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High Speed of Learning in Financial Markets

Laurent GERMAIN *

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*Groupe de Finance ESC Toulouse, SUPAERO and Europlace Institute of Finance, I acknowledge financial support from the European Union grant ERBFMRXCT 960054 part of this article was written while I was a faculty at the London Business School. I thank Bruno Biais, Gabrielle Demange, Bertrand Jacquillat, Michel Levasseur, Eric Renault, Jean-Charles Rochet as well as seminar participants at Toulouse University, EEA, ESEM, AFFI, Oxford University and London Business School for their comments. I am especially undebted to Nour Meddahi. I thank Emmanuel Schertzer for helpful research assistance. Correspondence to Laurent Germain ESC Toulouse 20 Boulevard Lascrosses 31068 Toulouse Cedex, email l.germain@esc-toulouse.fr, tel + 33 561 294 943.
Abstract

High Speed of Learning in Financial Markets

We analyze the role of liquidity and collection of information in order to measure the speed of revelation of information during the preopening of order-driven markets. We extend Vives (1995) model to the case where risk averse traders receive a new private signal before each round of quotation of the preopening. We show that price discovery takes place at high speed which is consistent with the empirical studies of Biais, Hillion and Spatt (1999).
Résumé

Forte Vitesse d’Apprentissage sur les Marchés Financiers

1 Introduction

How do prices aggregate dispersed information in financial markets? This is one of the fundamental questions in finance and in the theory of market microstructure. In his seminal paper Grossman (1976) proposes a theoretical framework to understand the role of prices as a vector of information over one period of trading. In a dynamic setting, Kyle (1985) and Holden and Subrahmanyam (1992) among others study the revelation of information during the trading day. Vives (1995) focuses on the speed at which agents learn from prices during the preopening of order-driven markets. Indeed, the Paris Bourse, Toronto, Tokyo or Madrid stock exchanges have set up a preopening period in order to facilitate the price discovery after the arrival of new information which occurs after a period of non quotation (as overnight). In Paris, for example, during one hour and 45 minutes the participants of the market can send buy or sell orders and observe the indicative clearing price. At any time before the opening of the market the agents can cancel their orders. At the opening of the market a price is set and the market is cleared.

In Vives (1995), risk averse traders are endowed with a noisy private signal $s_i$ about the liquidation value of a risky asset $v$. We extend Vives (1995) to the case where agents receive a new signal before each round of quotation. As a matter of fact, the flow of information received and collected by the market participants during the preopening of financial markets is important as the agents exchange information with each other, and try to obtain as more information as possible in order to anticipate the opening price. Then, it is likely the case that their private information evolves over the time of the preopening. At each period, traders observe the indicative price as well as their private signal and revise
their demand accordingly. Asymptotically, for a large number of quotations, the learning process converges to the rational expectations equilibrium and traders learn \( v \), the liquidation value of the risky asset. The speed of learning is defined as the speed at which prices converge to the fully revealing rational expectations equilibrium - REE. In Vives (1995), the speed of revelation of information when the liquidity of the market is provided by market makers is \( \sqrt{n} \) where \( n \) is the number of round of quotations. But Biais, Hillion and Spatt (1999) in their empirical financial study on the Paris Bourse reject this speed and find a speed of revelation of information which is much higher and is equal to \( \sqrt{n^3} \). In this paper, we show that such high speed of revelation of information can be attributed to the collection of new private information.  

As in Vives (1995), to characterize the learning process we focus on two parameters:

- the reaction of the traders to their private information,
- and the speed at which prices reveal private information.

With market makers supplying the liquidity in the financial market (or alternatively competitive limit orders) - endogenous price function - our results are the following:

- the speed of learning is \( n^{\frac{3}{2}} \),
- and the reaction of the traders to their private information goes to infinity.

In fact, the liquidity effect described by Vives (1995) and the informational effect, that we highlight in this model play the same symmetric role. It is why the
revelation of information happens very quickly when traders keep on collecting
information and the liquidity is endogenously provided by market makers.

In the next section, we characterize the equilibrium conditions. In section 3 we
study the asymptotic properties of the market. Section 4 states some concluding
remarks.

2 The model: financial market with competitive
market makers or limit orders

At each round \( n \) there is a probability \( \delta_n \) that \( v \) be revealed to the traders and
a probability \( 1 - \delta_n \) that the tatonement process continues, those probabilities
are independent over time. As in Vives (1995) we assume that the sequence \(( \delta_n )\)
is non decreasing. The probability that the tatonement process lasts until stage
\( n \) is \((1 - \delta_n)\)...\((1 - \delta_1)\) which goes to 0 as the sequence \(( \delta_n )\) is non-decreasing.
As Biais, Hillion and Spatt (1999) notice this is different from the Paris Bourse
where everybody knows the time of the opening. Nevertheless, this corresponds
to certain financial markets as the Frankfurt Bourse where the opening time is
random\(^3\).

