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Optimal predictive control of water transport systems: Arrêt-Darré/Arros case study


ABSTRACT

This paper proposes the use of predictive optimal control as a suitable methodology to manage efficiently transport water networks. The predictive optimal controller is implemented using MPC control techniques. The Arrêt-Darré/Arros dam-river system located in the Southwest region of France is proposed as case study. A high-fidelity dynamic simulator based on the full Saint-Venant equations and able to reproduce this system is developed in MATLAB/SIMULINK to validate the performance of the developed predictive optimal control system. The control objective in the Arrêt-Darré/Arros dam-river system is to guarantee an ecological flow rate at a control point downstream of the Arrêt-Darré dam by controlling the outflow of this dam in spite of the unmeasured disturbances introduced by rainfalls incomings and farmer withdrawals.

Key words | decision support, optimal control, predictive control, water systems

INTRODUCTION

Water transport systems are generally collection of components (networks of rivers, streams, channels and dams) which may stock and carry water from sources to receiving environments (sea, river basin, etc). These systems in general are very large (hundreds or thousands of kilometres) and during the transport of water many measured or unmeasured disturbances may occurred: rainfalls, irrigation needs for agriculture purposes as a result of the intensive cultivation of farmland, water supply for town needs, etc. These disturbances influence the flow rate of the system at different geographical points. Therefore the objective of controlling the outflow of the detention tanks to assure a minimum ecological flow rate in the river is a main issue.

Advanced water transport systems can benefit from the incorporation of optimal predictive control of dams since they allow guaranteeing the control objectives satisfaction. On the one hand, dams store important volumes of water with its associated inlet and outlet gates installed, as well as other flow-control devices, such as diversion gates, weirs and pumping stations. On the other, a telemetry and supervisory control system (SCADA) is required.
The telemetry system contains rain-gauges distributed in several areas of the networks, as well as flow or level meter (limnimeters) and quality meters in the main points of the transport systems, which periodically send information to a central dispatch. The supervisory control system allows operators to monitor the transport network and command the flow-control elements. Dams are used to store water during rain periods and gradually release it when the network is not overloaded. Flow-diversion and detention gates must be actuated so as to assure the ecological flow in different points of the river.

Optimal predictive control in water systems has been successfully applied to water supply and distribution in several applications. In the particular case of open-flow systems, Foss et al. (1989) defined a control algorithm for river flow regulation which uses an adaptive prediction model for the unregulated run-off to the river. The prediction model can take both measurements of precipitation and flow in streams in the local catchment area into consideration. Because the river flow is a process with a relatively long time delay, this controller has significantly better performance than a traditional PID controller. Georges (1994) designed a decentralized adaptive controller for a water distribution system. The primary goal is to satisfy the water demand at each pumping station, while guaranteeing a minimum water level, overflow avoidance and minimum wastes of water, in each section and reservoir of the system. The approach is divided into two main stages: An on-line identification procedure based both on a multi-input multi-output parameterized model and Kalman filtering. A state-space predictive control scheme using a Kalman state estimator defined on the basis of the on-line identified model. De Albuquerque & Labadie (1997) proposed the predictive canal operation method providing optimal real-time control of irrigation canals with incorporation of current and forecasted demands. The objective function for the canal control problem incorporates four weighted terms in decreasing order of priority: satisfaction of current and forecasted water delivery requirements along the canal; avoidance of rapid variation in controls resulting in unacceptable wave propagation; maintenance of ideal target storage levels in each control pool; and stability of final storage in control pools at the end of the forecast period, assuring recovery of canal pool storage in the next operational cycle. Sawadogo et al. (2001) designed a predictive controller for decentralized control of delivery canal which is decomposed into single pools separated by gate. The pools are inter-dependent. The objective of each local controller is to maintain the downstream end water level of each pool at constant target value under external perturbation. Gómez et al. (2002) a canal control scheme within the context of decentralized control. The global scheme is composed by a set of gates operating along a series of pools. Each pool is seen as a system controlled by its upstream gate. In order to derive each individual controller, a predictive control algorithm is formulated, which computes the desired upstream gate discharge to achieve a desired setpoint downstream level. The predictive control algorithm is based on the so-called Muskingum model to describe the hydraulic dynamics of each pool. In order to achieve the desired upstream gate discharge, a proportional-integral (PI) control is adopted to manipulate the gate opening by using local feedback information. Wahlin (2004) tested the model predictive control (MPC) algorithm on a testbed (ASCE canal 1), and shown that its performance was similar to that of other proposed controllers. When there were no minimum gate movement constraints, MPC was fairly robust because the controller performance did not significantly degrade under untuned conditions. In the presence of minimum gate movement constraints, the water levels continually oscillate around the water level setpoint. Using the configuration presented in this paper, the feedforward portion of MPC does not perform as well as other proposed feedforward routines. This under-performance is related to the simplifications made by the underlying process model and not to MPC itself.

