Knowledge and action: how should we combine their logics?

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FroCoS 2015, Wroclaw, September 2015

Outline

1. Motivation: simple logics of action and knowledge needed
2. Adding higher-order observability information
3. Application: the gossip problem
4. Adding public announcements
5. Application: the muddy children puzzle
6. Application: boolean games

Logics of knowledge and action

- fruitful in CS since 30+ years
  - epistemic temporal logics
    [Halpern et col., Lomuscio, . . . , ≥1990]
  - epistemic extension of the situation calculus
    [Scherl & Levesque, . . . , ≥1995]
  - Dynamic Epistemic Logics DEL
    [van Benthem, Moss, Baltag, van Ditmarsch, . . . , ≥2000]
- typically multi-dimensional modal logics
  - high complexity; often undecidable
- simplest combined logic of knowledge and action?
  - a typical question of philosophical logic
  - also relevant for computer science

Logics of knowledge and action

- idea [v.d.Hoek & Wooldridge, inspired from model checkers]:
  - ground action on propositional control
  - ground knowledge on propositional observability
- logics:

`ECL-PO`	=	"Epistemic Coalition Logic of Propositional Control with Partial Observability" [vdHTW11]
`LRC`	=	"Logic of Revelation and Concealment" [vdHIW12]

- this talk:
  - reduce to *Dynamic Logic of Propositional Assignments* `DL-PA`
  - overcome some limitations of the original approach

Grounding action on propositional control

agent i controls propositional variable p or not

- define accessibility relation for group of agents $J \subseteq Agt$:

$$R_J = \{(v, v') : v(p) = v'(p) \text{ if } p \in PVar \text{ not controlled by any } i \in J\}$$

- coalitional effectivity *ceteris paribus*:

vR_Jv' = at v, if the other agents don't act then J can guarantee that the next state of the world is v'

- interpret operator of coalitional effectivity:

$$v \models \Diamond_J \varphi \text{ iff } v' \models \varphi \text{ for every } v' \text{ such that } vR_Jv'$$

$\Longrightarrow$ Coalition Logic of Propositional Control

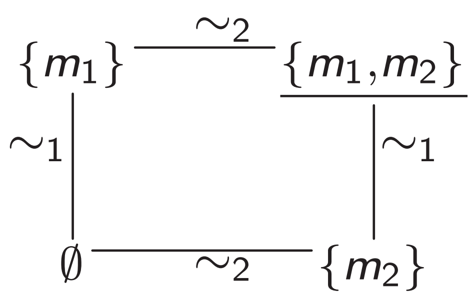
- approximates ATL/Pauly's operator of coalitional effectivity:

$$\langle\langle\{i\}\rangle\rangle X\varphi \approx \Diamond_{\{i\}} \Box_{Agt \setminus \{i\}} \varphi$$

Grounding knowledge on propositional observability

agent i observes whether propositional variable p is true or not

- muddy children: child 1 sees whether child 2 is muddy; doesn't see whether 1 is muddy
- define indistinguishability relation:



$$\sim_i \;=\; \{(v, v') : v(p) = v'(p) \text{ for every } p \in PVar \text{ observed by } i\}$$

$\Longrightarrow$ equivalence relation on the set of all valuations

- interpret epistemic operator as usual:

$$v \models \mathrm{K}_i \varphi \quad \text{iff} \quad v' \models \varphi \text{ for every } v' \text{ such that } v \sim_i v'$$

- pushes the envelope of the 'DEL philosophy' of replacing accessibility relations by model updates

  (while DELs still have accessibility relations for knowledge)

Propositional observability: properties

+ all axiom schemas of S5 valid
− observability is common knowledge:

$$(\mathtt{K}_i\, p \vee \mathtt{K}_i\, \neg p) \rightarrow \mathtt{K}_j\, (\mathtt{K}_i\, p \vee \mathtt{K}_i\, \neg p)$$
$$\neg(\mathtt{K}_i\, p \vee \mathtt{K}_i\, \neg p) \rightarrow \mathtt{K}_j\, \neg(\mathtt{K}_i\, p \vee \mathtt{K}_i\, \neg p)$$

− distributes over disjunction:

$$\mathtt{K}_i\, (p \vee q) \leftrightarrow (\mathtt{K}_i\, p \vee \mathtt{K}_i\, q)$$

so:

- initial situation of the muddy children puzzle can be modelled
- . . . but not the situation after the father's announcement "one of you is muddy"!

