Extension of Subspace Identification to LPTV Systems: Application to Helicopters

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Abstract In this paper, we focus on extending the subspace identification to the class of linear periodically time-varying (LPTV) systems. The Lyapunov-Floquet transformation is first applied to the system's state-space model in order to get the monodromy matrix (MM) and, thus, a necessary and sufficient condition for system stability. Then, given two successive covariance-driven Hankel matrices, the MM matrix is extracted by some calculus of a simultaneous singular value decomposition (SVD) and a least square optimization. The method is illustrated by a simulation application to the model of a hinged-blades helicopter.

1 INTRODUCTION

Over the last forty decades, subspace identification methods have enjoyed of some popularity and numerous applications of these methods have emerged in civil engineering, aeronautics and many other fields. The stochastic subspace identification (SSI) consists in obtaining the modal parameters (natural frequencies, modal damping ratios and mode shapes) of a system subject to vibrations, by some geometric manipulations and projections of data given by sensors' measurements or input measurements. There exist, then, two types of identification algorithms: output-only and input-output algorithms. A comprehensive overview of different approaches of SSI can be found in [3] Unfortunately, the most part of research interest has been given to linear time-invariant (LTI) systems. In contrast, the literature on time-varying (TV) case is not abundant.
The attempts to extend these methods to TV can be categorized into two main classes, as outlined in [2]. The first class consists in identifying the considered system recursively using adaptive algorithms which is appropriate only for slowly-varying dynamics and when a priori information about the variation behavior is available . Whereas, the second class suggests to find a set of output sequences that have the same time-varying behavior [6], which makes it possible to apply a

classical time-invariant identification algorithm to these sequences. We suggest a new algorithm of identification in this paper, with three main features: first the a covariance-based method is proposed, which makes the computation non cumbersome. Second, we combine the method proposed to the theory of Floquet that gives a simple criterion for the stability of a periodic system. With the Lyapunov-Floquet transformation, the state transition matrix is replaced by the monodromy matrix. The third feature is to find out this monodromy matrix without any approximation (not up to a matrix of similarity).
The paper is organized as follows: in Section 2, a typical SSI algorithm is presented. Section 3 gives the essential elements of the Floquet theory. Then, Section 4 is devoted to the design of the new LPTV-extended method. Finally, in Section 5 the efficiency of the method is tested on a numeric simulation of a model of a helicopter with a hinged-blades rotor.

2 Covariance-driven Subspace Identification

In this section, we present a typical SSI algorithm [3]. Let consider the discrete-time state space model of a given system:

$$\begin{cases} x_{k+1} = F\, x_k + v_{k+1} \\ y_k = H\, x_k + w_k \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^r$ the output vector or the observation, $F \in \mathbb{R}^{n \times n}$ the state transition matrix and $H \in \mathbb{R}^{r \times n}$ the observation matrix. The vectors v and w are noises that are assumed to be white Gaussian and zero-mean.
We choose the parameters p and q, and we build the covariance-driven Hankel matrix:

$$\mathscr{H}_{p+1,q} = \begin{bmatrix} R_0 & R_1 & \cdots & R_{q-1} \\ R_1 & R_2 & \cdots & R_q \\ \vdots & \vdots & \vdots & \vdots \\ R_p & R_{p+1} & \cdots & R_{p+q-1} \end{bmatrix}$$

The covariances of the output data write $R_i = \mathbf{E}(y_k y_{k-i}^T)$, where $\mathbf{E}$ is the expectation operator.

Remark 1. : R_i can be approximated by $\hat{R}_i = \frac{1}{N}\sum_{k=i+1}^{N} y_k y_{k-i}^T$ if N is the number of output measurements we have ($N \gg 1$). Then, the estimated Hankel matrix: $\hat{\mathscr{H}}_{p+1,q} = Hank(\hat{R}_i) = \frac{1}{N}\sum_{k=q}^{N} \mathscr{Y}_k^{+} \mathscr{Y}_k^{-T}$
Where: $\mathscr{Y}_k^{+} = (y_k \cdots y_{k+p})^T$, $\mathscr{Y}_k^{-} = (y_k \cdots y_{k-q+1})^T$