In this paper, a MPC controller is used to maintain the ecological flow along a river despite of measured (rain) or unmeasured disturbances of agricultural withdrawals. The paper shows that ecological flow can be guaranteed thanks to the constraint handling MPC capability. The case study to exemplify the proposed strategy is the Arrét-Darré/Arros dam-river system. This system located in the Southwest region of France. To assess the validity of the developed controller, a high fidelity dynamic MATLAB/SIMULINK simulator based on the full Saint-Venant equations has been developed and used to reproduce several scenarios in the proposed case study.
The structure of the paper is as follows: First, optimal predictive control and its application to water transport systems is revised. Then, the description of the case study dam-river system as well as the high-fidelity simulator developed is presented. Using identification techniques and historical data records available from this system, a control oriented model is developed either for the different river reaches as for the farmer demands and rainfall/runoff water incomings.

OPTIMAL PREDICTIVE CONTROL OF WATER TRANSPORT SYSTEMS

Control-oriented model

Complex non-linear hydrodynamics models based on Saint-Venant equations are very useful for off-line operations (calibration and simulation) of the water network, but for on-line computation purposes, such as the optimal predictive control, a simpler model structure must be selected with the following features, such as reproducibility of the network behaviour, simplicity, expandability and adaptability to on-line calibration from real data. In the literature several linear time-invariant (LTI) models for control have been proposed: the Hayami model (Litríco & Georges 1999a,b), the Muskingum model (Gómez et al. 2002), the IDZ model (Litríco & Fromion 2004) or black-box models identified using parameter estimation (Weyer 2001). In this paper, the modelling approach used to model the water transport system used as a case study is this last one.

Optimal predictive control strategy

The optimal control goals in transport water systems are generally concerned with environmental protection, in particular, ecological flow in all the points of the river. The objective of applying optimal control is to compute, ahead of time, feasible strategies for the actuators in the network which produce the best admissible states of the network, in terms of these objectives, during a certain horizon (Figure 1). The control period must be defined taking into account the telemetry system sampling time and the time constants of the actuators in the network. The optimization horizon must be selected considering the hydraulic time constants of water transport system. The computation of the optimal predictive control set-points to be applied at the actuators is based on model predictive control (MPC) (Camacho & Bordons 1999; Maciejowski 2001). In MPC, at each sampling time, starting at the current state, the following open-loop optimal control problem over a finite horizon $H_p$ is solved on-line:

\[
\min_{u(0|k), \ldots, u(H_p|k)} \sum_{i=0}^{H_p-1} \|y(k+i|k) - r(k+i|k)\|^2_{W_i(k)} \\
+ \sum_{i=1}^{H_p} \|\Delta u(k+i|k)\|^2_{W_{iu}(k)}
\]

subject to:

\[
\begin{align*}
x(i+1|k) &= Ax(i|k) + Bu(i|k), \quad i = 0, \ldots, H_p - 1 \\
y(i|k) &= Cx(i|k), \quad i = 0, \ldots, H_p \\
\min u_{\text{min}} &\leq u(i|k) \leq u_{\text{max}}, \quad i = 0, \ldots, H_p - 1 \\
\min y_{\text{min}} &\leq y(i+1|k) \leq y_{\text{max}}, \quad i = 0, \ldots, H_p - 1 \\
\Delta u_{\text{min}} &\leq \Delta u(i|k) \leq \Delta u_{\text{max}}, \quad i = 0, \ldots, H_p - 1 \\
\Delta u(i|k) &= 0, \quad i = m, \ldots, H_p - 1
\end{align*}
\]

As a result a virtual control input sequence $(u(0|k), \ldots, u(H_p|k))$ of present and future values, that optimizes an open-loop performance function using a prediction of the system evolution over the horizon $H_p$, is obtained. This prediction is performed assuming that disturbances (rain measures) and model parameters will keep constant during the horizon. Then, the receding horizon strategy is applied: only the first control input of sequence $(u(0|k))$ is actually applied to the system, until another sequence based on more recent data is computed (Figure 1). The same procedure is restarted at time $k+1$, using the new measurements obtained from sensors and the new model parameters obtained from the recursive parameter estimation algorithm that is working in parallel. The resulting controller belongs to the class called open-loop optimal-feedback control. As the name suggests, it is assumed that
feedback is used, but it is computed only on the basis of the
information available at the present time.

