Recovering the initial state of dynamical systems using observers

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The aim of this work is to show that the observer based algorithm proposed in [3] for solving initial data inverse problem allows to reconstruct the *observable part* of the initial data when observability assumption fails. This poster is an overview of the results obtained in [2].

Let X and Y be two Hilbert spaces, $A : \mathcal{D}(A) \to X$ a skew-adjoint operator, $C \in \mathcal{L}(X, Y)$ and $\tau > 0$. We consider the system

$$\begin{cases} \dot{z}(t) = Az(t), & \forall\, t \geq 0, \\ z(0) = z_0 \in X, \end{cases} \qquad y(t) = Cz(t), \qquad \forall\, t \in (0, \tau),$$

and we suppose that (A, C) is exactly observable in time τ, that is

$$\exists k_\tau > 0, \quad \int_0^\tau \|y(t)\|_Y^2 dt \geq k_\tau^2 \|z_0\|^2, \qquad \forall\, z_0 \in X. \tag{1}$$

We would like to reconstruct z_0 from the observation y, and we can use the following iterative algorithm to perform this task. Let $A^+ = A - \gamma C^* C$ (resp. $A^- = -A - \gamma C^* C$) be the generator of the exponentially stable C_0-semigroup $\mathbb{T}^+$ (resp. $\mathbb{T}^-$) for some $\gamma > 0$ (see for instance [3, Proposition 3.7]), and let $z_0^+ \in X$ be the initial guess (usually $z_0^+ = 0$). Then the algorithm reads: for all $n \in \mathbb{N}^*$

$$\begin{cases} \dot{z}_n^+(t) = A^+ z_n^+(t) + \gamma C^* y(t), & \forall\, t \in (0, \tau), \\ z_1^+(0) = z_0^+, \\ z_n^+(0) = z_{n-1}^-(0), & \forall\, n \geq 2, \end{cases} \tag{2}$$

$$\begin{cases} \dot{z}_n^-(t) = -A^- z_n^-(t) - \gamma C^* y(t), & \forall\, t \in (0, \tau), \\ z_n^-(\tau) = z_n^+(\tau), & \forall\, n \geq 1. \end{cases} \tag{3}$$

Therefore, one can easily obtain (working on the errors $e^+ = z^+ - z$ and $e^- = z^- - z$) that $z_n^-(0)$ converges exponentially to z_0: there exists a constant $\alpha \in (0, 1)$, namely $\|\mathbb{T}_\tau^- \mathbb{T}_\tau^+\|_{\mathcal{L}(X)}$, such that

$$\left\| z_n^-(0) - z_0 \right\| \leq \alpha^n \left\| z_0^+ - z_0 \right\|, \qquad \forall\, n \leq 1.$$

Using the framework of well-posed linear systems and a result of stabilization by collocated feedback of Curtain and Weiss [1], we prove in [2] that without any observability assumption (such as (1)), the state space X can be written as a direct sum of the observable and unobservable state, preserved by the algorithm (2)–(3). We obtain this result for both bounded and (some) unbounded observation operators C, and give a characterization of the exponential decay of the reconstruction error.

References

[1] R. F. Curtain and G. Weiss, *Exponential stabilization of well-posed systems by colocated feedback*, SIAM J. Control Optim., 45 (2006), pp. 273–297 (electronic).

[2] G. Haine, *Recovering the observable part of the initial data of an infinite-dimensional linear system with skew-adjoint operator*, In Revision.

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