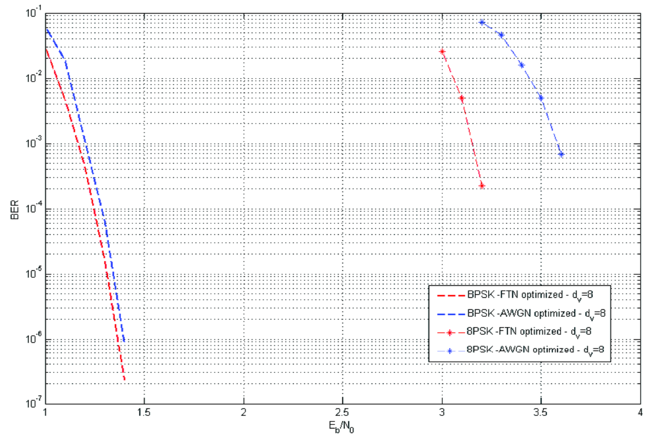

Fig. 4. Bit error rate for FTN signaling with $\beta = 0.35$ and $(1+\beta)\tau = 1$. For the BPSK case, bitwise SISO MAP is considered while linear SISO MMSE based detector are considered for the 8-PSK case.

V. Conclusion

In this paper, we have investigated on the design of sparse graph based codes for FTN signaling. Using an asymptotic approach based on EXIT charts, we have shown that good LDPC codes can be designed that perform well under iterative detection and decoding and that have better performance than a FTN scheme using a code optimized for the AWGN channel.

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