The cost function is the mathematical expression of the
water transport management goals. Usually, the main impor-
tant goal is to achieve an ecological flow in different points of
the river in order to preserve the life of flora and fauna.

WATER NETWORK CASE STUDY

Description

The case study concerns the Arrêt-Darré/Arros dam-river
system located in the Southwest region of France (Figure 2). The Arros River is about 100 km long with two gauging
stations at Villecomtal and Juillac. These enable to have
information for regulating the level of water. Along the river,
an ecological flow rate at least 2 m$^3$/s has to be maintained.
The only possible command is the gate of the Arrêt-Darré
dam (cap. 10,000,000 m$^3$). An intensive agricultural activity
takes place in the Arrow River valley, mainly, maize
cultures which consume a great amount of water. The irrigation is based on a set of numerous small pumping
units. A scheme of this case study is presented in Figure 2.
On the one hand five reach sections are considered: Gourgue-Mouflé, Mouflé-Confluence, Arrêt-Darré-
Confluence, Confluence-Villecomtal, Villecomtal-Juillac.
On the other hand a database containing: flow rates,
rainfalls, meteorological data, inflows and withdrawals for
several years are available. Currently, CACG (Compagnie
d’Aménagement des Coteaux de Gascogne) manages part of
the network shown in Figure 2 and provides data related
to geometry, transfer functions and withdrawals associated
with this area. Semboues and Anenos are the main with-
drawals points. At present, the flow rate control is a very
complex task especially during irrigation periods because
in certain places lack of water is often observed. Therefore,
external data (forecast) and estimation of withdrawals
are important issues to be addressed.
In this case study two main problems are considered: the first problem is related to the simulation of the river flow rate which is addressed by taking into account rainfalls, withdrawals and unknown or variable parameters depending on the variability of the flow rate profile. The second problem is linked to the use of additive information mainly rainfalls and withdrawals used for the calculation of a predictive command robust enough to handle the uncertainties of modelling and estimations. Finally, the predictive control objective for the Arrêt-Darré/Arros system is to meet the farmer demands at the points where water is extracted by using pumps while at the same time the minimum ecological flow rate is satisfied at given control river points.

High-fidelity simulator

To test the proposed optimal predictive controller to solve the Arrêt-Darré/Arros irrigation problem a high-fidelity dynamic simulator based on the well-known Saint-Venant equations that describe sub-critical, critical and supercritical flow has been developed and implemented. As it is well known, these equations allow the reproduction of effects such as inertial phenomena, backwater effects and the attenuation of wave flow through time and space. The implemented hydraulic model used in this case study is based on the 1-D Saint-Venant equations for the description of continuity (Equation 2) and momentum conservation (Equation 3) (Abbot & Minns 1998; Martin et al. 2006).

\[
\frac{\partial Q}{\partial t} + \frac{\partial A}{\partial x} = q
\]  
\[
\frac{\partial Q}{\partial t} + \frac{\partial (QU)}{\partial x} + gA \frac{\partial y}{\partial x} + gA(S_l - S_o) = 0
\]

where, \( Q \) denotes stream flow (m\(^3\)/s), \( A \) cross-sectional area (m\(^2\)), \( q \) lateral flow (inflow is positive, outflow is negative) per unit length (m\(^2\)/s), \( U \) average flow velocity (m/s), \( g \) gravitational acceleration (m/s\(^2\)), \( S_o \) bed slope (\( - \)), \( S_l \) friction slope (\( - \)), \( y \) channel depth (m), \( x \) longitudinal coordinate (m), and \( t \) time coordinate (s). The case study (Figure 2) is composed by four branches (5 sections). Each branch is discretized into a number of computational elements of equal lengths \( \Delta x \). The model computes flow rates and water levels for each element using the full non-linear hyperbolic partial differential Equations (2) and (3). The numerical method implemented is based on a four-point implicit finite difference scheme where the partial differential equations are replaced by the evaluation of functions at the discrete points. For each iteration step a sparse matrix is computed.

