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Recovering the initial state of dynamical systems using observers

Ghislain Haine*

The aim of this work is to show that the observer based algorithm proposed in [3] for solving initial data inverse problem allows to reconstruct the observable part of the initial data when observability assumption fails. This poster is an overview of the results obtained in [2].

Let $X$ and $Y$ be two Hilbert spaces, $A : \mathcal{D}(A) \to X$ a skew-adjoint operator, $C \in \mathcal{L}(X,Y)$ and $\tau > 0$. We consider the system

\[ \begin{align*}
\dot{z}(t) &= Az(t), & \forall \ t \geq 0, \\
z(0) &= z_0 \in X, & \dot{y}(t) = Cz(t), & \forall \ t \in (0, \tau),
\end{align*} \]

and we suppose that $(A,C)$ is exactly observable in time $\tau$, that is

\[ \exists k_\tau > 0, \int_0^\tau \|y(t)\|^2 dt \geq k_\tau^2 \|z_0\|^2, \quad \forall \ z_0 \in X. \tag{1} \]

We would like to reconstruct $z_0$ from the observation $y$, and we can use the following iterative algorithm to perform this task. Let $A^+ = A - \gamma C^*C$ (resp. $A^- = -A - \gamma C^*C$) be the generator of the exponentially stable $C_0$-semigroup $T^+$ (resp. $T^-$) for some $\gamma > 0$ (see for instance [3, Proposition 3.7]), and let $z_0^+ \in X$ be the initial guess (usually $z_0^+ = 0$). Then the algorithm reads: for all $n \in \mathbb{N}^*$

\[ \begin{align*}
\dot{z}_n^+(t) &= A^+ z_n^+(t) + \gamma C^*y(t), & \forall \ t \in (0, \tau), \\
z_n^+(0) &= z_0^+, \\
\dot{z}_n^-(t) &= -A^- z_n^-(t) - \gamma C^*y(t), & \forall \ t \in (0, \tau), \\
z_n^-(\tau) &= z_n^+(\tau), \quad \forall \ n \geq 1.
\end{align*} \tag{2} \tag{3} \]

Therefore, one can easily obtain (working on the errors $e^+ = z^+ - z$ and $e^- = z^- - z$) that $z_n^-(0)$ converges exponentially to $z_0$: there exists a constant $\alpha \in (0, 1)$, namely $\|T_{\tau}^- T_{\tau}^+\|_{\mathcal{L}(X)}$, such that

\[ \|z_n^-(0) - z_0\| \leq \alpha^n \|z_0^+ - z_0\|, \quad \forall \ n \leq 1. \]

Using the framework of well-posed linear systems and a result of stabilization by collocated feedback of Curtain and Weiss [1], we prove in [2] that without any observability assumption (such as (1)), the state space $X$ can be written as a direct sum of the observable and unobservable state, preserved by the algorithm (2)–(3). We obtain this result for both bounded and (some) unbounded observation operators $C$, and give a characterization of the exponential decay of the reconstruction error.

References


*Institut Supérieur de l’Aéronautique et de l’Espace, Département DMIA, 10 avenue Édouard Belin - B.P. 54 032, 31 055 Toulouse, Cedex 4, France. E-mail: ghislain.haine@isae.fr