Micromechanical modeling of brittle damage in composite materials: primary anisotropy, induced anisotropy and opening-closure effects

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Various degradation mechanisms

- Matrix cracking in SiC-SiC [Guillaumat 94]
- Fiber-matrix debonding in carbon-epoxy [Aussedat-Yahia 97]
- Fiber breakage in glass-epoxy [François 04]
**DAMAGE IN LAMINATED COMPOSITES**

- Influence on the macroscopic behavior:
  - non linearity
  - degradation of elastic properties
  - induced anisotropy
  - unilateral effect

*Carbon-epoxy [Goidescu 11]*
DAMAGE IN LAMINATED COMPOSITES

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Carbon-epoxy [Moffat 94]

SiC-SiC [Gasser 94]
CDM MODELING APPROACH

Thermodynamic of irreversible processes

1. Damage variables
2. Thermodynamic potential
3. Damage evolution law
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**Thermodynamic of irreversible processes**
- Micromechanics-based formulation

1. **Damage variables**
2. **Thermodynamic potential**
3. **Damage evolution law**

- rigorous, physical meaning
- arbitrary microcracks orientation
- interactions between:
  - initial and induced anisotropies
  - opening-closure effects

\[ E(P) = \int_\nu \varepsilon \, dV \]
**Assumptions**

- Small transformations, rate-independent and isothermal conditions
- 2D framework (fracture mechanics solutions)
- Initial orthotropic media
- Dilute concentration, no interaction
- Flat microcracks (unit normal $\mathbf{n}$), open or closed (no friction)

**General framework**

- Damage variables
- Thermodynamic potential – State laws
- Damage evolution law

**RVE = square cell area**

 Virgin material  

$$A = e_1 \otimes e_1, \quad C^0$$

 Microcracked material
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1 Damage variables

Discrete description

- Internal variables = damage densities $d_i$ of $N$ families of parallel microcracks
- Orientations $n_i$ regularly spaced

$$d = (d_i)_{i=1,N}$$
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Thermodynamic potential

Micromechanical direct approach – Goidescu et al. 12

- Extension of works by:
  - Andrieux et al. 86 (anisotropic context)
  - Gruescu 04 (closure effects)
- Based on jump displacements
  \[ \mathbf{u} = \mathbf{u}^+ - \mathbf{u}^- = [u_n] \mathbf{n} + [u_t] \mathbf{t} \]

- Formulation in strain (free energy):
  \[ W(\mathbf{E}, \beta, \gamma) \]
  - Open state: \( \beta \neq 0 \)
  - Closed state: \( \beta = 0 \)

\[ \beta = \mathcal{N} \int_{\omega} [u_n] dx \] (opening)
\[ \gamma = \mathcal{N} \int_{\omega} [u_t] dx \] (sliding)
2 Thermodynamic potential

Expression of the free energy

\[
W = W_0 + \sum_{i=1}^{N} d_i 
\]

\[
= c_1^{(i)} tr^2 E + c_2^{(i)} tr^2 (E \cdot A) + c_3^{(i)} tr E tr(E \cdot A) 
+ c_4^{(i)} tr^2 (E \cdot n_i \otimes n_i) + c_5^{(i)} tr E tr(E \cdot n_i \otimes n_i) + c_6^{(i)} tr(E \cdot E \cdot n_i \otimes n_i) 
+ c_7^{(i)} tr E tr(E \cdot n_i \otimes n_i \cdot A) + c_8^{(i)} tr(E \cdot A) tr(E \cdot n_i \otimes n_i) 
+ c_9^{(i)} tr(E \cdot A) tr(E \cdot n_i \otimes n_i \cdot A) + c_{10}^{(i)} tr(E \cdot n_i \otimes n_i) tr(E \cdot n_i \otimes n_i \cdot A) 
\]

with \( \{c_p^{(i)}(C^0, n_i, A)\}_{p=1,10} \)

- Closed-form expression accounting for interaction between initial and induced anisotropies
  - « isotropic » coupling (preserves initial orthotropy)
  - weak anisotropic coupling (similar to isotropic context)
  - strong anisotropic coupling
2 Thermodynamic potential

Unilateral effects

✓ open state: \( g(E, n_i, A) > 0 \)  \( \Rightarrow \) \( \left\{ c_p^{(i)}(C^0, n_i, A) \right\}_{p=1,10} = \left\{ c_p^{\text{open}}(C^0, n_i, A) \right\}_{p=1,10} \)

✓ closed state: \( g(E, n_i, A) \leq 0 \)  \( \Rightarrow \) \( \left\{ c_p^{(i)}(C^0, n_i, A) \right\}_{p=1,10} = \left\{ c_p^{\text{clos}}(C^0, n_i, A) \right\}_{p=1,10} \)

opening-closure criterion \( (\beta=0) \):

\[
g(E, n_i, A) = \eta_1 (n_i \cdot E \cdot n_i) + \eta_2 tr(E) + \eta_3 tr(E \cdot A) + \eta_4 tr(E \cdot n_i \otimes n_i \cdot A)
\]

with \( \left\{ \eta_p(C^0, n_i, A) \right\}_{p=1,4} \)

- Mathematical consistence: \( W \) of class \( C^1 \)
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3 Damage evolution law

Standard framework

- Systematic satisfaction of the 2nd principle of thermodynamics
- Dissipation potential (Marigo 85):

$$\mathcal{D}(\dot{d}_i, d_i) = \mathcal{G}(d_i) \dot{d}_i = \Phi_i$$

material strength damage rate

$$\mathcal{G}(d_i) = k_0 (1 + \eta d_i)$$

$$\dot{d}_i = \begin{cases} 0, & \text{si} \ f(F^{d_i}) = F^{d_i} - \mathcal{G}(d_i) \leq 0, \dot{f} < 0 \\ \frac{\dot{F}_{d_i}}{k_0 \eta}, & \text{si} \ f(F^{d_i}) = 0, \dot{f} = 0 \end{cases}$$

How? When?
**ELASTIC PROPERTIES**

**Elongation and volumetric moduli**

- one family of microcracks along principal axis

\[
L(m) = m \otimes m : C : m \otimes m
\]

\[
\kappa(m) = I : C : m \otimes m
\]

- « isotropic-like » effect (open and closed states)
- recovery mode identical to isotropic context (Welemane 02)

SiC-SiC [Aubard 92]

- virgin state (orthotropic)
- open state
- closed state

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Elongation and volumetric moduli

- one family of microcracks with arbitrary orientation

\[ L(m) = m \otimes m : C : m \otimes m \]

\[ \kappa(m) = I : C : m \otimes m \]

- strong anisotropic coupling (open and closed states)
- complex recovery mode
Influence of initial orthotropy

- stress-strain response
Influence of initial orthotropy

- density distribution

\[ d = (d_i)_{i=1,N} \]
Influence of unilateral effects

- Dissymetry between tension and compression
Influence of unilateral effects

opening-closure domains ($\delta=0^\circ$)

Elastic properties
Dissipative behavior

PREDICTIVE ABILITY – DISSIPATIVE BEHAVIOR
CONCLUSION AND PERSPECTIVES

- Micromechanics-based model of brittle damage in 2D-orthotropic materials
- Rigorous approach
  - Verification of mathematical and thermodynamical principles
- Account of main features of microcracking
  - interaction of initial and induced anisotropy
  - opening-closure effects for arbitrarily oriented microcracks
- More complete validation on experimental results
- Account of other dissipative mechanisms: dissipative sliding (closed microcracks), viscosity, plasticity
Thanks for your attention!

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