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# Recovering the initial state of dynamical systems using observers

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The aim of this work is to show that the observer based algorithm proposed in [3] for solving initial data inverse problem allows to reconstruct the *observable part* of the initial data when observability assumption fails. This poster is an overview of the results obtained in [2].

Let  $X$  and  $Y$  be two Hilbert spaces,  $A : \mathcal{D}(A) \rightarrow X$  a skew-adjoint operator,  $C \in \mathcal{L}(X, Y)$  and  $\tau > 0$ . We consider the system

$$\begin{cases} \dot{z}(t) = Az(t), & \forall t \geq 0, \\ z(0) = z_0 \in X, \end{cases} \quad y(t) = Cz(t), \quad \forall t \in (0, \tau),$$

and we suppose that  $(A, C)$  is exactly observable in time  $\tau$ , that is

$$\exists k_\tau > 0, \quad \int_0^\tau \|y(t)\|_Y^2 dt \geq k_\tau^2 \|z_0\|^2, \quad \forall z_0 \in X. \quad (1)$$

We would like to reconstruct  $z_0$  from the observation  $y$ , and we can use the following iterative algorithm to perform this task. Let  $A^+ = A - \gamma C^*C$  (resp.  $A^- = -A - \gamma C^*C$ ) be the generator of the exponentially stable  $C_0$ -semigroup  $\mathbb{T}^+$  (resp.  $\mathbb{T}^-$ ) for some  $\gamma > 0$  (see for instance [3, Proposition 3.7]), and let  $z_0^+ \in X$  be the initial guess (usually  $z_0^+ = 0$ ). Then the algorithm reads: for all  $n \in \mathbb{N}^*$

$$\begin{cases} \dot{z}_n^+(t) = A^+ z_n^+(t) + \gamma C^* y(t), & \forall t \in (0, \tau), \\ z_1^+(0) = z_0^+, \\ z_n^+(0) = z_{n-1}^+(0), & \forall n \geq 2, \end{cases} \quad (2)$$

$$\begin{cases} \dot{z}_n^-(t) = -A^- z_n^-(t) - \gamma C^* y(t), & \forall t \in (0, \tau), \\ z_n^-(\tau) = z_n^+(\tau), & \forall n \geq 1. \end{cases} \quad (3)$$

Therefore, one can easily obtain (working on the errors  $e^+ = z^+ - z$  and  $e^- = z^- - z$ ) that  $z_n^-(0)$  converges exponentially to  $z_0$ : there exists a constant  $\alpha \in (0, 1)$ , namely  $\|\mathbb{T}_\tau^- \mathbb{T}_\tau^+\|_{\mathcal{L}(X)}$ , such that

$$\|z_n^-(0) - z_0\| \leq \alpha^n \|z_0^+ - z_0\|, \quad \forall n \leq 1.$$

Using the framework of well-posed linear systems and a result of stabilization by collocated feedback of Curtain and Weiss [1], we prove in [2] that without any observability assumption (such as (1)), the state space  $X$  can be written as a direct sum of the observable and unobservable state, preserved by the algorithm (2)–(3). We obtain this result for both bounded and (some) unbounded observation operators  $C$ , and give a characterization of the exponential decay of the reconstruction error.

## References

- [1] R. F. CURTAIN AND G. WEISS, *Exponential stabilization of well-posed systems by collocated feedback*, SIAM J. Control Optim., 45 (2006), pp. 273–297 (electronic).
- [2] G. HAINE, *Recovering the observable part of the initial data of an infinite-dimensional linear system with skew-adjoint operator*, In Revision.
- [3] K. RAMDANI, M. TUCSNAK, AND G. WEISS, *Recovering the initial state of an infinite-dimensional system using observers*, Automatica, 46 (2010), pp. 1616–1625.

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