− related:

- logic only accounts for observation but not for *communication*

Embedding into DL-PA

- can be captured in *Dynamic Logic of Propositional Assignments* DL-PA
  1. introduce new propositional variables
     $$\mathtt{C}_i\, p \quad = \quad \text{“}i \text{ controls } p\text{”}$$
     $$\mathtt{S}_i\, p \quad = \quad \text{“}i \text{ sees } p\text{”}$$
  2. identify $\Diamond_i$ and $\mathtt{K}_i$ with assignment programs:
     for φ boolean with $PVar(\varphi) = \{p_1, \ldots, p_n\}$,
     $$\Diamond_i \varphi \leftrightarrow \Big\langle \big(\neg \mathtt{C}_i\, p_1? \sqcup (\mathtt{C}_i\, p_1?; (+p_1 \sqcup -p_1))\big); \ldots; \big(\neg \mathtt{C}_i\, p_n? \sqcup (\mathtt{C}_i\, p_n?; (+p_n \sqcup -p_n))\big) \Big\rangle \varphi$$
     $$\mathtt{K}_i\, \varphi \leftrightarrow \Big[\big(\mathtt{S}_i\, p_1? \sqcup (\neg \mathtt{S}_i\, p_1?; (+p_1 \sqcup -p_1))\big); \ldots; \big(\mathtt{S}_i\, p_n? \sqcup (\neg \mathtt{S}_i\, p_n?; (+p_n \sqcup -p_n))\big) \Big] \varphi$$
     $\Longrightarrow$ start with innermost modal operators!
  3. axiomatize exclusive and exhaustive control
     $$\Big(\bigwedge_{i \neq j} \neg(\mathtt{C}_i\, p \wedge \mathtt{C}_j\, p) \Big) \wedge \Big(\bigvee_{i \in Agt} \mathtt{C}_i\, p \Big)$$

DL-PA

- assignment programs built by the PDL program operators from

$$\begin{aligned} +p &= \text{“make } p \text{ true”} \\ -p &= \text{“make } p \text{ false”} \end{aligned}$$

- generalizes QBF:

$$\forall p.\varphi \leftrightarrow [+p \sqcup -p]\varphi$$

- compact models
  - valuations of classical propositional logic
- PSPACE complete (both model checking and SAT)
- uniform substitution does not preserve validity

Adding higher-order observability information

Higher-order observability

- idea: introduce higher-order visibility atoms

  $S_i\, p$ = "i sees the value of p"

  $S_i\, S_j\, p$ = "i sees whether j sees the value of p"

  $S_i\, S_j\, S_k\, p$ = "..."

- general schema as before:

$$K_i\, \varphi \leftrightarrow [\pi_{i,Atm(\varphi)}]\varphi$$

$$\text{where } \pi_{i,Atm(\varphi)} = \big(S_i\, \alpha_1? \sqcup (\neg S_i\, \alpha_1?; (+\alpha_1 \sqcup -\alpha_1))\big); \ldots$$

examples:

$$K_i\, p \leftrightarrow p \land S_i\, p$$

$$K_i\, \neg p \leftrightarrow \neg p \land S_i\, p$$

$$K_i\, K_j\, p \leftrightarrow K_i\, (p \land S_j\, p)$$

$$\leftrightarrow K_i\, p \land K_i\, S_j\, p$$

$$\leftrightarrow p \land S_i\, p \land S_j\, p \land S_i\, S_j\, p$$

DEL-PAO = DEL of Propositional Assignment and Observation

Language of DEL-PAO

- visibility atoms:

$$\alpha ::= p \mid \mathtt{S}_i\, \alpha \mid \mathtt{JS}\, \alpha$$

with p propositional variable and i agent

$$\begin{array}{rcl} p & = & \ldots \\ \mathtt{S}_i\, \alpha & = & \ldots \\ \mathtt{JS}\, \alpha & = & \text{“all agents \textit{jointly} see whether } \alpha\text{”} \end{array}$$

