Direction detector for distributed targets in unknown noise and interference

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Adaptive detection of distributed radar targets in homogeneous Gaussian noise plus subspace interference is addressed. It is assumed that the actual steering vectors lie along a fixed and unknown direction of a preassigned and known subspace, while interfering signals are supposed to belong to a unknown subspace, with directions possibly varying from one resolution cell to another. The resulting detection problem is formulated in the framework of statistical hypothesis testing and solved using an ad hoc algorithm strongly related to the generalised likelihood ratio test. A performance analysis, carried out also in comparison to natural benchmarks, is presented.

Introduction: Adaptive radar detection of distributed targets requires proper strategies taking into account the nature of the targets as shown in [1,2]. In those papers returns from the target are modelled as signals known up to multiplicative factors, namely they are supposed to belong to a one-dimensional subspace of the observables. The case of returns modelled in terms of signals having the same direction which is not a priori known, but for the fact that it belongs to a given subspace of the observables, has also been considered [3,4]. Subsequently, in [5] it has been assumed that the target is also buried by interference belonging to a known subspace linearly independent of the signal subspace. In this Letter, we attack the detection problem addressed in [5], but assuming that the interference subspace is unknown (but for its rank). Since a closed form of the generalised likelihood ratio test (GLRT) is not available, we derive an ad hoc algorithm capable of effectively dealing with the considered scenario.

Problem formulation: Assume that an array of $N$ (possibly space-time) sensors probes $K_p$ range cells. Denote by $r_{k}, k \in \Omega_p = \{1, \ldots, K_p\}$, the $N$-dimensional vector containing returns from the $k$th cell. We want to decide between the $H_0$ hypothesis that the $r_k$ contain disturbance only and the $H_1$ hypothesis that they also contain signals backscattered from target scattering centres. We assume that the overall disturbance is the sum of coloured noise and deterministic interference. In symbols, the detection problem to be solved can be formulated in terms of the following binary hypothesis test:

$$
\begin{align*}
H_0: r_k &= 1_q_j + n_k, \quad k \in \Omega_p, \\
H_1: r_k &= a_k H_p + 1_q_j + n_k, \quad k \in \Omega_p, \\
&= n_k, \quad k \in \Omega_p
\end{align*}
$$

(1)

where the useful signals $a_k H_p$ and the interference signals $1_q_j$ are supposed to belong to the range spaces of the full-column-rank matrices $H \in \mathbb{C}^{N \times d}$ and $J \in \mathbb{C}^{N \times d}$, respectively, with $p \in \mathbb{C}^{N \times d}$, $q \in \mathbb{C}^{N \times d}$, and $q^r < N$. In signals the assumption that $1_q_j$ is not (but for its rank) $q$. The noise vectors $n_k, k \in \Omega_p$, are modelled as $N$-dimensional complex normal random vectors, i.e. $n_k \sim \mathcal{CN}(0, \mathbf{M})$, $k \in \Omega_p$, with $\mathbf{M}$ being in turn a positive-definite matrix; we assume that $M$ is unknown. We suppose that $K_p \geq N$ secondary data, $n_k, k \in \Omega_p = \{K_p + 1, \ldots, K_p + K_s\}$, containing noise only, are available and that these returns share the same statistical modelisation of the noise components in the primary data. Finally, we assume that the $n_k, k \in \Omega_p \cup \Omega_s$, are independent random vectors.

Detector design: Denote by $R = [r_1, r_2, \ldots] \in \mathbb{C}^{N \times K}$ the overall data matrix, with $r_k = [r_{k,1}, \ldots, r_{k,N}] \in \mathbb{C}^{N \times K}$ being the primary data matrix, $R_s = [r_{K_p+1}, \ldots, r_{K_p+K_s}] \in \mathbb{C}^{N \times K}$ the secondary data matrix, and $K = K_p + K_s$. Let us also introduce the following matrices $Q = [q_{1}, \ldots, q_{K_s}] \in \mathbb{C}^{K \times K_s}$ and $a = [a_1, \ldots, a_{K_s}] \in \mathbb{C}^{K \times 1}$. The GLRT for the above hypothesis testing problem can be written as

$$
\begin{align*}
\max_{p} \max_{a} \max_{J} \max_{Q, M} \frac{\mathcal{J}(R, p, a, J, Q, M)}{\mathcal{J}_0} \gamma
\end{align*}
$$

(2)

where $f(R, \gamma)$ is the probability density function of $R$ under the $H_1$, $\gamma > 0$, hypothesis and $\gamma$ the threshold value to be set in order to ensure $\delta_0$ of (2) has been computed in [6] and it is given by

$$
\frac{[K / e^\pi]^{1/2}}{[S / \|S\|_2^{1/2}]} \sum_{k=1}^{K} \left[1 + \lambda_k(S^{-1/2} R_p R_p^H S^{-1/2})\right]^{-1/2}
$$

(3)

where $S = R_s R_s^H$, $\| \cdot \|$ is the determinant of the matrix argument, $\lambda_k(\cdot)$ are the eigenvalues of the matrix argument arranged in decreasing order, and $\gamma$ denotes conjugate transpose. As to the numerator of (2), it is possible to reach the following intermediate step

$$
\max_{p} \max_{J} \frac{[K / e^\pi]^{1/2}}{[S / \|S\|_2^{1/2}]} \left|I_k + R_k R_k^H S^{-1/2}(I - P_{R_k}) S^{-1/2} R_p^H\right|^{-1/2}
$$