Let $G = \mathbf{E}(x_k y_k)$ be the correlation between the state and the observation. $\mathscr{O}_{p+1} = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^p \end{bmatrix}$ and $\mathscr{C}_q = \begin{bmatrix} G \ FG \cdots \ FG^{q-1} \end{bmatrix}$ are the $(p+1)$th order observability matrix and the qth order controllability matrix.
The computation of the R_i's leads to the decomposition:

$$\mathscr{H}_{p+1,q} = \mathscr{O}_{p+1}\mathscr{C}_q$$

Therefore, the observability matrix $\mathscr{O}_{p+1}$ can be obtained with a thin SVD of the Hankel matrix $\mathscr{H}_{p+1,q}$ and its truncation at the desired model order n: $\mathscr{H}_{p+1,q} = \begin{bmatrix} U_1 \ U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1{}^T \\ V_2{}^T \end{bmatrix}$ and:

$$\mathscr{O}_{p+1} = U_1 \Sigma_1{}^{\frac{1}{2}}$$

Where Σ_1 contains the first n singular values.
The observation matrix H is extracted from the first r rows of the observability matrix $\mathscr{O}_{p+1}$. The state transition matrix F is obtained from a least squares solution of:

$$\mathscr{O}^{\uparrow}{}_{p+1} F = \mathscr{O}^{\downarrow}{}_{p+1}$$

Where:

$$\mathscr{O}^{\uparrow}{}_{p+1} = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{p-1} \end{bmatrix}, \ \mathscr{O}^{\downarrow}{}_{p+1} = \begin{bmatrix} HF \\ HF^2 \\ \vdots \\ HF^p \end{bmatrix}$$

We get the eigenstructure of the system (1) from the resolution of $det(F - I\lambda) = 0$ and $F\phi_\lambda = \lambda\phi_\lambda$, (λ, ϕ_λ) denote the eigenvalues and the eigenvectors of the system.

3 Floquet Theory

The theory of Floquet is a mathematical theory of ordinary differential equations (ODEs). Introduced in 1883, it is the first complete theory for the class of periodically time-varying systems. In this section, we briefly review some of its essential elements that are related to our study. More details can be found in [1].
Let consider the periodic differential system:

$$\dot{x}(t) = A(t)x(t) \tag{2}$$

Where $x \in \mathbb{R}^n$ is the state vector. The state transition matrix $A \in \mathbb{R}^{n\times n}$ is continuous in time (or at least, piecewise continuous) and periodic, of period $T > 0$. If an initial

condition $x(t_0) = x_0$ is fixed, a solution of (2) is guaranteed to exist.
Let $\Phi(t)$ be the matrix whose n columns are n linearly independent solutions of (2), $\Phi(t)$ is known as the *Fondumental Transition Matrix* (FTM). It has the properties below:

$$\dot{\Phi}(t) = A(t)\Phi(t),\ \Phi(t+T) = \Phi(t)\Phi(T), \forall t$$

3.1 Stability Analysis

The value of the fundemantal matrix at $t = T$, $Q = \Phi(T)$ is called the *Monodromy Matrix*.
Let $R = \frac{1}{T} log(Q)$. According to Floquet's theory, the dynamical system (2) is stable if and only if the eigenvalues of R are negative or, similarly, if the norm of the eigenvalues of Q is less than one.

3.2 Floquet Transformation

The Floquet transformation, also called the Lyapunov-Floquet transformation, gives an underlying *autonomous system* (a system with a constant state transition matrix) that is equivalent to the initial periodic system *i.e*: the transformation is invertible.
If we make the change of variable $x(t) = \Phi(t)e^{-Rt}z(t)$, the theory insures that:

$$\dot{z}(t) = Rz(t) \tag{3}$$

Now if the equation of observation of the considered system is: $y(t) = Cx(t)$, the equivalent equation for the new variable z is:

$$\dot{y}(t) = Cx(t) = C\Phi(t)e^{-Rt}z(t) \tag{4}$$

Remark 2. : It is easy to demonstrate that $\Phi(t+T)e^{-R(t+T)} = \Phi(t)e^{-Rt}$ using the fact that $R = \frac{1}{T} log(\Phi(T))$. Therefore, any periodic system can be transformed into an equivalent autonomous system with an equivalent periodic matrix of observation.