- formulas and programs as in PDL:

$$\varphi ::= \alpha \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathtt{K}_i\, \varphi \mid \mathtt{CK}\, \varphi \mid [\pi]\varphi$$

$$\pi ::= +\alpha \mid -\alpha \mid \pi;\pi \mid \pi \sqcup \pi \mid \varphi?$$

with i agent and α visibility atom

DEL-PAO: valuations

- valuation = sets of visibility atoms v
- define indistinguishability relations:

$$v \sim_i v' \quad \text{iff} \quad \forall\alpha, \text{ if } \mathtt{S}_i\,\alpha \in v \text{ then } v(\alpha) = v'(\alpha)$$

$$v \sim_{Agt} v' \quad \text{iff} \quad \forall\alpha, \text{ if } \mathtt{JS}\,\alpha \in v \text{ then } v(\alpha) = v'(\alpha)$$

- problem: are reflexive, but neither transitive nor symmetric
  - $\emptyset \sim_i v$ for every v
  - $v \not\sim_i \emptyset$ as soon as $p \in v$ and $\mathtt{S}_i\, p \in v$

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- solution: valuations must be introspective

DEL-PAO: introspective valuations

Definition

v is introspective iff

1. $\mathtt{S}_i\,\mathtt{S}_i\,\alpha \in v$
2. $\mathtt{JS}\,\mathtt{JS}\,\alpha \in v$
3. $\mathtt{JS}\,\mathtt{S}_i\,\mathtt{S}_i\,\alpha \in v$
4. if $\mathtt{JS}\,\alpha \in v$ then $\mathtt{S}_i\,\alpha \in v$
5. if $\mathtt{JS}\,\alpha \in v$ then $\mathtt{JS}\,\mathtt{S}_i\,\alpha \in v$

Theorem

introspective valuations contain all atoms of form "$\cdots \mathtt{S}_i\,\mathtt{S}_i \cdots p$" and "$\cdots \mathtt{JS}\,\mathtt{JS} \cdots p$"

Theorem

$\sim_i$ *and* $\sim_{Agt}$ *are equivalence relations on introspective valuations*

DEL-PAO: interpretation of formulas

- interpretation of formulas:

$v \models \alpha$	iff	$\alpha \in v$
$v \models \mathtt{K}_i\,\varphi$	iff	$v' \models \varphi$ for every $v \sim_i v'$
$v \models \mathtt{CK}\,\varphi$	iff	$v' \models \varphi$ for every $v \sim_{Agt} v'$
$v \models [\pi]\varphi$	iff	$v' \models \varphi$ for every $vR_\pi v'$

- interpretation of programs:

$vR_{+\alpha}v'$	iff	$v' = v \cup \{\alpha$ and its introspective consequences$\}$
$vR_{-\alpha}v'$	iff	α is not an introspectively valid atom
		and $v' = v \setminus \{\alpha$ and its causes$\}$
$vR_{\pi_1;\pi_2}v'$	iff	there is v'' such that $vR_{\pi_1}v''R_{\pi_2}v'$
$vR_{\pi_1 \sqcup \pi_2}v'$	iff	$vR_{\pi_1}v'$ or $vR_{\pi_2}v'$
$vR_{\varphi?}v'$	iff	$v = v'$ and $v \models \varphi$

Valid in introspective valuations

- S5 axiom schemas valid for $\mathtt{K}_i$:

$$\mathtt{K}_i\,\varphi \to \varphi$$
$$\mathtt{K}_i\,\varphi \to \mathtt{K}_i\,\mathtt{K}_i\,\varphi$$
$$\neg\mathtt{K}_i\,\varphi \to \mathtt{K}_i\,\neg\mathtt{K}_i\,\varphi$$

- fixed-point axiom schema valid for $\mathtt{CK}$:

$$\mathtt{CK}\,\varphi \leftrightarrow \varphi \land \bigwedge_i \mathtt{K}_i\,\mathtt{CK}\,\varphi$$

- induction axiom schema invalid for $\mathtt{CK}$:

$$\varphi \land \mathtt{CK}\,(\varphi \to \bigwedge_i \mathtt{K}_i\,\mathtt{CK}\,\varphi) \not\to \mathtt{CK}\,\varphi$$

Properties of DEL-PAO, ctd.

