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Appendix A

Eulerian-Lagrangian simulations with the code NTMIX-2Φ

A.1 The code NTMIX-2Φ

NTMIX-2Φ is a parallel code. It solves the compressible Navier-Stokes equations in three-dimensions and non-dimensional form. The temporal advancement uses third-order Runge-Kutta scheme and it uses a sixth-order compact difference scheme on cartesians grids (Lele (1992)). Direct Particle Simulation by means of Lagrangian tracking is performed through the Newton’s equations. The dispersed phase simulations use the same time advancement scheme that the gaseous phase. Two-way coupling between the gas and the dispersed phase is taken into account. The interpolation of the gaseous properties at the particle’s position is done by means of a third-order Lagrangian polynomial algorithm.

In the configuration of Chapter 5 boundary conditions are periodic in all directions. However, NTMIX-2Φ can use non-reflecting boundary conditions (Poinsot et al. (1992)) if needed. The calculations performed by Masi (2010) in the particle-laden temporal turbulent planar jet used a domain decomposition method (Vermorel (2003)) with MPI message passing protocol. More details about the code and its characteristics can be found in Masi (2010).

A.2 Projection algorithm

Eulerian fields are obtained from Lagrangian quantities by means of a projection algorithm, that projects the Lagrangian quantities into an Eulerian grid. Kaufmann & Moreau (2008) performed comparisons of different projection algorithms. The retained projector is a Gaussian filter (Eq. (A.1), Kaufmann & Moreau (2008), Moreau & Desjardins (2008)):

\[ w(x_p^{(k)} - x) = \frac{(2\Delta_p)^3}{\text{erf}(6^{3/2})} \left( \frac{6}{\pi\Delta_p^2} \right) \exp \left( -\frac{6|x_p^{(k)} - x|^2}{\Delta_p^2} \right), \]

where \( w(x_p^{(k)}) \) is the weight function, \( \Delta_p \) is the filter width or the size of the projection, which is taken equal to the grid spacing. \( x_p^{(k)} \) is the particle position and \( x \) is the coordinates of each grid node in the mesh.
The projected Eulerian particle density and velocity read:

\[
\tilde{n}_p(x, t) = \frac{1}{(2\Delta_p)^3} \sum_k w(x^{(k)}_p(t) - x), \tag{A.2}
\]

\[
\tilde{n}_p(x, t) \tilde{u}_p(x, t) = \frac{1}{(2\Delta_p)^3} \sum_k w(x^{(k)}_p(t) - x) \tilde{u}^{(k)}_p(t). \tag{A.3}
\]

Problems may appear in regions of the flow where the number of particles is not sufficient. This may lead to discontinuities in the Eulerian projected fields. This problem can be overcome with an interpolation procedure taking the values in the cells around the problematic point. In the simulations performed by Masi (2010), low-inertia cases presented a higher level of preferential concentration, leading to more empty zones in the flow and thus, the simulations of low Stokes numbers (between 0.1 and 0.5) were the most affected by this problem.
Appendix B

Gaseous phase validation for particle-laden slab. Additional graphs.

B.1 High turbulence case (HR_St1_#).

Figure B.1: Comparison of Eulerian and Lagrangian carrier phase velocities in X-direction. HR_St1_# case. (a) Mean velocity ($U_f$) and (b) RMS velocity times the fluid density ($\rho_f U_{f,RMS}$) at 5 and 40$t_{ref}$. Simulations performed with AVBP (—) and NTMIX-2Φ (–•–).
Figure B.2: Comparison of Eulerian and Lagrangian carrier phase velocities in Y-direction. HR_St1_# case. (a) Mean velocity ($V_f$) and (b) RMS velocity times the fluid density ($\rho_f V_{f,RMS}$) at 5 and 40$t_{ref}$. Simulations performed with AVBP (—) and NTMIX-2Φ (–•–).

Figure B.3: Comparison of Eulerian and Lagrangian carrier phase velocities in Z-direction. HR_St1_# case. (a) Mean velocity ($W_f$) and (b) RMS velocity times the fluid density ($\rho_f W_{f,RMS}$) at 5 and 40$t_{ref}$. Simulations performed with AVBP (—) and NTMIX-2Φ (–•–).
Figure B.4: Comparison of Eulerian and Lagrangian carrier phase turbulent kinetic energy ($q_f^2$) at 5 and 40$\tau_{ref}$. HR_St1_# case. Simulations performed with AVBP (—) and NTMIX-2Φ (–•–).
Gaseous phase validation for particle-laden slab. Additional graphs.
Appendix C

Particle-laden slab. Case LR_St1_#. Additional data.