4 Subspace Identification for LPTV

We have presented above, a subspace method for time-invariant systems. In this section, we suggest an extension of this method to the linear periodically time-varying case. Let consider the periodic state-space system, of a period T:

$$\begin{cases} \dot{x}(t) = A(t)x(t) \\ y(t) = Cx(t) \end{cases} \tag{5}$$

As outlined in Section 3, an equivalent representation can be given by the Floquet's transformation:

$$\begin{cases} \dot{z}(t) = Rz(t) \\ y(t) = C\Phi(t)e^{-Rt}z(t) \end{cases} \tag{6}$$

Sampling at rate $\frac{1}{\tau}$ ($\tau > 0$) yields the discrete time model below:

$$\begin{cases} z_{k+1} = F\, z_k + v_{k+1} \\ \quad y_k = H_k\, z_k + w_k \end{cases}, \; z_k = z(k\tau) \tag{7}$$

The discrete state transition and observation matrices are: $F = e^{\tau R}$ and $H_k = C\Phi(k\tau)e^{-Rk\tau}$. The output and the state are corrupted by the noises v and w assumed to be white Gaussian and zero means.
If μ is an eigenvalue of the continuous system R, the correspondent eigenvalue λ of F is such that: $\lambda = e^{\tau\mu}$. According to the theory of Floquet, the continuous system is stable when the eigenvalues μ are negative. Therefore, the discrete system is stable when $\lambda < 1$ (or, if λ is complex, its real part is under 1). The discrete observation matrix H_k is periodic of period $T_d = \lfloor \frac{T}{\tau} \rfloor + 1$ ($\lfloor\ \rfloor$ denotes the floor operator).
We build the instantaneous product $\mathscr{Y}_k^{+}\mathscr{Y}_k^{-T}$:

$$\mathscr{Y}_k^{+}\mathscr{Y}_k^{-T} = \begin{bmatrix} H_k \\ H_{k+1}F \\ \vdots \\ H_{k+p}F^p \end{bmatrix} \begin{bmatrix} z_k y_k^T & F z_{k-1} y_{k-1}^T & \cdots \\ & \cdots & F z_{k-q+1} y_{k-q+1}^T \end{bmatrix}$$

Following the lines of [4], the ensembles $\left(z_k \; z_{k+T_d} \; z_{k+2T_d} \; z_{k+3T_d} \; \cdots \right)$ and $\left(y_k \; y_{k+T_d} \; y_{k+2T_d} \; y_{k+3T_d} \; \cdots \right)$ are time-invariant series. If we have, for example, NT_d output measurements ($N \gg 1$), the output-input covariance of the k-th invariant ensemble can be estimated by $G^{(k)} = \mathbf{E}(z_k y_k^T) = \frac{1}{N}\sum_{i=0}^{N-1} z_{k+iT_d} y_{k+iT_d}^T$.
A Hankel matrix is formed from these cross-products, at sample k:

$$\begin{aligned}\mathscr{H}_{p+1,q}^{(k)} &= \sum_{i=0}^{N-1} \mathscr{Y}_{k+iT_d}{}^{+}\mathscr{Y}_{k+iT_d}{}^{-T} \\ &= \sum_{i=0}^{N-1} \begin{bmatrix} H_{k+iT_d} \\ H_{k+1+iT_d}F \\ \vdots \\ H_{k+p+iT_d}F^p \end{bmatrix} \Big[z_{k+iT_d} y_{k+iT_d}^T \\ &\quad \cdots \quad F^{q-1} z_{k-q+1+iT_d} y_{k-q+1+iT_d}^T \Big]\end{aligned} \tag{8}$$

We know that the matrix of observation H is periodic. Then $H_{k+iT_d} = H_k, \;\; \forall i > 0$. Using this property, the matrix of Hankel writes:

$$\begin{aligned}\mathscr{H}_{p+1,q}^{(k)} &= \begin{bmatrix} H_k \\ H_{k+1}F \\ \vdots \\ H_{k+p}F^p \end{bmatrix} \Big[\sum_{i=0}^{N-1} z_{k+iT_d} y_{k+iT_d}^T \\ &\quad \cdots \quad F^{q-1} \sum_{i=0}^{N-1} z_{k-q+1+iT_d} y_{k-q+1+iT_d}^T \Big] \\ &= \begin{bmatrix} H_k \\ H_{k+1}F \\ \vdots \\ H_{k+p}F^p \end{bmatrix} \Big[G^{(k)} \quad \cdots \\ &\quad \cdots \quad F^{q-1} G^{(k-q+1)} \Big]\end{aligned}$$