- sound and complete axiomatization
  1. reduction axioms for $\mathtt{K}_i$, $\mathtt{CK}$, $[\pi]$

$$\mathtt{K}_i\,\varphi \leftrightarrow [\pi_{i,ATM(\varphi)}]\varphi$$
$$\mathtt{CK}\,\varphi \leftrightarrow [\pi_{Agt,ATM(\varphi)}]\varphi$$
$$[\pi \sqcup \pi']\varphi \leftrightarrow \ldots$$
$$\ldots$$
$$[+\alpha]\varphi \leftrightarrow \ldots$$
$$[-\alpha]\varphi \leftrightarrow \ldots$$

  2. introspection axioms:

$$\mathtt{S}_i\,\mathtt{S}_i\,\alpha$$
$$\mathtt{JS}\,\mathtt{JS}\,\alpha$$
$$\mathtt{JS}\,\mathtt{S}_i\,\mathtt{S}_i\,\alpha$$
$$\mathtt{JS}\,\alpha \rightarrow \mathtt{S}_i\,\alpha$$
$$\mathtt{JS}\,\alpha \rightarrow \mathtt{JS}\,\mathtt{S}_i\,\alpha$$

  3. modus ponens
  4. rules of equivalence for $\mathtt{K}_i$, $\mathtt{CK}$, $[\pi]$

Properties of DEL-PAO, ctd.

- complexity: SAT and MC both PSPACE-complete
  1. MC can be polynomially reduced to SAT
  2. SAT can be polynomially reduced to MC
  3. lower bound for MC: polynomial encoding of QBF

     $$v \models \forall p.\varphi \text{ iff } v \models [+p \sqcup -p]\varphi$$

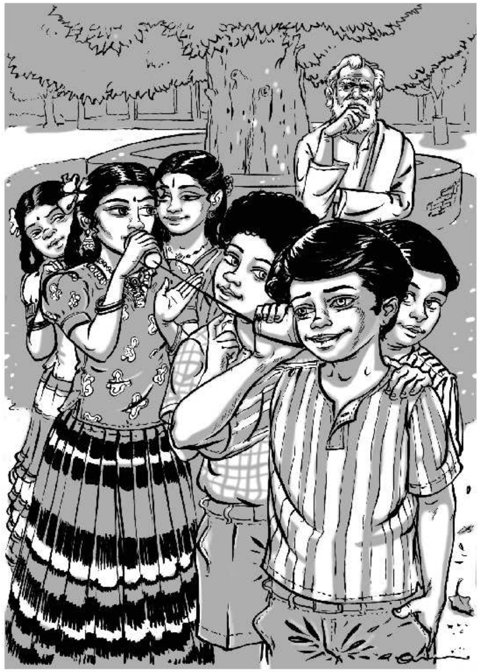
  4. upper bound for MC: polynomial encoding into Dynamic Logic of Propositional Assignments DL-PA [HLTM11, BHT13]

$\Longrightarrow$ better than SAT for $\mathtt{S5}_{\mathtt{n}}^{\mathtt{CK}}$ (EXPTIME-complete)

Application: the gossip problem

The gossip problem [Baker&Shostak, Discrete Mathematics 1972]

- six friends each with a secret σ_i
- they can call each other to exchange every secret they know
- how many calls to spread all secrets among all friends?



(picture from [vDK15])

The gossip problem

- goal: shared knowledge
$$\mathtt{EK}\,\varphi = \bigwedge_{i \in Agt} \mathtt{K}_i\,\varphi$$
('everybody knows')
- optimal algorithm: 8 calls to obtain $\mathtt{EK}\,(\sigma_1 \wedge \cdots \wedge \sigma_6)$
  - for n agents: $2(n-1)$ calls
- versatile:
  - reasoning about social networks, disease spreading, ...
    $\Longrightarrow$ take some network structure into account
  - different kinds of protocols
    $\Longrightarrow$ distributed vs. centralized
- hot topic in the DEL community:
  - [AvDGvdH14, vDK15]
  - ongoing work by v.Ditmarsch, v.Eijck, v.d.Hoek, Grossi, Apt
- multiagent planning's blocksworld?