Figure C.1: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle number density ($N_p$) at $5t_{ref}$, LR_St1_# case.
Figure C.2: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle number density ($N_p$) at $40t_{ref}$. LR_St1_# case.

Figure C.3: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle velocity magnitude ($U_p$) at $5t_{ref}$. LR_St1_# case.
Figure C.4: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle velocity magnitude ($U_p$) at $40t_{ref}$. LR_St1_# case.

Figure C.5: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian Random Uncorrelated Energy at $5t_{ref}$. LR_St1_# case.
Figure C.6: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian Random Uncorrelated Energy at 40t_{ref}. LR_St1_# case.

Figure C.7: Comparison of AV sensor levels at 5t_{ref}. LR_St1_# case.
Figure C.8: Comparison of AV sensor levels at $40t_{ref}$. LR_St1_# case.
Appendix D

Particle-laden slab. Case LR_St3_#. Additional data.

D.1 Dispersed phase statistics at $40t_{ref}$

Figure D.1: Comparison of Eulerian and Lagrangian (a) mean particle number density ($\langle \tilde{n}_p \rangle$, normalized by the initial particle number density at the center of the slab) and (b) mean particle velocity in X-direction ($\langle \tilde{u}_p \rangle_p$, normalized by the initial particle velocity in X-direction at the center of the slab) at $40t_{ref}$, LR_St3_# case.
Figure D.2: Comparison of Eulerian and Lagrangian (a) RMS particle number density ($\bar{n}_{p,RMS}$, normalized by the initial particle number density at the center of the slab) and (b) RMS particle velocity in X-direction ($\bar{u}_{p,RMS}$, normalized by the initial particle number density at the center of the slab) at 40$t_{ref}$, LR_St3_# case.

Figure D.3: Comparison of Eulerian and Lagrangian RMS particle segregation ($\bar{n}_{p}^2 / \bar{n}_{p}^2$) at 40$t_{ref}$, LR_St3_# case.
Figure D.4: Comparison of Eulerian and Lagrangian (a) mean Random Uncorrelated Energy \( \langle \delta q_{p}^2 \rangle_p \) and (b) mean mesoscopic \( \langle q_{p}^2 \rangle_p \) and mesoscopic \( \langle q_{meso}^2 \rangle_p \) particle energies at 40\( t_{ref} \). Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St3_# case.
Particle-laden slab. Case LR_St3_#. Additional data.

Figure D.5: Comparison of Eulerian and Lagrangian (a) mean production of RUM energy by shear components \(\langle p_{\text{RUM, shear}} \rangle_p \) and (b) mean productions of RUM energy by compression \(\langle p_{\text{RUM, compression}} \rangle_p \) at 40\(t_{\text{ref}}\). Normalized by the square of the initial particle velocity in X-direction at the center of the slab and the reference time \(t_{\text{ref}}\). LR_St3_# case.

Figure D.6: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor XX component \(\langle \delta R_{p,11}^* \rangle_p \) and (b) mean deviatoric RUM stress tensor XY component \(\langle \delta R_{p,12}^* \rangle_p \) at 40\(t_{\text{ref}}\). Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St3_# case.
Figure D.7: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor YY component \( <\delta \hat{\mathbf{R}}_{p,22}^* >_p \) and (b) mean deviatoric RUM stress tensor ZZ component \( <\delta \hat{\mathbf{R}}_{p,33}^* >_p \) at \( 40t_{ref} \). Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St3_# case.

Figure D.8: Mean Artificial Viscosity sensor activation at \( 40t_{ref} \). LR_St3_# case.
D.2 Dispersed phase statistics at 80$t_{ref}$

Figure D.9: Comparison of Eulerian and Lagrangian (a) mean particle number density ($<\bar{n}_p>$, normalized by the initial particle number density at the center of the slab) and (b) mean particle velocity in X-direction ($<\bar{u}_p>$, normalized by the initial particle velocity in X-direction at the center of the slab) at 80$t_{ref}$. LR_St3_# case.