As in the time-invariant case, the matrix of Hankel $\mathscr{H}_{p+1,q}^{(k)}$ can be decomposed in a product of two matrices:

$$\mathscr{H}_{p+1,q}^{(k)} = \mathscr{O}_{p+1}^{(k)} \mathscr{C}_q^{(k)} \tag{9}$$

Where:

$\mathscr{O}_{p+1}^{(k)} = \begin{bmatrix} H_k \\ H_{k+1}F \\ \vdots \\ H_{k+p}F^p \end{bmatrix}$ and $\mathscr{C}_q^{(k)} = \begin{bmatrix} G^{(k)} & \cdots & F^{q-1}G^{(k-q+1)} \end{bmatrix}$ The observability matrix $\mathscr{O}_{p+1}^{k}$ can be obtained as in Section 2 via an SVD of $\mathscr{H}_{p+1,q}^{(k)}$ and its truncation at the desired model order n. The observation matrix H_k, at k, is obtained from the first r rows of $\mathscr{O}_{p+1}^{(k)}$.

The extraction of the transition matrix from one observability matrix is no longer possible. In order to get F, we have to compute two successive matrices of Hankel. At $(k+1)$, $\mathscr{H}_{p+1,q}^{(k+1)}$ writes:

$$\mathscr{H}_{p+1,q}^{(k+1)} = \begin{bmatrix} H_{k+1} \\ H_{k+2}F \\ \vdots \\ H_{k+p+1}F^p \end{bmatrix} \Big[G^{(k+1)} \dots \\ \dots \quad F^{q-1}G^{(k-q+2)} \Big]$$

Now if we have the two successive observability matrices $\mathscr{O}_{p+1}^{(k)}$ and $\mathscr{O}_{p+1}^{(k+1)}$, the transition matrix F is -as in Section 2- the Moore-Penrose (least square) solution of:

$$\mathscr{O}\uparrow_{p+1}^{(k+1)} F = \mathscr{O}\downarrow_{p+1}^{(k)} \tag{10}$$

Where:

$$\mathscr{O}_{p+1}^{(k)} \downarrow = \begin{bmatrix} H_{k+1}F \\ H_{k+2}F^2 \\ \vdots \\ H_{k+p+1}F^p \end{bmatrix}, \mathscr{O}_{p+1}^{(k+1)} \uparrow = \begin{bmatrix} H_{k+1} \\ H_{k+2}F \\ \vdots \\ H_{k+p+1}F^{p-1} \end{bmatrix}$$

Remark 3. : Notice that the range space of $\mathscr{O}_{p+1}^{(k)}$ and $\mathscr{O}_{p+1}^{(k+1)}$ must be in the same basis. In fact, the left part of the singular value decomposition gives the observability matrix up to some similarity matrix *i.e* for example: in the time-invariant case, $\mathscr{O}_{p+1} = TU_1\Sigma_1^{\frac{1}{2}}$ where T is a unitary matrix (we have omitted this discussion until now, for simplicity). This matrix will be simplified by its inverse, with the Moore-Penrose resolution. And the obtained F corresponds, then, to the real transition matrix with no similarity.

In the case of time-variant systems, if the two successive SVD are done separately, we will have two different similarity matrices T_k and T_{k+1}. When it comes to the least square resolution, no simplification will be possible:

$$F = (\mathscr{O}\uparrow_{p+1}^{(k+1)})^{\dagger} T_{k+1}^{-1} T_k \mathscr{O}\downarrow_{p+1}^{(k)} \tag{11}$$

The symbol † denotes the Moore-Penrose pseudo-inverse.
None of the similarity matrices T_k or T_{k+1} is known. A solution for this is to force an SVD in the same basis for the two successive Matrices of Hankel:

$$\begin{bmatrix} \mathscr{H}_{p+1,q}^{(k)} \\ \mathscr{H}_{p+1,q}^{(k+1)} \end{bmatrix} = \begin{bmatrix} U_{1,k} & U_{2,k} \\ U_{1,k+1} & U_{2,k+1} \end{bmatrix} \begin{bmatrix} \Sigma_{1,k} & 0 \\ 0 & \Sigma_{2,k} \end{bmatrix} V_{k,k+1}^T \tag{12}$$