In this market there are three types of agents - it is a competitive version of
Kyle (1985):

• Market makers who set the price conditionally on past volumes \( \omega^n \),

\[
p_n = E(v|\omega^n),
\]

(1)

where \( \omega^n = (\omega_1, ..., \omega_{n-1}, \omega_n)\)\(^4\).
A continuum of informed agents in the interval \([0, 1]\) maximizing CARA utility \(U(x) = -e^{-\rho x}\) where \(\rho\) is the risk aversion coefficient\(^5\).

Each agent \(i\) at round \(n\) is endowed with a set of signals \((s^n_i = s_{i1}, ..., s_{in})\) with

\[s_{in} = v + \varepsilon_{in}\]

where \(v\) is the liquidation value of the normally distributed risky asset \(\tilde{v} \sim N(\bar{v}, \sigma^2_v)\) and \((\varepsilon_i)_i \sim N(0, \sigma^2_{\varepsilon_i})\) are independent normally distributed variables and independent to \(v\) too\(^6\).

Even if we consider that those private signals are independent over time, we will see that a correlation does not change any of the results and that one could consider the case of an AR(1) where \(\varepsilon_{in} = \gamma \varepsilon_{i(n-1)} + h_{in}\) and \((h_{in})_{i,n}\) variables are independent with \(v\) and \(h_{in} \sim N(0, \sigma^2_{h_{in}})\).

At each round, noise traders submit inelastic demand,

\[u_t \sim N(0, \sigma^2_u),\]

where \(u_t\) are independent with \((\varepsilon_i)_i\) and with \(v\).

Informed agents maximize their expected utility at each period conditional to \(I_{in}\) their information both private and public. Past prices \((p_{\tau}, \tau < n)\) and past volumes \((\omega_{\tau}, \tau < n)\) are public information. Therefore \(I_{in}\) is equal to \((s^n_i, p^{n-1})\) which are the past signals received from \(t = 1\) to \(t = n\) and past prices until \(n - 1\) \(^7\).

We define \(X_{in}\) as the quantity demanded by the \(i^{th}\) agent and \(\pi_{in} = X_{in}(v - p_n)\) his profit. Each agent solves the following program :

\[
\max_{X_{in}} \mathbb{E}(U(X_{in}(v - p_n))|I_{in}) \text{ with } U(x) = -\exp(-\rho x)
\]
Therefore, the optimal quantity is:

\[ X_{in}^* = \frac{E(v - p_n|I_{in})}{\rho \text{Var}(v - p_n|I_{in})} \] (2)

We focus on linear equilibria:

\[ X_{in} = a_{in}f_{in} + F_n(p^{n-1}) \]

where \( f_{in} \) is the OLS estimator of \( v \) over \( s^n_i \) (the mean of the private signals) and \( F_n \) is a deterministic function of past prices.

By the SLLN we have \( \int_0^1 f_{in}di = v \) and therefore the aggregate volume is:

\[ \omega_n = a_n v + F_n(p^{n-1}) + u_n \] (3)

The following proposition characterizes the linear equilibrium.

**Proposition 1** There is a unique linear equilibrium where:

\[ X_n(s^n, p^{n-1}) = a_n(f_{in} - p_{n-1}) \] (4)

and \( \omega_n \) the total volume is equal to:

\[ \int_0^1 X_n di + u_n = \int_0^1 a_n(f_{in} - p_{n-1}) di + u_n = a_n(v - p_{n-1}) + u_n \]

The equilibrium price function is:

\[ p_n = p_{n-1} + \lambda_n \omega_n \]

where \( a_n \) and \( \lambda_n \) are defined in the proof in the Appendix.
3  Asymptotic properties: financial market with competitive market makers or limit orders

The speed of revelation of information is captured by the asymptotic variance 
\[ \tau_n = \text{Var}(v|p^n)^{-1} \] which is the informativeness of prices\(^8\). When \( \tau_n = +\infty \), \( v \) is revealed.

We compute \( \nu \) such that \( n^{-\nu}\tau_n \) goes to a constant \( A\tau_\infty \) as \( n \) goes to infinity. \( A\tau_\infty \) is the asymptotic precision. By Amemya (1985) theorem:

\[
\sqrt{n^{-\nu}\tau_n} (p_n - v) \sim \mathcal{N}(0, (A\tau_\infty)^{-1})
\]

Vives (1995) shows that the presence of competitive market makers allow the traders to react increasingly to their private information. In this case the speed of convergence is \( \sqrt{n} \), and \( \sqrt{n}(p_n - v) \sim \mathcal{N}(0, \sigma_u^2 \rho^2 \sigma_\varepsilon^4) \). The following proposition characterizes the reaction of the traders to their signals and the speed of the price discovery process in our general set up.