Figure D.10: Comparison of Eulerian and Lagrangian (a) RMS particle number density ($<\bar{n}_{p,RMS}>$, normalized by the initial particle number density at the center of the slab) and (b) RMS particle velocity in X-direction ($<\bar{u}_{p,RMS}>$, normalized by the initial particle number density at the center of the slab) at 80$t_{ref}$. LR_St3_# case.
Figure D.11: Comparison of Eulerian and Lagrangian RMS particle segregation ($\bar{n}_p^2 / \langle \bar{n}_p \rangle^2$) at $80t_{ref}$. LR_St3_# case.
Figure D.12: Comparison of Eulerian and Lagrangian (a) mean Random Uncorrelated Energy ($\langle \delta \tilde{q}^2_p \rangle_p$) and (b) mean total ($\langle \tilde{q}^2_p \rangle_p$) and mesoscopic ($\langle \tilde{q}^2_p \rangle_p$) particle energies at $80t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St3_# case.
Figure D.13: Comparison of Eulerian and Lagrangian (a) mean production of RUM energy by shear components 
\(< P_{\text{Shear, RUM}} >_p \) and (b) mean productions of RUM energy by compression \(< P_{\text{Compression, RUM}} >_p \) at 80\( t_{ref} \).
Normalized by the square of the initial particle velocity in X-direction at the center of the slab and the reference
time (\( t_{ref} \)). LR_St3_# case.

Figure D.14: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor XX component \(< \delta \hat{R}_{p,11}^* >_p \)
and (b) mean deviatoric RUM stress tensor XY component \(< \delta \hat{R}_{p,12}^* >_p \) at 80\( t_{ref} \). Normalized by the square
of the initial particle velocity in X-direction at the center of the slab. LR_St3_# case.
Particle-laden slab. Case LR_St3_#. Additional data.

Figure D.15: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor YY component ($\langle \delta \tilde{R}_{p,22}^* \rangle_p$) and (b) mean deviatoric RUM stress tensor ZZ component ($\langle \delta \tilde{R}_{p,33}^* \rangle_p$) at 80$t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St3_# case.

Figure D.16: Mean Artificial Viscosity sensor activation at 80$t_{ref}$. LR_St3_# case.
D.3 Instantaneous fields at 40 and $80t_{\text{ref}}$

Figure D.17: Comparison of Lagrangian (NTMIX-2$\Phi$) and Eulerian particle number density ($N_p$) at $40t_{\text{ref}}$. LR_St3_# case.

Figure D.18: Comparison of Lagrangian (NTMIX-2$\Phi$) and Eulerian particle number density ($N_p$) at $80t_{\text{ref}}$. LR_St3_# case.
Figure D.19: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle velocity magnitude ($U_p$) at $40t_{ref}$. LR_St3_# case.

Figure D.20: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle velocity magnitude ($U_p$) at $80t_{ref}$. LR_St3_# case.
Particle-laden slab. Case LR_St3_#. Additional data.

Figure D.21: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian Random Uncorrelated Energy at 40\(t_{ref}\). LR_St3_# case.

Figure D.22: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian Random Uncorrelated Energy at 80\(t_{ref}\). LR_St3_# case.
Figure D.23: Comparison of AV sensor levels at $40t_{ref}$. LR_St3_# case.

Figure D.24: Comparison of AV sensor levels at $80t_{ref}$. LR_St3_# case.
Appendix E

Particle-laden slab. Case LR_St0.33_#. Additional data.

E.1 Dispersed phase statistics at $20t_{ref}$

![Comparison of Eulerian and Lagrangian (a) mean particle number density ($\bar{n}_p$, normalized by the initial particle number density at the center of the slab) and (b) mean particle velocity in X-direction ($\bar{u}_p$, normalized by the initial particle velocity in X-direction at the center of the slab) at $20t_{ref}$. LR_St033_# case.](image)

Figure E.1: Comparison of Eulerian and Lagrangian (a) mean particle number density ($\bar{n}_p$, normalized by the initial particle number density at the center of the slab) and (b) mean particle velocity in X-direction ($\bar{u}_p$, normalized by the initial particle velocity in X-direction at the center of the slab) at $20t_{ref}$. LR_St033_# case.
Figure E.2: Comparison of Eulerian and Lagrangian (a) RMS particle number density \(\langle \tilde{n}_p, \text{RMS} \rangle\), normalized by the initial particle number density at the center of the slab) and (b) RMS particle velocity in X-direction \(\langle \tilde{u}_p, \text{RMS} \rangle, \text{normalized by the initial particle number density at the center of the slab}\) at 20\(t_{ref}\). LR_St033_# case.

