Incentive Compatible Contracts for the Sale of Information

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Abstract

An informed financial institution can trade private information and also sell it to clients through a managed fund. To incentivize the informed agent to trade in the interest of her client, the optimal contract requires that she be compensated as an increasing function of the profits of the fund. The optimal contract is also designed to limit the aggressiveness of the sum of the fund’s trade and the proprietary trade. This reduces information revelation, and thus leads to greater overall trading profits than if the informed agent only conducts proprietary trades.
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1 Introduction

An important dimension of the workings of financial markets is the agency relation between investors and such financial intermediaries as brokers, securities houses and investment banks. These financial institutions are likely to have private information about the value of the securities they trade. In their dealings with their customers it is therefore a crucial issue whether they retain this information private or communicate it to their clients. One way to communicate information is to sell or transmit it to clients directly, as in newsletters and investment advisory services or by issuing buy or sell recommendations.\footnote{Empirical evidence in Michaely and Womack (1996) highlights the conflict of interests faced by intermediaries. On the one hand they have incentives to transmit reliable information for reputation considerations. On the other hand they may also pursue other} The informed party can also sell information indirectly,
by setting up a fund, selling its shares to clients and managing it on the basis of its private information.\textsuperscript{2} This paper analyzes optimal, incentive compatible contracts governing the indirect sale of private information by financial intermediaries to their customers.

Admati and Pfleiderer (1986) analyze direct sales of information, in which the informed party does not trade but sells a signal to his customer. They show that the informed party adds noise to the signal he sells in order to reduce the sensitivity of prices to trades. \textsuperscript{3} Admati and Pfleiderer (1986) assume that the precision of the signal is contractible, and analyze contracts whereby the fees paid to the seller of information are a function of this precision. As noted by Brennan and Chordia (1993), it is unclear however, how contracts can be written on the precision of the signal.\textsuperscript{4}

Brennan and Chordia (1993) study the case where the seller of information is not compensated as a function of the precision of the signal but by goals, such as serving the interests of their corporate customers, giving them incentives to distort information.

\textsuperscript{2}Admati and Pfleiderer (1990) analyze the difference between direct and indirect sales of information. Dow and Gorton (1997) analyze the agency relation between the investor and the agent managing his funds and the incentives of the agent to appropriately use his private information.

\textsuperscript{3}See Naik (1993) for a generalization of this analysis in an intertemporal framework.

\textsuperscript{4}A related issue is whether the intermediary does really have private information or just pretends he does. Bhattacharya and Pfleiderer (1985) and Allen (1990) analyze this reliability problem.
brokerage commissions, which are a function of the trading volume of the information purchaser. In their analysis, however, the information seller does not trade on his own account. Once one considers the possibility that the investment bank or securities firm selling the information also trades for its own account, additional issues arise. Indeed, the informed party could be tempted to transmit noisy or even wrong information to his clients, since this would make the aggregate order flow noisy, and thus would reduce the market impact of his own trades. This possibility is discussed in Admati and Pfleiderer (1986) who write (page 408) 5:

“If the seller traded, his incentives to reveal truthfully the information he promised would be severely distorted”

One approach to this problem is to introduce reputation effects. Benabou and Laroque (1992) assume that the informed party can be honest or opportunistic. Opportunistic agents face a trade-off between the one-shot trading profit obtained when disseminating false information and thus manipulating prices, and the reputational loss caused by such lies.

We take an alternative route, by analyzing how, in a one shot model, optimal contracts can mitigate the conflict of interests arising between the seller.

5In Admati and Pfleiderer (1988) the fund manager can buy shares of the fund for his own account but there is no incentive problems since the fund manager is assumed not to trade the shares outside the fund.
of information and his customer and give rise to optimal trading strategies. We show that these optimal trading strategies can involve a noisy component, similar to the noise component optimally injected in the signal sold by informed agents in Admati and Pfleiderer (1986).

In the next section, we present the simple trading game we analyze. There is a risk–neutral strategic agent (securities house or investment bank), who observes a private signal about the final value of the risky asset. She can trade on it in a financial market where risk neutral competitive market makers supply liquidity at prices equal to the expectation of the final value of the asset, conditional on the order flow. By mimicking the orders placed by uninformed liquidity traders the strategic informed agent reduces information revelation.