The gossip problem in DEL-PAO

call = program:

$$\begin{aligned} call_{ij} = {} & \big((\mathtt{K}_i\, \sigma_1?; +\mathtt{S}_j\, \sigma_1) \sqcup \neg\mathtt{K}_i\, \sigma_1?\big); \cdots ; \big((\mathtt{K}_i\, \sigma_6?; +\mathtt{S}_j\, \sigma_6) \sqcup \neg\mathtt{K}_i\, \sigma_6?\big); \\ & \big((\mathtt{K}_j\, \sigma_1?; +\mathtt{S}_i\, \sigma_1) \sqcup \neg\mathtt{K}_j\, \sigma_1?\big); \cdots ; \big((\mathtt{K}_j\, \sigma_6?; +\mathtt{S}_i\, \sigma_6) \sqcup \neg\mathtt{K}_j\, \sigma_6?\big) \end{aligned}$$

The gossip problem in DEL-PAO

call = program:

$$call_{ij} = \big((\mathtt{K}_i\,\sigma_1?; +\mathtt{S}_j\,\sigma_1) \sqcup \neg\mathtt{K}_i\,\sigma_1?\big); \cdots ; \big((\mathtt{K}_i\,\sigma_6?; +\mathtt{S}_j\,\sigma_6) \sqcup \neg\mathtt{K}_i\,\sigma_6?\big);$$
$$\big((\mathtt{K}_j\,\sigma_1?; +\mathtt{S}_i\,\sigma_1) \sqcup \neg\mathtt{K}_j\,\sigma_1?\big); \cdots ; \big((\mathtt{K}_j\,\sigma_6?; +\mathtt{S}_i\,\sigma_6) \sqcup \neg\mathtt{K}_j\,\sigma_6?\big)$$

For valuation v such that $\sigma_i \in v$ and such that $\mathtt{S}_i\,\sigma_j \in v$ iff $i{=}j$:

$$v \models \big[call_{12}; call_{34}; call_{56}; call_{13}; call_{45}; call_{16}; call_{24}; call_{35}\big]\ \mathtt{EK}\left(\bigwedge_{1\le j\le 6}\sigma_j\right)$$

$$v \models \Big\langle\Big(\bigsqcup_{1\le i,j\le 6}\neg\mathtt{S}_i\,\sigma_j?; call_{ij}\Big)\Big\rangle^6\ \mathtt{EK}\left(\bigwedge_{1\le j\le 6}\sigma_j\right)$$

$$v \models \Big[\Big(\bigsqcup_{1\le i,j\le 6}\neg\mathtt{S}_i\,\sigma_j?; call_{ij}\Big)\Big]^5\ \neg\mathtt{EK}\left(\bigwedge_{1\le j\le 6}\sigma_j\right)$$

The gossip problem: attaining higher-order shared knowledge

- attain shared knowledge of level 2:

$$\mathtt{EK}\,\mathtt{EK}\left(\bigwedge_{1\leq j\leq 6}\sigma_j\right)$$

- attain shared knowledge of level k:

$$\mathtt{EK}^k\left(\bigwedge_{1\leq j\leq 6}\sigma_j\right)$$

- algorithm with $(k+1)\times(n-1)$ calls to attain shared knowledge of order 2 [Herzig & Maffre, submitted]
  - optimal?

Adding public announcements

Semantics: add current info state

[Herzig et al., ongoing]

- idea: evaluate epistemic formulas not only wrt agents' observations, but also wrt the current information state [CS15]
  - current information state = set of valuations W
  - pointed model = information state W + valuation v
- language: add public announcements
- truth conditions:

$$W, v \models [\psi!]\varphi \quad \text{iff} \quad W, v \models \psi \text{ implies } \|\psi\|_W, v \models \varphi$$

$$W, v \models \mathrm{K}_i\, \varphi \quad \text{iff} \quad W, v' \models \varphi \text{ for every } v' \in W \text{ s.th. } v \sim_i v'$$

- properties:
  - reduction axioms $\Longrightarrow$ decidable
  - PSPACE complete

Application: the muddy children puzzle

Application: the muddy children puzzle

for v such that $\mathtt{S}_i\, m_j \in v$ iff $i \neq j$ and $\mathtt{JS}\,\mathtt{S}_i\, m_j \in v$ for all i, j:

- ignorance persists for $n-2$ rounds

$$v \models \mathit{Ignorance}$$

$$v \models [(\bigvee_i m_i)\,!]\mathit{Ignorance}$$

$$v \models [(\bigvee_i m_i)?\,!][\mathit{Ignorance}?\,!]\mathit{Ignorance}$$

$$v \models [(\bigvee_i m_i)?\,!][\mathit{Ignorance}?\,!]^{n-2}\mathit{Ignorance}$$