**Proposition 2** The reaction \( (a_n) \) to private information is increasing and goes to infinity.

The speed of convergence is \( n^{\frac{3}{2}} \):

\[
n^{\frac{3}{2}}(P_n - v) \sim \mathcal{N}(0, \sigma_u^2 \rho^2 \sigma_\varepsilon^4)
\]

**Proof:** see Appendix

The speed of revelation of information is therefore the cube of that in Vives (1995). In fact, in our model informed agents have an unbiased estimator of \( v f_{in} \) on top of the equilibrium price which is too an unbiased estimator of \( v \). Informed agents can compute a new unbiased estimator the precision of which
is a weighted average of the two others $f_{in}$ and $y_n$. As a consequence, the precision of their information is higher than in the case where they had only one private signal over all the preopening period which accelerates the learning process.

Whereas in Vives (1995), the reaction to the private information ($a_n$) is increasing, and goes to a limit $(\rho \sigma^2 \epsilon)^{-1}$, in this model there is no limit to the reaction of the agents to their private information. $a_n$ is not bounded because the depth of the market $\lambda_n = \frac{a_n \tau_n}{\tau_n}$ is of the order of $n^{-3}$ whereas the order is $n^{-1}$ with only one private signal. This shows the role of competitive market makers or limit orders in inducing revelation of information in a financial market where there is an important collection of information. ($a_n$) goes to infinity does not mean that the demand of the informed goes to infinity. This demand is finite and $E|X|$ goes to $0^9$. Indeed, each agent is competitive and the reaction to their private information is infinite because at the limit they know $v$ perfectly.

4 Conclusion

We have shown that if the traders collect private information at each round of quotation during the preopening period of an order-driven market then more information is incorporated into prices and the speed of convergence towards the fully revealing REE is consistent with empirical studies. Indeed, in the case where the liquidity is endogenous the speed of learning predicted by our model is the same as the speed estimated by Biais, Hillion and Spatt (1999) that is to say $n^{3}$. 


APPENDIX

PROOF OF PROPOSITION 1

Using (3) the price function is given by:

\[ p_n = E(v|\omega^n) = E(v|a_1v + u_1 + F_1(p_0), ..., a_nv + u_n + F_n(p^{n-1})). \]

Let define \( z_t = a_tv + u_t \) and \( y_t \) the OLS estimator of \( v \) over \( (z^t) \). Vives (1995) shows that \( p_t = E(v|z^t) \) (consequently to the fact that the functions \( F(.) \) are deterministic) and so that \( p_t = E(v|y_t) \) by definition of \( y_t \).

Note that the main difference between the model presented in this article and the one described in Vives (1995) is that the trader receives sequential signals. We know that the information collected to evaluate \( v \) can be summarized by a single variable, which is the OLS estimator of \( v \) over the \( (s_i) \).

We note \( f_{in} \) this variable.

Therefore, \( I_{in} = (s_i^n, p^{n-1}) \) can be summarized by \( f_{in} \), the OLS estimator of \( s_i^n \), and \( y_{n-1} \), the OLS estimator of \( p^{n-1} \).

As in Vives (1995), we assume that the demand of a client is a linear function of his private information (summarized by \( f_{in} \)) and the public information (summarized by \( y_n \)), and we find that:

\[ X_{in} = a_n(f_{in} - p_{n-1}) \]

As in Vives (1995) we can write \( p_n = p_{n-1} + \lambda_n\omega_n \) where the aggregate volume \( \omega_n = a_n(v - p_{n-1}) + u_n \) and the depth of the market is \( \lambda_n = \frac{a_n\tau_n}{\tau_n} \) with \( \tau_n = \)
\( \tau_v + \tau_u A_n \) and \( A_n = \sum_{t=1}^{n} a_t^2 \) and we show that \( a_n \) is defined as the solution of the following cubic equation:

\[
F(a_n) = (\rho \sigma^2 \varepsilon^{-1} a_n - 1) \tau_{n-1} + \rho a_n^3 \frac{\tau_n}{\tau_n} = 0
\]

Moreover, we will show in the next proof that this maximum is unique.

\[ \text{QED} \]

**PROOF OF PROPOSITION 2**

From the proposition above, \( a_n \) and \( \tau_n \) can be seen as the roots of a cubic equation, more precisely:

\[
F(a_n, \tau_n) = 0
\]

Where \( F(a_n, \tau_n) = (\rho \sigma^2 n^{-1} a_n - 1) \tau_{n-1} + \rho a_n^3 \frac{\tau_n}{\tau_n} \).

On the interval \( \mathbb{R}^+ \times \mathbb{R}^+ \), the function \( F \) is \( C^1 \) and \( \frac{\partial F}{\partial a} \) is strictly positive.