The impact of orders on prices depends on the beliefs of the market makers about the strategy of the informed agent. To reduce this impact, the informed agents might find it desirable to pre-commit to a relatively unaggressive trading strategy. Such a strategy would not be optimal ex–post, given the realization of the signal and the beliefs of the market makers, when the informed agent would like to trade rather aggressively. Yet, it could enhance the ex–ante expected profits of the informed agent, by inducing market makers to limit the responsiveness of their prices to the order flow. In Section 2 we characterize conditions under which the ability to commit to a strategy leads to strictly greater expected profit, and in particular analyze
how this strategy requires injecting noise in the trades of the informed agent.\textsuperscript{6}

In Section 3, we turn to the analysis of the case where the informed agent can engage in indirect sales of information to customers, by setting up an investment fund that trades on the basis of the private information.\textsuperscript{7} At the same time she also can conduct proprietary trades. By this we mean that the securities firm (or its subsidiary or parent company) can trade in the same markets as the fund, rather than that the manager is trading for his personal account while he manages the fund. The customers of the informed financial intermediary are uninformed, rational and risk neutral investors who can purchase (indirectly) the private information by investing in the fund.\textsuperscript{8} The compensation of the informed party, and the net profits of the customer, are defined in a contract, contingent on the funds’ profits.\textsuperscript{9} The monopolistic informed agent designs the contract to maximize expected profit subject to

\textsuperscript{6}In the linear normal context analyzed by Kyle (1985) it turns out that even if the informed agent can precommit to a trading strategy he finds it optimal not to add noise to his trades. In the present paper we present other distributions for which this is not the case.

\textsuperscript{7}Fishman and Hagerty (1995) analyze competition between informed agents in the case of direct sales of information. Germain (1997) analyze competition between sellers of information who also conduct proprietary trades.

\textsuperscript{8}We consider a simplified case whereby, in contrast with Admati Pfleiderer (1990), customers have no private information.

\textsuperscript{9}This differs from Admati and Pfleiderer (1986, 1990) since we do not assume that the fees are directly contingent on the precision of the signal.
individual rationality and incentive compatibility conditions. In this set-up, the client can be seen as the principal, to the extent that he delegates the management of his investments to the informed agent. We study the optimal contract designed to regulate this agency relation. The contract determines the compensation of the fund manager as a function of the fund’s profits, or, to put it differently, how the profits of the fund are split between the client and the manager.

We find that, in order to align the interests of the seller and the buyer of information, the compensation of the informed party is increasing in the fund’s profits. This gives incentives to the informed with good (resp. bad) news to buy (resp. sell) on behalf of the fund. The optimal contract, and the corresponding compensation scheme for the fund manager, are also designed to induce the informed agent to optimally introduce noise in the overall informed trades, equal to the sum of the fund’s trades and the proprietary trades of the informed agent. This optimally noisy trading strategy reduces information revelation, and maximizes the total profitability of informed trades. In fact, through the information sales contract, the informed agent can manipulate her own incentives in the subsequent trading game, and thus enhance her ability to commit to a relatively unaggressive trading strategy.

We show that the optimal contract can be implemented by a transfer function involving penalties for the fund manager if the fund profits are
non-positive and an attractive compensation if the fund profits are strictly positive.\textsuperscript{10} This is in the same spirit as the existing hedge fund managers contracts described in Fung and Hsieh (1997):

\begin{quote}
"Hedge fund managers derive a great deal of their compensation from incentive fees, which are paid only when these managers make a positive return. In addition a high water mark feature in their incentives contracts requires them to make up all previous losses before an incentive fee is paid."
\end{quote}

Finally, in Section 4 we offer some concluding comments. The proofs are in the appendix.

\section{The case where there are no sales of information}

\subsection{Model}

Consider the market for a risky asset. There are three possible realizations of its final liquidation value $v$: $u$, $m$, and $d$. For simplicity, assume the distribution is symmetric around the medium state. More precisely, $u = m + \epsilon$ \textsuperscript{10}Das and Sundaram (2000) also analyze theoretically incentive fees for hedge fund managers.
and occurs with probability $\frac{1}{3} + \frac{\delta}{2}$, $m$ occurs with probability $\frac{1}{3} - \delta$, and $d = m - \epsilon$ and occurs with probability $\frac{1}{3} + \frac{\delta}{2}$, with $\delta \in [0, \frac{1}{3}]$. For simplicity the risk free rate is normalized to 0.