- shared and even common knowledge comes after $n-1$ rounds

$$v \models [(\bigvee_i m_i)?\,!][\mathit{Ignorance}?\,!]^{n-1}\mathtt{EK}\bigwedge_i m_i$$

$$v \models [(\bigvee_i m_i)?\,!][\mathit{Ignorance}?\,!]^{n-1}\mathtt{CK}\bigwedge_i m_i$$

with $\mathit{Ignorance} = \bigwedge_i(\neg\mathtt{K}_i\, m_i \wedge \neg\mathtt{K}_i\,\neg m_i)$

Application: boolean games

Putting things together: accounting for epistemic boolean games

- boolean games
  - exclusive and exhaustive propositional control:

$$\Big(\bigwedge_{i \neq j} \neg(\mathtt{C}_i\, p \land \mathtt{C}_j\, p)\Big) \land \Big(\bigvee_{i \in Agt} \mathtt{C}_i\, p\Big)$$

  - strategy of agent i = truth values of i's variables
    $\Longrightarrow$ strategy profile = valuation
  - goal of agent i = propositional formula γ_i
    $\Longrightarrow$ utility of strategy profile v is 1 if $v \models \gamma_i$; is 0 otherwise
  - strategy profile v is a Nash equilibrium iff

$$v \models \bigwedge_{i \in Agt} (\Diamond_i \gamma_i \to \gamma_i)$$

Putting things together: accounting for epistemic boolean games

- epistemic boolean games:
  - generalize propositional variables to atoms: $\mathtt{S}_i\,\mathtt{C}_j\,p$, ...
  - generalize goals to epistemic formulas
  - same definitions: strategy, Nash equilibrium, ...

example:

- agent 1 has a secret, s_1, and 2 has a secret, s_2
- agent i may privately communicate his secret to j: $+\mathtt{S}_j\,s_i$
- both have goal of 'fair division of information':

$$\gamma_1 = \gamma_2 = \mathrm{K}_1\,s_2 \leftrightarrow \mathrm{K}_2\,s_1$$

example:

- ... and agent 3 shouldn't learn anything:

$$\gamma_1 = \gamma_2 = \big(\mathrm{K}_1\,s_2 \leftrightarrow \mathrm{K}_2\,s_1\big) \wedge \neg\mathrm{K}_3\,s_1 \wedge \neg\mathrm{K}_3\,\neg s_1 \wedge \neg\mathrm{K}_3\,s_2 \wedge \neg\mathrm{K}_3\,\neg s_2$$

Conclusion and future work

- DEL-PAO = dynamic epistemic logic based on visibility
  - higher-order observations
    - no common knowledge of who sees what
- add public announcements
  - information state
- add propositional control: DEL-PAOC
- interesting complexity
- future work:
  - ?? from knowledge to belief
    - problem: guarantee introspection

- Maduka Attamah, Hans van Ditmarsch, Davide Grossi, and Wiebe van der Hoek.
  Knowledge and gossip.
  Proceedings of 21st ECAI, pages 21–26, 2014.

- Philippe Balbiani, Andreas Herzig, and Nicolas Troquard.
  Dynamic logic of propositional assignments: a well-behaved variant of PDL.
  In *Proceedings of the 28th Annual IEEE/ACM Symposium on Logic in Computer Science*, pages 143–152, 2013.

- Tristan Charrier and Francois Schwarzentruber.
  Mental programs and arbitrary announcements.
  In *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems*. IFAAMAS, 2015.

- Andreas Herzig, Emiliano Lorini, and Faustine Maffre.
  A poor man's epistemic logic based on propositional assignment and higher-order observation.
  In *International Conference on Logic, Rationality and Interaction (LORI), Taipei, October 28-31, 2015*. Springer Verlag, 2015.

Andreas Herzig, Emiliano Lorini, Frédéric Moisan, and Nicolas Troquard.
A dynamic logic of normative systems.
In *Proceedings of the 22nd International Joint Conference on Artificial Intelligence*, pages 228–233, 2011.

Wiebe van der Hoek, Petar Iliev, and Michael Wooldridge.
A logic of revelation and concealment.
In *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems*, pages 1115–1122. IFAAMAS, 2012.

Wiebe van der Hoek, Nicolas Troquard, and Michael Wooldridge.
Knowledge and control.
In *Proceedings of the 10th International Conference on Autonomous Agents and Multiagent Systems*, pages 719–726. IFAAMAS, 2011.

Hans van Ditmarsch and Barteld Kooi.
Epistemic Puzzles.
Springer Verlag, 2015.