Description of the simulator-COM Architecture

The simulation platform (Simulator-Manager) has been designed and implemented based on the COM programming model and is composed by: a) a topology component, b) a hydraulic component and c) an interface component as shown in Figure 3. The topology component (Topology) denotes the database that represents the characteristics and contains all information (node definitions, link lengths, bed slopes, Manning coefficients, cross-sectional areas, etc.) of the network necessary for the hydraulic simulation. The hydraulic component (HDModel) is based on the resolution of the Saint-Venant equations that are applied to the network, and then for each time step a sparse matrix is solved. The platform provides the simulator manager: flow rate \( (Q) \), water level \( (y) \), velocity \( (U) \), etc., for each discretization mesh point. The interface component (Matlab–Simulink) implemented in MATLAB/SIMULINK (Figure 3) allows the user to interact with the topology and hydraulic components by defining the initial and boundary conditions (BCs), lateral flows (tributaries, withdrawals and rainfalls). Furthermore, this modular architecture allows: direct on line comparison between real data and simulated results, simple building and simulation of different scenarios, easy model calibration and realistic exploration of control strategies.

Simulated network

Figure 2 shows the network used for simulation. The studied area was divided into four main branches: Gourgue-Moulédois, Moulédois-Confluence, Confluence-Villecomtal, Villecomtal-Juillac, with different lengths and slopes depending on the characteristics of the river basin and on the location of gauging stations. Each branch is subdivided into discretization lengths for a total of 50 computational elements (51 nodes). Gourgue and Juillac...
nodes denote the upstream and downstream Boundary Conditions respectively. All nodes but the Boundary Conditions allow lateral flow (either positive or negative). The flow from the Arret-Darré Dam can be considered either a lateral flow or a main stream into the main channel of the river. In the first case a hydrograph is used for simulation, however, in the second case dynamic Boundary Conditions are used for simulation. The simulator was verified by comparing its outputs (many execution runs) with the ones given by commercial packages such as: MIKE-11 and QUAL-2E. The obtained errors among all profiles were not important and therefore, the implemented high-fidelity dynamic simulator was considered to be a verified tool. After the verification, the model was calibrated against experimental data obtained from the gauging station in the whole stretch of the study.

For the verification process the simulator was run with constant upstream Boundary Condition and the confluence point (Arret-Darré Dam) was considered to be a lateral inflow. Concerning Boundary Condition downstream the simulator was tested using two different strategies: either rating curve \(Q - h\) data at Juillac station or a dynamic computed level. In this case study the second strategy is been used. In this way at each iteration step, normal and critical heights are computed and the Boundary Condition downstream is properly selected to satisfy the gradually varied unsteady flow profile. Once the steady-state was reached, it was considered to be the initial condition for the calibration process and for the dynamic simulation. For the calibration process, October and November of 2005 were simulated. In this period of time there were no withdrawals and the rainfalls were no taken into account. A uniform Manning’s coefficient was applied throughout the river network and was subject to calibration. A range \((0.03–0.075)\) of Manning’s coefficient was used for the calibration process being \(n = 0.06\) the one that reproduced a good agreement. Therefore \(n = 0.06\) was set in the model.

**SIMULATION RESULTS**

**Control oriented model**

To implement the optimal predictive controller to allow optimally managing the Arret-Darré/Arros system, a control oriented model has been developed using a black box modeling approach and systems identification techniques. The whole control oriented-model includes: an upstream/downstream flow model of each river pool, a rainfall/runoff transformation model to predict the inflow rain water entering in to each river pool and a water demand model in each extraction.

**River pool models**

The Arret-Darré/Arros system can be decomposed in three pools: Gourgue-Moulédous, Arret-Darré-Villecomtal and Villecomtal-Juillac. Using input/output data obtained from the high-fidelity simulation and the MATLAB Identification Toolbox (in particular, the “ident” tool), the best model structure and parameters has been found for each pool. The three linear models obtained are presented in Table 1.
Rainfall-runoff prediction-models

The objective of a rainfall-runoff model is to characterize the behaviour of a particular watershed in case of rain. The relationship between rainfalls and runoffs is a very complex task. There are numerous kinds of models but the main use tank theories. Each tank represents a phenomenon. The number of used tank depends on the importance of the different phenomenon and there influence on the resulting river discharge. For the case study, a three-tank daily model has been developed. The first represents the soil behaviour influenced by the vegetal cover and its evapotranspiration (Smith 2000; Oudin 2006). The second is the hydrological tank corresponding to the infiltration phenomenon. The third is the pure runoff tank. This model is fed with Meteo France data enabling a good characterisation of the Arrêt-Darré/Arros inflows.