The size of covariance-driven matrices of Hankel is $(p+1)r \times qr$. Therefore, the complexity of the SVD does not depend on N, and the computation is not cumbersome.
The range spaces of the observability matrices, up to the same similarity matrix, are

$$\mathscr{O}_{p+1}^{(k)} = U_{1,k}\Sigma_{1,k}{}^{\frac{1}{2}}, \quad \mathscr{O}_{p+1}^{(k+1)} = U_{1,k+1}\Sigma_{1,k}{}^{\frac{1}{2}} \tag{13}$$

Once F and H_k are computed, the modal structure of the system (7) are obtained from the resolution of $det(F - I\lambda) = 0$, $F\phi_\lambda = \lambda\phi_\lambda$ and $\varphi_\lambda = H_k\phi_\lambda$.

Algorithm: To sum up, here are the steps of the suggested LPTV identification method:

- NT_d output-measurements are available. We set the data in ensembles of time-invariant series $\left(y_k\ y_{k+T_d}\ y_{k+2T_d}\ y_{k+3T_d}\ \cdots\right)$ for each sample k
- we compute, at k and $(k+1)$, the two successive matrices of Hankel $\mathscr{H}_{p+1,q}^{(k)}$ and $\mathscr{H}_{p+1,q}^{(k+1)}$ using the formula (8)
- we make a simultaneous singular value decomposition of those two matrices as in (12)
- we compute the range spaces of the observability matrices, at k and $(k+1)$, as in (13)
- we resolve the equation (10) using the Moore-Penrose pseudo-inverse
- H_k is obtained as the first r rows of $\mathscr{O}_{p+1}^{(k)}$
- given F and H_k, we compute the eigenstructure of the system

5 Application

In this section, we give an illustrative example in order to test the new suggested identification algorithm. The system we study herein is a helicopter with a hinged-blades rotor. The chosen modeling is the modeling for the analysis of the ground resonance phenomenon. We present the dynamical equations of motion that describe this phenomenon, and develop the linear periodically time-varying mechanical model in a similar way as described in [5], but adding damping to the structure this time.

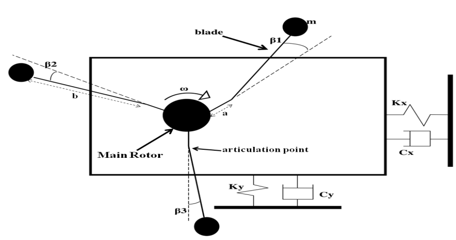

Fig. 1 Mechanical model of a helicopter with 3 blades

The helicopter's fuselage is considered to be a rigid body with mass M, attached to a flexible LG (landing gear) which is modeled by two springs K_x and K_y, and two viscous dampers C_x and C_y as illustrated in Fig. 1. The rotor spinning with a velocity ω, is articulated and the offset between the MR (main rotor) and each articulation is noted a. The blades are modeled by a concentrated mass m at a distance b of the articulation point. Torque stiffness and a viscous damping K_β and C_β are present into each articulation. The moment of inertia around the articulation point is I_z. The degrees of freedom are the lateral displacements of the fuselage x and y, and the out-of-phase angles $\beta_{k=1\cdots N_b}$, with N_b the number of blades.

Let $\mathscr{Z} = \begin{bmatrix} x \; y \; \beta_1 \; \cdots \; \beta_{N_b} \end{bmatrix}^T$. A linear model for the considered system under free vibrations is defined by:

$$\mathscr{M}(t)\ddot{\mathscr{Z}}(t) + \mathscr{C}(t)\dot{\mathscr{Z}}(t) + \mathscr{K}(t)\mathscr{Z}(t) = 0 \tag{14}$$