There are three types of agents. Liquidity traders submit one of three equiprobable market orders $\omega_l \in \{L, 0, -L\}$ independent of $v$. The monopolistic risk neutral informed trader perfectly observes the realization of $v$ in advance (but not the liquidity trade) and submits a market order $X(v)$. Risk neutral competitive market makers observe the pair of orders submitted by the informed and liquidity traders, but cannot ascertain which of the two orders comes from which agent, i.e., trading is anonymous. Hence, on observing a pair of orders $(a, b)$, the market makers do not know if it corresponds to $X(v) = a$ and $\omega_l = b$ or to $X(v) = b$ and $\omega_l = a$. For convenience we shall always write the order flow as $(X(v), \omega_l)$, but it is important to keep in mind our anonymity assumption. Conditional on the order flow, the market–makers set the price equal to the expectation of the value of the security.

The market structure we consider, whereby the market makers observe the pair of orders stemming from the informed and uninformed agents is slightly different from that of Kyle (1985), whereby the market makers observe the sum of the informed and uninformed orders.\(^\text{11}\) Our approach is similar to

\(^{11}\)Of course, in addition to the difference in terms of market structure, another difference between our set–up and that of Kyle (1985) is that we consider a discrete distribution.
that of Dow and Gorton (1997), who also consider the case where market
makers can observe two orders, one stemming from uninformed hedgers, and
the other submitted by the informed trader. Also, since we assume that
the informed agent and the liquidity trader are simultaneously present, the
market structure we consider is somewhat different from that analyzed by
Glosten and Milgrom (1985), whereby either one informed trader or one
liquidity trader submits an order.

2.2 Equilibrium

We fi

rst analyze the perfect Bayesian equilibrium of this game, where af-

fter observing the private signal (\(v\)) the insider optimally chooses his trade

(\(X(v)\)), but cannot commit to a trading strategy before observing \(v\). The

liquidity trade and the order of the strategic informed agent are transmitted
to the market, and on observing this order flow the market makers quote the

price:

\[ P(X, \omega_l) = E(v|X, \omega_l). \]

In equilibrium: given the rational beliefs of the market makers, and the
corresponding price function (\(P(., .)\)), the insider chooses \(X\) to maximize her
expected profits:

\[ X(v) \in Argmax_{x} E[(v - P(x, \omega_l))x|v]. \]
To avoid being spotted, the insider chooses $X(v)$ in $\{-L, 0, L\}$. Further, in state $u$ she buys $L$, in state $d$, she trades $-L$, and in state $m$ she does not trade. Indeed, selling (resp. buying) in state $u$ (resp. $d$), or trading in $m$, would generate losses. Also, submitting an order for 0 shares is never optimal: it generates no profit while buying or selling can generate strictly positive profits.

### 2.3 Commiting to add noise

Now, suppose that the insider could commit ex ante to a certain strategy, mapping his signal into a (possibly random) trade. As in the previous case, to avoid being spotted, the insider will always trade $L$, 0 or $-L$. But, in contrast to the previous case, the insider can also commit to trading 0 or $-L$ in state $u$, and 0 or $L$ in state $d$. More precisely we consider strategies whereby, in state $u$ the insider trades 0 with probability $\alpha$, $-L$ with probability $\beta$, and $L$ with probability $\gamma$. This is supported by the following out–of–equilibrium beliefs: if the market makers observe a buy (resp. sell) order for any other amount than $L$ they update their probability of state $u$ (resp. $d$) to 1. This indeed deters the informed agent from deviating from $-L, 0, L$. Consider the case of purchases. Suppose the informed agent has observed that state $u$ is realized. Then buying any other amount than $L$ would generate 0 profits. Now consider the case of the insider in state $d$. Any purchase would generate losses. Similar arguments apply in the case of sales.

The out–of–equilibrium beliefs supporting this are similar to those described in the no–commitment case.
bility $\beta$ and $L$ with probability $1 - \alpha - \beta$. By symmetry, in state $d$ the insider trades 0 with probability $\alpha$, $L$ with probability $\beta$ and $-L$ with probability $1 - \alpha - \beta$.