(4)

where $I_k$ denotes the identity matrix of dimension $n \in \mathbb{N}$ and $P_{R_k}$ is the projector onto the span of $R_k = S^{-1/2} H_p = [ H_p, J_s ]$ which is assumed to be full rank (i.e. $q + 1$). Combining (3) and (4) into (2) provides the following equivalent decision rule

$$
\min_{p} \left|I_k + R_k R_k^H S^{-1/2}(I - P_{R_k}) S^{-1/2} R_p^H\right|
$$

(5)

where $\lambda_k$ is the eigenvalue of $\lambda_k(S^{-1/2} R_p R_p^H S^{-1/2})$. In order to evaluate the thresholds $\mathcal{J}_0$ we solve the problem $\mathcal{J}_0$ with respect to $J$. For this reason, we choose to construct the set of candidate estimates of the vector $H_p$ as the $r$ eigenvectors of $E$ corresponding to the largest eigenvalues. To be quantitative, let $v_1, \ldots, v_r$ be the eigenvectors corresponding to the largest eigenvalues of $E$, for each $v_i$, $\ell = 1, \ldots, r$, we solve the problem

$$
\min_{p} \left|I_k + R_k R_k^H S^{-1/2}(I - P_{R_k}) S^{-1/2} R_p^H\right|
$$

(7)

with the matrix $E$ that is now given by $E_{v_i} = [v_i, J_s]$. The solution to such problem is known (see [6]) and it is given by

$$
\mathcal{J}'_{v_i} = \prod_{\ell=1}^{N} \left[1 + \lambda_i(S^{-1/2} R_p R_p^H S^{-1/2})\right]^{1/2}
$$

(8)

where $\mathcal{J}'_{v_i}$ is the projector onto the orthogonal complement of the span of $v_i$. Summarising, we propose to replace the statistic in (2) with the one computed as follows. First: extract the $r$ eigenvectors corresponding to the largest eigenvalues of $E$ (given by (6)) and denote them by $v_1, \ldots, v_r$; secondly, for each $v_i$, $\ell = 1, \ldots, r$, construct the statistic

$$
\Lambda_i(R) = \prod_{\ell=1}^{N} \left[1 + \lambda_i(S^{-1/2} R_p R_p^H S^{-1/2})\right]^{1/2}
$$

(9)

thirdly, compute the maximum of $\Lambda_i(R)$ with respect to $\ell = 1, \ldots, r$. The resulting detection detector for unknown $J$ will be denoted by DD-UJ.

Performance assessment: We carry out a Monte Carlo simulation to evaluate the performance of the proposed algorithm, also in comparison to the direction detector that assumes perfect knowledge of the interference subspace $J$ [5] (denoted by DD-J) and the subspace detector for unknown $J$ [6] (denoted by SD-J). In order to evaluate the thresholds necessary to ensure a preassigned value of $P_{fa}$ and the probabilities of detection $P_d$ we resort to 100 $P_d$ and 100 independent trials, respectively. We assume $N = 16$, $K_p = 32$, $r = 6$, $q = 2$, $P_{fa} = 10^{-4}$. Matrices $H$ and $J$ are randomly generated at each Monte Carlo run as matrices whose entries are independent and identically distributed.
(LLD) random variables taking on values ±1/√N with equal probability. Vector \( p \) is generated as \( p \sim CN(0,I) \). Moreover, at each run of the Monte Carlo simulation, we check the condition that the matrix [\( Hp \ J \)] is full rank (i.e. \( q + 1 \)). We also assume \( |\alpha_k| = |\alpha| \), \( k \in \Omega_p \), (\(| \cdot \) being the modulus of a complex number). The signal-to-noise ratio (SNR) is defined as \( \text{SNR} = \frac{K_p|\alpha|^2}{\text{tr}(M^(-1))} \) where \( \text{tr}(\cdot) \) is the trace of the matrix argument. In addition, the interference coefficients \( \alpha_k \), \( k \in \Omega_p \), are LLD and \( \alpha_k \sim CN(0, \sigma_k^2 I_p) \). The noise vectors \( n_q \), \( k = 1, \ldots, K_p \), are LLD and \( n_q \sim CN(0, \sigma_q^2) \) with the \((i,j)\)th element of \( M \) given by \( \sigma_q^2 0.95^{\min(i,j)} \), \( \sigma_q^2 = 1 \). Finally, the interference-to-noise ratio (INR), defined as \( \sigma_q^2/\sigma_n^2 \), is set to \( \text{INR} = 20 \, \text{dB} \). Figs. 1 and 2 report \( P_d \) against SNR for DD-uJ, DD-kJ, and SD-uJ for \( K_p = 9 \) and \( K_p = 32 \), respectively. From the Figures it is seen that, on one hand, the proposed DD-uJ can provide a performance better than that of the SD-uJ and, on the other hand, that its loss with respect to its non-adaptive counterpart (i.e. the DD-kJ) is limited. Such relations are emphasised as \( K_p \) increases; in fact, a higher value of \( K_p \) means that more data are available to estimate \( Hp \) and \( J \).

**Conclusion:** We have addressed adaptive detection of distributed targets in Gaussian noise plus interference, proposing a heuristic procedure to approximate the GLRT for the case that the interference subspace is unknown. The analysis has shown that the DD-uJ can guarantee a better performance than the more conventional SD-uJ and that it ‘converges’ to its non-adaptive counterpart, the DD-kJ, as the number of data increases.

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