Where:

$$\mathscr{M}(t) = \begin{bmatrix} (M+Nm) & 0 & -mbsin(\omega t) & -mbsin(\omega t+\alpha) & \cdots & -mbsin(\omega t+(N-1)\alpha) \\ 0 & (M+Nm) & mbcos(\omega t) & mbcos(\omega t+\alpha) & \cdots & mbcos(\omega t+(N-1)\alpha) \\ -mbsin(\omega t) & mbcos(\omega t) & (mb^2+I_z) & 0 & \cdots & 0 \\ -mbsin(\omega t+\alpha) & mbcos(\omega t+\alpha) & 0 & (mb^2+I_z) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\mathscr{C}(t) = \begin{bmatrix} C_x & 0 & -2mb\omega cos(\omega t) & -2mb\omega cos(\omega t+\alpha) & \cdots & -2mb\omega cos(\omega t+(N-1)\alpha) \\ 0 & C_y & -2mb\omega sin(\omega t) & -2mb\omega sin(\omega t+\alpha) & \cdots & -2mb\omega sin(\omega t+(N-1)\alpha) \\ 0 & 0 & C_\beta & 0 & \cdots & 0 \\ 0 & 0 & 0 & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & & & \end{bmatrix}$$

and

$$\mathscr{K}(t) = \begin{bmatrix} K_x & 0 & mb\omega^2 sin(\omega t) & mb\omega^2 sin(\omega t+\alpha) & \cdots & mb\omega^2 sin(\omega t+(N-1)\alpha) \\ 0 & K_y & -mb\omega^2 cos(\omega t) & -mb\omega^2 cos(\omega t+\alpha) & \cdots & -mb\omega^2 cos(\omega t+(N-1)\alpha) \\ 0 & 0 & K_{\beta_0} & 0 & \cdots & 0 \\ 0 & 0 & 0 & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

The system can be written as in (1), with $x = \begin{bmatrix} \mathscr{Z}^T & \dot{\mathscr{Z}}^T \end{bmatrix}^T$ and $A(t) = \begin{bmatrix} 0 & \mathscr{I} \\ -\mathscr{M}(t)^{-1}\mathscr{K}(t) & -\mathscr{M}(t)^{-1}\mathscr{D}(t) \end{bmatrix}$, $A(t) = A(t+\frac{2\pi}{\omega})$. It is periodic of period $T = \frac{2\pi}{\omega}$.

The numerical values used for the application are reported in Fig. 2.

Structural variable	Value
m	$31.9Kg$
M	$2902.9Kg$
K_β	$200N/m$
K_x	$3200N/m$
K_y	$3200N/m$
C_β	$15Ns/m$
C_x	$300Ns/m$
C_y	$300Ns/m$
a	$0.2m$
b	$2.5m$
I_z	$259Kg/m^2$

Fig. 2 Structural properties for hinged-blades helicopter with 4 blades

The helicopter model is simulated for $\omega = 4rad.s^{-1}$. To get precise results, the subspace identification should be processed on very large dataset (to overcome the problem of bias due to the noise *i.e* $\frac{1}{N}\sum_k v_k \approx 0$ and $\frac{1}{N}\sum_k w_k \approx 0$). A total of $N = 80000$ data points are generated, with a sampling frequency of $50Hz$. The order of the system is known and is equal to $n = 12$.

The identification algorithm is applied to the data as explained in Section 4. The summary of the identified eigenvalues, and the difference between them and the real values, are given in Fig. 3. The differences in the obtained real part (damping ratios) are less than 0.1 %. For the imaginary part (frequencies), the differences are larger, but still not significant considering the large uncertainty on the estimates. The identification is more accurate when N is larger.

The most important variable the real part of the eigenvalues, because it is the criterion for deciding whether the system is stable or not. Therefore, the suggested method of identification is sufficiently accurate for the purpose of stability analysis.

Identified mode		Real mode	
damping	frequency	damping	frequency
0.9974	0.0766	0.9998	0.0200
0.9974	-0.0766	0.9998	-0.0200
0.9989	0.0702	0.9998	0.0214
0.9989	-0.0702	0.9998	-0.0214
0.9995	0.0556	0.9994	0.0359
0.9995	-0.0556	0.9994	-0.0359
0.9985	0.0130	0.9995	0.0330
0.9985	-0.0130	0.9995	-0.0330
0.9971	0.0310	0.9994	0.0335
0.9971	-0.0310	0.9994	-0.0335
0.9999	0.0471	0.9994	0.0335
0.9999	-0.0471	0.9994	-0.0335

Fig. 3 Identified modes vs. real modes

6 Conclusion

The problem of identification for linear periodically time-varying is addressed. An extension of the covariance-driven SSI algorithm is proposed and tested with simulation data. Future works encompasses a generalization of the subspace detection methods to the same class of systems.

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