Clearly, such trades are not optimal ex post, since they generate negative or 0 profits. But it can be optimal ex-ante to commit to follow them, to the extent that they lead to a less reactive price function and hence generate larger profits for the insider when he trades $L$ in state $u$ or $-L$ in state $d$.

We understand it is not plausible that in practice the insider can thus commit. We only consider the commitment case as a thought experiment, in order to highlight the benefits derived from adding noise. Thus we set the stage for the analysis in the next section, which establishes that optimal contracts can enable the insider to credibly commit to add noise.

In this context we obtain the following result:

Consider the case where the informed agent can commit to a trading strategy. There exists $\delta_1 \in [0, 1/3]$ and $\alpha \in [0, 1]$ such that, when $\delta > \delta_1$, the profit maximizing strategy of the informed agent is the following:

- In state $u$: buy $L$ with probability $1 - \alpha$ and 0 with the complementary probability.

- In state $m$: never trade.

- In state $d$: sell $L$ with probability $1 - \alpha$ and 0 with the complementary probability.
(The exact values of $\alpha$ and $\delta_1$ as a function of the parameters of the game are given in the proof in appendix.)

The proposition states that adding noise to the order flow is optimal if the probability of state $m$ is low enough, compared to the probabilities of $u$ and $d$. The intuition is the following. The advantage for the informed agent of committing to trade 0 in state $d$ is that this reduces the informational content of his purchases in state $u$, by making it possible that the pair of orders $(L,0)$ could have stemmed from state $d$. The larger the probability (from the point of view of the market makers) that $(L,0)$ stems from $d$, the lower the corresponding price. This effect is less pronounced if the probability of state $m$ is high, because in this case $(0,L)$ is quite likely to stem from $m$. This helps explain the fact that in the Kyle (1985) normal distribution case random strategies are not optimal, even if the informed agent has commitment power. Indeed, in Proposition 2.3 randomization is optimal if the probability of the medium state is low relative to the probability of the extreme states. In contrast, in the case of the normal distribution, the mode is equal to the mean and the mass of the medium states is large relative to that of the extreme states.

In our simple framework, adding noise to trades is optimal when the distribution of the final value of the asset is bimodal. Although this situation is admittedly not typical, it can arise when the private information corresponds to very important news, with pervasive consequences on the value of the asset.
(possibly relative to whether a take-over bid will be launched, or whether a risky venture in which the firm was engaged will be successful). It is indeed at such times that private information is very valuable and that clever trading strategies, designed to exploit it optimally, are particularly useful.

2.4 The two-point case

For simplicity, in the rest of the paper, we will only consider the case where there are just two possible final values for the risky asset: \( u \) and \( d \), occurring with equal probability, (i.e., \( \delta = 1/3 \)). When the informed agent cannot commit to trading strategies, conditional expectations and equilibrium prices are:

\[
\begin{align*}
  P(L, L) = P(L, 0) &= m + \epsilon, \\
  P(L, -L) = P(-L, 0) &= m, \\
  P(-L, -L) &= m - \epsilon,
\end{align*}
\]

and the expected profit of the informed agent is: \( \frac{L}{3}\epsilon \). If, in contrast, the informed trader could commit to add noise to her trades, she would obtain greater expected profits, as shown by Proposition 1. In the two point distribution case the results of Proposition 1 boil down to the following:

**Corollary 1:**

*In the two-point distribution case, the optimal mixing probability is:*

\[
\alpha^* = -1 + (2/3)\sqrt{3} = 0.1547,
\]

*(as shown in appendix).*
In this case the equilibrium prices are:

\[ P(L, L) = u, P(L, 0) = \frac{1}{1 + \alpha^*}u + \frac{\alpha^*}{1 + \alpha^*}d, P(-L, L) = m. \]

3 Sales of information

3.1 The contract between the informed agent and his client

In addition to the three agents considered in the previous section, we now assume there is a risk neutral, rational investor: the customer of the investment bank or securities house. The latter engages in indirect sales of information to its customer. It does so by setting up a fund. This corresponds to indirect sales of information, since the informed agent manages the fund, i.e., decides if the fund will buy or sell, on the basis of her private information. This raises an agency issue however, since the informed agent may not always have the incentive to fully reflect her private information in the fund’s trades. To cope with this incentive problem, the relationship between the informed financial intermediary and its customer must be governed by an incentive compatible contract. The contract specifies the trading volume of the fund, i.e., how many shares can be bought or sold, and the compensation of the seller of information. The latter involves a variable part, i.e. transfers contingent on the fund’s profits, as well as an upfront payment. Denote the latter by \( c \). It
corresponds to the amount the customer has to pay initially to the financial intermediary, in order to set up the fund, i.e., an entry fee.\textsuperscript{14}