Figure E.3: Comparison of Eulerian and Lagrangian RMS particle segregation \(\langle \tilde{n}_p^2 \rangle / \langle \tilde{n}_p \rangle^2\) at 20\(t_{ref}\). LR_St033_# case.
Particle-laden slab. Case LR_St0.33_#. Additional data.

Figure E.4: Comparison of Eulerian and Lagrangian (a) mean Random Uncorrelated Energy ($\langle \delta \dot{\mathbf{u}}_p \rangle_p$) and (b) mean mesoscopic ($\langle \dot{q}_p^2 \rangle_p$) particle energies at $20t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St033_# case.

Figure E.5: Comparison of Eulerian and Lagrangian (a) mean production of RUM energy by shear components ($\langle P_{\text{shear}}^{\text{RUM}} \rangle_p$) and (b) mean productions of RUM energy by compression ($\langle P_{\text{Compression}}^{\text{RUM}} \rangle_p$) at $20t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab and the reference time ($t_{ref}$). LR_St033_# case.
Figure E.6: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor XX component ($\langle \delta \hat{R}_{p,11}^* >_p$) and (b) mean deviatoric RUM stress tensor XY component ($\langle \delta \hat{R}_{p,12}^* >_p$) at $20t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St033_# case.

Figure E.7: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor YY component ($\langle \delta \hat{R}_{p,22}^* >_p$) and (b) mean deviatoric RUM stress tensor ZZ component ($\langle \delta \hat{R}_{p,33}^* >_p$) at $20t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St033_# case.
E.2 Dispersed phase statistics at $40t_{ref}$

Figure E.8: Comparison of Eulerian and Lagrangian (a) mean particle number density ($\langle \hat{n}_p \rangle$, normalized by the initial particle number density at the center of the slab) and (b) mean particle velocity in X-direction ($\langle \hat{u}_p \rangle_p$, normalized by the initial particle velocity in X-direction at the center of the slab) at $40t_{ref}$. LR_St0.33_# case.

Figure E.9: Comparison of Eulerian and Lagrangian (a) RMS particle number density ($\langle \hat{n}_{p,RMS} \rangle$, normalized by the initial particle number density at the center of the slab) and (b) RMS particle velocity in X-direction ($\langle \hat{u}_{p,RMS} \rangle_p$, normalized by the initial particle number density at the center of the slab) at $40t_{ref}$. LR_St0.33_# case.
Figure E.10: Comparison of Eulerian and Lagrangian RMS particle segregation \( \left( \frac{\bar{n}_p^2}{\bar{n}_p^2} \right) \) at 40\( t_{ref} \). LR_St033_# case.

Figure E.11: Comparison of Eulerian and Lagrangian (a) mean Random Uncorrelated Energy \( \langle \delta q_{p}^2 \rangle_p \) and (b) mean mesoscopic \( \langle q_{p}^2 \rangle_p \) particle energies at 40\( t_{ref} \). Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St033_# case.
Figure E.12: Comparison of Eulerian and Lagrangian (a) mean production of RUM energy by shear components 
\(<\mathcal{P}^\text{Shear}_{\text{RUM}}>_p\) and (b) mean productions of RUM energy by compression \(<\mathcal{P}^\text{Compression}_{\text{RUM}}>_p\) at \(40t_{\text{ref}}\). Normalized by the square of the initial particle velocity in X-direction at the center of the slab and the reference time \(t_{\text{ref}}\). LR_St033_# case.

Figure E.13: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor XX component \(<\delta R^\ast_{p,11}>_p\) and (b) mean deviatoric RUM stress tensor XY component \(<\delta R^\ast_{p,12}>_p\) at \(40t_{\text{ref}}\). Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St033_# case.
Figure E.14: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor YY component ($\langle \delta R_{p,22}^* \rangle_p$) and (b) mean deviatoric RUM stress tensor ZZ component ($\langle \delta R_{p,33}^* \rangle_p$) at $40t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. LR_St033_# case.
E.3 Instantaneous fields at 40 and 20\(t_{\text{ref}}\)

Figure E.15: Comparison of Lagrangian (NTMIX-2\(\Phi\)) and Eulerian particle number density (\(N_p\)) at 20\(t_{\text{ref}}\). LR_St033_# case.