Farmer demand models

In order to forecast irrigation needs, several models have been developed. The first approach is based on the principle of soil’s water balance (Wallach et al. 2001; Popova et al. 2006). However, the tank model parameters are difficult to tune and need a lot of information. The second approach consists in stochastic simulation (Bergez et al. 2004) or behavioural simulation (Leenhardt et al. 2004a,b). In this case, the state of the plants and their evolution during a season in each exploitation are also difficult to estimate. For the case study, a behavioural model of farmer’s withdrawals has been implemented using the rules of good practices of irrigation. It allows for estimation of the irrigation quantities and the times of withdrawal.

Optimal predictive controller implementation

Using the MATLAB MPC toolbox and, in particular, the “mpctool”, an optimal predictive controller has been implemented. The MPC parameters have been configured in order to achieve the objectives of the control system by tuning the weights through several simulations using a “trial” and “error” approach. The MPC parameters selected according canal dynamics were: $T_s$ (Sampling Time) 1 hour, Prediction horizon 21 $T_s$ and Control horizon 7 $T_s$. These parameters has been tuned taking into account that prediction horizon captures the settling time of system, while control horizon has been selected to 1/3 of the prediction horizon as is standard done in MPC.

The MPC objective function has been tuned to maintain a set-point flow at Juillac control point. At the intermediate control points (as in Villecomtal) the ecological flow using soft-constraints, that is, a slack variable $e$ is introduced such that the minimum flow bound is relaxed as follows: $y_{\text{min}} - e \leq y(k)$. The value of the slack variable is chosen in the optimization process as the minimum value necessary to make the MPC optimization problem feasible.

MPC simulation results

Three different simulations scenarios have been prepared to show how the MPC controller performs corresponding to several farmer withdrawals and rainfall/runoff supplies affecting the river flow.

Scenario 1

This scenario shows how the MPC controller is able to track the set-point flow at Juillac control point while guaranteeing...
at an intermediate point (Villecomtal) the ecological flow constraint of 2 m$^3$/s. Figure 4 shows the evolution of flow at Juillac and Villecomtal, as well as, the control variable (flow released at the output of the Arrêt-Darré dam) as well as the upstream flow.

In order to show the benefits of using a MPC controller instead of a PID controller, the second plot in this figure shows the control results in case of using the PID. It can be seen that in this case although the flow at Juillac control point can be controlled (although with more oscillations than the MPC controller since the PID can not anticipate transport delays as MPC does thanks to the use of a prediction horizon). Moreover, the MPC controller can guarantee the ecological level at any intermediate point (Villecomtal, in this case), while the PID controller can not. This clearly shows the benefits of MPC regarding constraint handling. Finally, using the set-point anticipation capability of MPC (using the set-point look-ahead facility of the MPC control toolbox), the MPC controller can start reacting before the set-point effectively changes. This allows MPC a faster reaction in the set-point tracking compared to the PID controller.

**Scenario 2**

This scenario shows how the MPC controller is able to maintain the set-point flow at Juillac control point while guaranteeing at any intermediate point (Villecomtal in this case) the ecological flow constraint of 2 m$^3$/s when a rain incoming appears. Figure 5 shows the evolution of flow at Juillac and Villecomtal, as well as, the control variable (flow released at the output of the Arrêt-Darré dam) as well as the upstream flow.

**Scenario 3**

This scenario shows how the MPC controller is able to maintain the set-point flow at Juillac control point while guaranteeing at any intermediate point (Villecomtal in this case) the ecological flow constraint of 2 m$^3$/s when farmer withdrawals appear. Figure 6 shows the evolution of flow at Juillac and Villecomtal, as well as, the control variable (flow released at the output of the Arrêt-Darré dam) as well as the upstream flow.
CONCLUSIONS

This paper has proposed the use of predictive optimal control as a suitable methodology to manage efficiently transport water networks. The predictive optimal controller has been implemented using MPC control techniques. The Arrêt-Darré/Arros dam-river system located in the Southwest region of France has been proposed as case study. A high-fidelity dynamic simulator based on the full Saint-Venant equations and able to reproduce this system has been developed in MATLAB/SIMULINK to validate the performance of the developed predictive optimal control system. The control objective in the Arrêt-Darré/Arros dam-river system is to guarantee an ecological flow rate at a control point downstream of the Arrêt-Darré dam by controlling the outflow of this dam in spite of the unmeasured disturbances introduced by rainfalls and farmer demands. The control results obtained by the high-fidelity dynamic simulator in three typical scenarios are very satisfactory. Currently the application of this controller to the real system is in progress.

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