Denote by $y$ the number of shares to be traded by the fund as specified in the contract. We consider the case where the trade of the fund can only take two values. That is, the fund can either buy $+y$ or sell $-y$ shares.\textsuperscript{15} The customer does not monitor the trading process. It is left to the informed fund manager to decide (on the basis of her private information) whether to buy or sell. This choice corresponds to the management of the fund. The securities house, or investment bank in charge of managing the fund, may simultaneously trade in the market for its own account.

In contrast to the trades, the trading profit of the fund: $\pi_f$, is observed by the customer. The compensation ($t(\pi_f)$) of the fund manager is contingent on this profit. In fact, we will show below that it is increasing in these profits, in order to give the informed agent incentives to reflect his private information.

\textsuperscript{14}Note that, since we consider a one-shot model, with a single opportunity to trade, $c$ could also be interpreted as a per trade fee.

\textsuperscript{15}In fact, in our simple two-point distribution framework, as we will show below, there is no need to consider more complex contracts: the financial intermediary would not be better off allowing for a larger set of possible trades for the fund. In a richer continuous framework, however, more complex contracts would have to be designed. In this case, instead of two possible trades ( $\{-y, y\}$ ) there would be a continuum of possible trades: $[-y, y]$. In both cases $-y$ and $+y$ can be interpreted as the maximum risky position that the fund manager is allowed to take. Such bounds on the maximum risky positions that fund managers can take are actually observed in practice.
in the trades of the fund.

At the end of the period the client receives the profits of the fund, minus the compensation of the fund manager:

\[ \pi_f - t(\pi_f). \]

Before observing her private signal, the informed agent makes a take it or leave it contract offer \( \{c, y, t(.)\} \) to her client. The latter accepts the offer if it gives rise to expected net profits larger than her (known) reservation level, \( k \). \( k \) can be interpreted as reflecting the bargaining power of the customer in her negotiation with the informed agent.

As shown in the previous section, if the informed agent does not sell her private information, and only conducts proprietary trades (while being unable to commit to a trading strategy), her expected profit is \( G = \frac{L^2}{3} \). On the other hand, as shown above, if the informed agent can commit to a trading strategy, she obtains larger expected gains: \( G^* > G \). With sales of information, the total expected gains of the coalition formed by the informed agent and the customer cannot be higher than \( G^* \). If \( k \) were greater than \( G^* - G \), then clearly the informed agent would prefer to trade alone. To concentrate on the case where sales of information can be attractive, we focus on the case where \( k < G^* - G \).

The individual rationality condition of the customer is:

\[ E(\pi_f - t(\pi_f)) \geq k + c. \] (1)
Since the informed agent makes a take it or leave it offer and the customer has no private information, this individual rationality constraint will hold as an equality in equilibrium.\textsuperscript{16}

\subsection{The trading game}

The informed financial intermediary transmits to the market an aggregate order equal to the sum of its proprietary trades and the fund’s trade. Indeed, banks, collecting customers orders, customarily aggregate these with their

\textsuperscript{16}We consider the case where it is the agent, i.e., the fund-manager, who designs the contract. This differs somewhat from the standard approach, see e.g., Harris and Raviv (1979) and Holmstrom (1979). In this context, we study the optimal contract, which maximizes the informed party’s expected profit, subject to the individual rationality condition of the uninformed customer, and the individual rationality and incentive compatibility conditions of the informed party. One might wonder if our results would be qualitatively altered if one made the opposite assumption that it is the client, i.e., the principal who designs the contract. In fact, as we show below, the optimal contract is designed to maximize the sum of the expected profits of the informed agent and his client. If the client designed the contract, then the optimal contract (as well as the trading and information revelation patterns) would be exactly the same, except that a positive constant would be subtracted from the transfer paid to the agent. In this context the participation constraint of the uninformed investor would no longer be binding, while the participation constraint of the institution would have to be taken into account.
own trades.\textsuperscript{17} Alternatively, order aggregation can take place at the level of
the broker.\textsuperscript{18}

For the coalition of the informed agent and the customer to always make
strictly positive profits, this aggregate order must be a purchase in state $u$,
while in state $d$ it must be a sale. Yet, as shown below, to maximize overall
expected profits, the contract between the informed agent and the customer
induces the intermediary to sometimes choose an aggregate order equal to
0. This reduces information revelation and price impact and consequently
increases expected profits. Also, as in the previous section, there is liquidity
trading, taking the values $L, 0$ and $-L$ with equal probabilities.