Figure E.16: Comparison of Lagrangian (NTMIX-2\(\Phi\)) and Eulerian particle number density (\(N_p\)) at 40\(t_{\text{ref}}\). LR_St033_# case.
Figure E.17: *Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle velocity magnitude (U_p) at 20t_{ref}. LR/St033_# case.*

Figure E.18: *Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle velocity magnitude (U_p) at 40t_{ref}. LR/St033_# case.*
Particle-laden slab. Case LR_St0.33_. Additional data.

Figure E.19: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian Random Uncorrelated Energy at 20t_{ref}, LR_St033_# case.

Figure E.20: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian Random Uncorrelated Energy at 40t_{ref}, LR_St033_# case.
Figure E.21: Comparison of AV sensor levels at $20t_{ref}$. LR_St033_# case.

Figure E.22: Comparison of AV sensor levels at $40t_{ref}$. LR_St033_# case.
Figure E.23: Comparison of AV sensor levels at 80\textsubscript{tref}. LR\_St0.33\_# case.
Particle-laden slab. Case LR_St0.33 #. Additional data.
Appendix F

Particle-laden slab. Case HR_St1_.#. Additional data.

F.1 Dispersed phase statistics at $5t_{ref}$

![Figure F.1: Comparison of Eulerian and Lagrangian (a) mean particle number density ($\langle \tilde{n}_p \rangle$, normalized by the initial particle number density at the center of the slab) and (b) mean particle velocity in X-direction ($\langle \tilde{u}_p \rangle_p$, normalized by the initial particle velocity in X-direction at the center of the slab) at $St_{ref}$. HR_St1_.# case.](image_url)
Figure F.2: Comparison of Eulerian and Lagrangian (a) RMS particle number density ($\langle \hat{n}_{p,RMS} \rangle$, normalized by the initial particle number density at the center of the slab) and (b) RMS particle velocity in X-direction ($\langle \hat{u}_{p,RMS} \rangle_p$, normalized by the initial particle number density at the center of the slab) at $St_{ref}$. HR_St1_# case.

Figure F.3: Comparison of Eulerian and Lagrangian RMS particle segregation ($\langle \hat{n}_p^2 \rangle / \langle \hat{n}_p \rangle^2$) at $St_{ref}$. HR_St1_# case.
Particle-laden slab. Case HR_St1_. Additional data.

Figure F.4: Comparison of Eulerian and Lagrangian (a) mean Random Uncorrelated Energy ($\langle \delta q_p \rangle_p$) and (b) mean mesoscopic ($\langle q_{\text{p}}^2 \rangle_p$) particle energies at $5t_{\text{ref}}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. HR_St1_# case.

Figure F.5: Comparison of Eulerian and Lagrangian (a) mean production of RUM energy by shear components ($\langle P_{\text{RUM}}^{\text{shear}} \rangle_p$) and (b) mean productions of RUM energy by compression ($\langle P_{\text{RUM}}^{\text{compression}} \rangle_p$) at $5t_{\text{ref}}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab and the reference time ($t_{\text{ref}}$). HR_St1_# case.
Figure F.6: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor XX component ($< \delta \tilde{R}_{p,11}^e >_p$) and (b) mean deviatoric RUM stress tensor XY component ($< \delta \tilde{R}_{p,12}^e >_p$) at $t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. HR_St1_# case.

Figure F.7: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor YY component ($< \delta \tilde{R}_{p,22}^e >_p$) and (b) mean deviatoric RUM stress tensor ZZ component ($< \delta \tilde{R}_{p,33}^e >_p$) at $t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. HR_St1_# case.
F.2 Dispersed phase statistics at 40\(t_{\text{ref}}\)

Figure F.8: Comparison of Eulerian and Lagrangian (a) mean particle number density \(\langle \tilde{n}_p \rangle\), normalized by the initial particle number density at the center of the slab) and (b) mean particle velocity in X-direction \(\langle \tilde{u}_p \rangle_p\), normalized by the initial particle velocity in X-direction at the center of the slab) at 40\(t_{\text{ref}}\). HR_St1_# case.