Applying the theorem of implicit functions, at each point \((a^*_n, \tau^*_n)\) solution of the equation above, there exists a function \( \psi \) defined on an open interval \( I_{\tau^*_n} \) (containing \( \tau^*_n \)) which goes to an open interval \( I_{a^*_n} \) (containing \( a^*_n \)) such that:

- For every \( \tau \) included in \( I_{\tau^*_n} \), if \( a \) is solution of \( F(\tau, a) = 0 \), then \( a = \psi(\tau) \).
- \( \psi \) is \( C^1 \) and

\[
\psi' = -\frac{\partial F}{\partial \tau} \frac{\partial F}{\partial a}
\]

It is straightforward to show that on \( \mathbb{R}^+ \times \mathbb{R}^+ \) : \( \frac{\partial F}{\partial a} > 0 \) and \( \frac{\partial F}{\partial \tau} < 0 \). Hence, at every point \((a^*_n, \tau^*_n)\) the differential of the function \( \psi \) locally defined is always positive.
Therefore $a_n$ is an increasing function of $\tau_n$ and $a_n$ is unique. Moreover, $\tau_n$ is obviously increasing with $n$, therefore $(a_n)$ is increasing with $n$ too. Hence, as $\tau_n > na_1$, the informativeness of prices, goes to $\infty$. Moreover we have:

$$a_n\rho(\sigma^2 \epsilon n^{-1} + a_n \lambda \tau_{n-1}^{-1}) = 1$$

If $a_n$ goes to a finite limit, the equation above is not verified when $n$ goes to $\infty$ as the first term goes to 0. Therefore, as $(a_n)$ increases, it goes to $\infty$.

Let $f$ be a function of $n$ such that $a_n = n(\rho \sigma^2 \epsilon)^{-1} + f(n)$. By substituting $a_n$ to this new expression in the equation above, it can easily be shown that

$$\lim_{n \to \infty} f(n)/n = 0$$

Therefore,

$$a_n \sim n(\rho \sigma^2 \epsilon)^{-1}$$

Now let determine the speed of convergence.

It is straightforward that the standard deviation of the estimator $y_n$ is equal to $\sigma_u \sum_{t=1}^{n} a_t / \sum_{t=1}^{n} a_t^2$.

After computing the equivalent of the numerator and the denominator of this expression when $n$ goes to $\infty$ we find that the standard deviation of $y_n$ is:

$$\sigma_{y_n} \sim (\sqrt{3} \sigma^2 \epsilon)$$

To apply the general central limit theorem (see Amemiya (1985)) we have to check that the following condition is fulfilled:

$$\lim_{n \to \infty} \frac{\text{Max} \{a_k | k = 1...n\}}{A_n} = 0$$

We know that $a_n \sim n(\rho \sigma^2 \epsilon)^{-1}$ and $A_n = \sum_{t=1}^{n} a_t^2 \sim n^{3/2}(1/\rho \sigma^2 \epsilon)^2$. Because $a_n$ is increasing

$$\frac{\text{Max} \{a_k | k = 1...n\}}{A_n} = \frac{a_n}{A_n}$$
which goes to zero as n goes to $\infty$

Hence, we use the theorem Amemiya (1985) and we have:

$$\sigma_{y_n}^{-1} \left( \sum_{t=1}^{t=n} a_t^2 \right)^{\frac{1}{2}} (y_n - f_{in}) \sim N(0, 1)$$

and that $\sqrt{n^3} (y_n - f_{in}) \sim N(0, 3\rho^2 \sigma_u^2 \sigma^4)$

QED
Footnotes

1 This has been related to us by market practitioneers.

2 Biais and Martinez (1999) study the preopening in Frankfurt and in Paris.

3 It would be of interest to measure the speed of learning in Frankfurt as well in order to see if this difference could change the result.

4 In this paper we consider competitive market makers or alternatively competition between traders who place limit orders. As Biais, Hillion, Spatt (1995) notice “the conditional probability that investors place limit orders (rather than hitting the quotes) is larger when the bid-ask spread is larger or the order book is thin.”


6 One could have considered that at each round of quotation the traders receive a new signal with a positive probability. This would have diminished the speed of convergence. Nevertheless we also consider the case where the private signals follow an AR(1) and show that any correlation would have changed our results.

7 As prices and volumes are observationally equivalent it is not necessary to include the past volumes in the information set.

8 $\tau_i$ is the inverse of the variance $\sigma^2_i$.

9 It is straightforward that $E|X_n| = (2/\pi)^{1/2} a_n \tau_{n-1}$ where $\frac{a_n}{\tau_v + \tau_u A_{n-1}}$ goes to 0.
Consequently, there is a very simple isomorphism between the two models.

In fact, in this proposition, it is sufficient to substitute $f_i$ to $s_i$ in Vives (1995) to derive the new results of this proof.
References