The market makers observe the order flow ($\omega$), which is equal to the pair
of orders stemming from the liquidity traders and the financial intermediary.
As in the previous section, trading is anonymous, i.e., the market makers do
not know which order comes from whom. The market makers set the price
to be equal to the expectation of the value of the asset, conditional on the
order flow: $P = E(v|\omega)$.

The informed agent chooses the size and direction of her aggregate trade
without knowing the realization of the liquidity trade. Once the aggregate

\textsuperscript{17}This was mentioned to us in discussions with bank managers operating in Europe as
well as in the US.

\textsuperscript{18}We have checked with exchange officials that such netting is legal and customary on
Nasdaq and on the NYSE. On the latter it used to be forbidden for member firms for
stocks listed before April 26, 1979 (SEC Rule 19c 3). But this rule has now been repealed.
trade is conducted, and the informed agent has observed the price, she has to decide what trade to report for the fund, i.e., she must choose between reporting the purchase of \( y \) shares, or the sale of \( y \) shares. This discretion in the choice of the trade to be reported to the customer reflects the delegation of the trading process by the uninformed investor to the fund manager.

3.3 Trades, prices and profits with sales of information

On observing that the final value of the asset is \( u \), if the informed agent decides to submit an aggregate order to buy shares, in order to avoid full information revelation she sets the size of this order to \( L \). If the informed agent chooses to report that the fund purchased \( y \) shares, then the profits of the fund are:

\[
y(u - P(L, \omega_l)),
\]

where \( \omega_l \) is the liquidity trade, and the total gains of the informed party are:

\[
(L - y)(u - P(L, \omega_l)) + t(y(u - P(L, \omega_l))).
\]

On the other hand, if the informed agent chooses to report that the fund sold shares, then the profits of the fund are:

\[
-y(u - P(L, \omega_l)),
\]

while the total gains of the informed party are:

\[
(L + y)(u - P(L, \omega_l)) + t(-y(u - P(L, \omega_l))).
\]
The respective gains of the informed agent and of his customer can be similarly computed, when the aggregate trades of the intermediary is 0. The case where the asset value is $d$ is symmetric.

Denote by $\alpha$ the probability with which the informed submits an aggregate order for 0 shares (in state $u$ or $d$), while $1 - \alpha$ denotes the probability with which the informed in state $u$ (resp. $d$) submits an aggregate order to buy (resp. sell) $L$ shares. Relying on the corresponding trading strategy, and applying the conditional expectation condition, one can easily compute the prices arising in the market place for the different realizations of the order flow. They are given in Table 1.

3.4 Incentive compatibility conditions

We focus on the situation where the contract between the informed agent and the uninformed investor is structured in such a way that the financial intermediary always has incentives to trade for the fund in the interest of the customer. When the final value of the asset is $u$, the intermediary purchases $y$ shares for the fund, while she sells $y$ shares (i.e., trades $-y$) in state $d$. For this to be the case, in spite of the fact that the uninformed investor cannot monitor the trading process, the incentive compatibility conditions spelled out below must hold.

Suppose that in state $u$, the informed agent sent an aggregate order for $L$
shares, the informed agent prefers to report that the fund purchased $y$ shares rather than to report that he sold $y$ shares if:

$$(L - y)(u - P(L, \omega_l)) + ty(u - P(L, \omega_l))$$

$$\geq (L + y)(u - P(L, \omega_l)) + t(-y(u - P(L, \omega_l))), \forall \omega_l \in \{-L, 0, L\}.$$ 

Now suppose that, in state $u$, the informed agent submitted an aggregate order for 0 shares. The informed agent prefers to report that the fund bought rather than sold, irrespective of what the liquidity trade was, if:

$$-y(u - P(0, \omega_l)) + ty(u - P(0, \omega_l))$$

$$\geq y(u - P(0, \omega_l)) + t(-y(u - P(0, \omega_l))), \forall \omega_l \in \{-L, 0, L\}.$$ 

The conditions arising in the case where the final value of the asset is $d$ are symmetric.