Figure F.9: Comparison of Eulerian and Lagrangian (a) RMS particle number density \(\langle \tilde{n}_{p,RMS} \rangle\), normalized by the initial particle number density at the center of the slab) and (b) RMS particle velocity in X-direction \(\langle \tilde{u}_{p,RMS} \rangle_p\), normalized by the initial particle number density at the center of the slab) at 40\(t_{\text{ref}}\). HR_St1_# case.
Figure F.10: Comparison of Eulerian and Lagrangian RMS particle segregation ($\langle \hat{n}_p^2 \rangle / \langle \hat{n}_p \rangle^2$) at $40t_{ref}$. HR_St1_# case.

Figure F.11: Comparison of Eulerian and Lagrangian (a) mean Random Uncorrelated Energy ($\langle \delta q^2_p \rangle_p$) and (b) mean mesoscopic ($\langle q^2_p \rangle_p$) particle energies at $40t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. HR_St1_# case.
Particle-laden slab. Case HR_St1_#. Additional data.  

Figure F.12: Comparison of Eulerian and Lagrangian (a) mean production of RUM energy by shear components ($<P_{\text{RUM}}^{\text{shear}}>_p$) and (b) mean productions of RUM energy by compression ($<P_{\text{RUM}}^{\text{compression}}>_p$) at $40t_{\text{ref}}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab and the reference time ($t_{\text{ref}}$). HR_St1_# case.

Figure F.13: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor XX component ($<\delta \bar{R}^{*}_{\text{p,11}}>_p$) and (b) mean deviatoric RUM stress tensor XY component ($<\delta \bar{R}^{*}_{\text{p,12}}>_p$) at $40t_{\text{ref}}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. HR_St1_# case.
Figure F.14: Comparison of Eulerian and Lagrangian (a) mean deviatoric RUM stress tensor YY component ($\langle \delta \hat{R}_{p,22}^* \rangle_p$) and (b) mean deviatoric RUM stress tensor ZZ component ($\langle \delta \hat{R}_{p,33}^* \rangle_p$) at $40t_{ref}$. Normalized by the square of the initial particle velocity in X-direction at the center of the slab. HR_St1_# case.
F.3 Instantaneous fields at 5 and 40$t_{ref}$

![Image showing comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle number density ($N_p$) at 5$t_{ref}$, HR_St1_# case.]

Figure F.15: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle number density ($N_p$) at 5$t_{ref}$. HR_St1_# case.

![Image showing comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle number density ($N_p$) at 40$t_{ref}$. HR_St1_# case.]

Figure F.16: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle number density ($N_p$) at 40$t_{ref}$. HR_St1_# case.
Figure F.17: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle velocity magnitude ($U_p$) at $5t_{ref}$. HR_St1_# case.

Figure F.18: Comparison of Lagrangian (NTMIX-2Φ) and Eulerian particle velocity magnitude ($U_p$) at $40t_{ref}$. HR_St1_# case.
Particle-laden slab. Case HR_St1_#. Additional data.

Figure F.19: Comparison of Lagrangian ($NTMIX-2\Phi$) and Eulerian Random Uncorrelated Energy at $5t_{ref}$. HR_St1_# case.

Figure F.20: Comparison of Lagrangian ($NTMIX-2\Phi$) and Eulerian Random Uncorrelated Energy at $40t_{ref}$. HR_St1_# case.
Figure F.21: Comparison of AV sensor levels at $5t_{ref}$, HR_St1_# case.

Figure F.22: Comparison of AV sensor levels at $40t_{ref}$, HR_St1_# case.
Figure F.23: Comparison of AV sensor levels at $70t_{ref}$, HR_St1_# case.
Particle-laden slab. Case HR_St1_#. Additional data.
Appendix G

MERCATO configuration. Additional graphs.

Figure G.1: Mean (a) and RMS (b) droplet diameter profiles.
Figure G.2: Mean (a) and RMS (b) liquid axial velocity profiles.

Figure G.3: Mean (a) and RMS (b) liquid radial velocity profiles.
Figure G.4: Mean (a) and RMS (b) liquid tangential velocity profiles.

Figure G.5: Mean liquid volume fraction (a) and liquid volume flux (b).