In addition to these incentive compatibility conditions, we impose the condition that the informed agent be willing to add noise to trades, to reduce information revelation. This requires that she be indifferent in state $u$ between submitting an aggregate order for 0 shares and an aggregate order for $L$ shares. This amounts to:

$$\sum_{\omega_l \in \{L, 0, -L\}} (L - y)(u - P(L, \omega_l)) + ty(u - P(L, \omega_l)) = \sum_{\omega_l \in \{L, 0, -L\}} -y(u - P(0, \omega_l)) + t(y(u - P(0, \omega_l))).$$
3.5 Optimal contract

An optimal contract maximizes the expected gains of the informed agent, subject to the individual rationality condition of the customer, the rational pricing rule of the market makers and the incentive compatibility conditions of the informed agent, characterized in the previous subsection. The following proposition states that the trading strategy implemented by the optimal contract is the same as that arising in the full commitment case, which involved mixing between purchases and 0 trades in state $u$.

There exists an optimal contract whereby the informed agent submits an aggregate order to buy (resp. sell) $L$ shares in state $u$ (resp. $d$) with probability $1 - \alpha^*$, and chooses the null aggregate trade with the complementary probability (where $\alpha^*$ is as given in Corollary 1).

Remarkably, by optimally designing the information sale contract, the informed agent is able to manipulate her own incentives in the trading game, so that she follows the same trading strategy as in the full commitment case, analyzed in the previous section. Thus, she can credibly commit to sometimes trade 0 shares, and consequently pass up ex-post profit opportunities, in order to reduce information revelation, and maximize ex-ante expected profits. Our result contrasts with Admati and Pfleiderer (1990), where the fund manager cannot commit to add noise into prices. The difference stems from the fact that in Admati and Pfleiderer (1990) there is no simultaneous
proprietary trading by the fund manager.

As shown in the proof of the proposition, the optimal contract can be implemented by a transfer schedule with the following payoff structure:¹⁹ When the profits of fund are strictly positive, the fund manager receives a fraction \((\gamma < 1)\) of these profits. When the profits of the fund are non-positive, the transfer to the manager is negative. This structure is appropriate to deal with the following two major constraints: (i) the contract must ensure that the informed agent is willing to mix between a large trade \((L)\) and the null aggregate trade (corresponding to two mutually offsetting transactions, to buy and to sell \(y\)), and (ii) the contract must also ensure that in such circumstances the informed agent chooses to report the profitable trade as that of the customer and the unprofitable trade as her own proprietary transaction.

The intuition is the following:

As shown in the proof (in equation (9)), the reporting condition (ii) imposes that:

\[ t(\pi) - t(-\pi) \geq 2\pi, \]

for all possible positive values of the fund profit \((\pi)\). Intuitively, the left-hand-side of this inequality is the incremental transfer received by the fund manager when he reports the profitable trade as that of the client, while the right-hand-side is the reduction in proprietary trading profit it entails.

¹⁹There also exists other transfer schedules implementing the optimal contract.
While, roughly speaking the inequality imposes that, across 0, transfers rise faster than profits, it can be satisfied while keeping transfers lower than profits in the positive domain by imposing large penalties for the out-of-equilibrium cases where the fund’s trades are loss-making.

The mixing condition (i) imposes that the informed agent be indifferent between conducting a large trade ($L$) and share the corresponding profits with the client, or conduct a 0 aggregate trade, and incur proprietary trading losses. This has two consequences:

- To ensure that the informed agent does not always prefer the large trade ($L$), the optimal contract imposes penalties when the fund profits are 0, which can happen only if the aggregate trade is $L$. Note that, while the optimal contract is linear (with slope $\gamma$) for strictly positive profits, such negative transfers when the fund profit is 0 imply a discontinuity at 0.

- To offset the proprietary losses of the informed agent, and ensure that she earns positive profits overall (while keeping transfers below the funds profits), the contract involves an upfront entry fee, paid by the client to the financial intermediary. The value of this fee is set so as to saturate the participation constraint of the client.
4 Conclusion

This paper analyzes indirect sales of information, whereby the informed agent sets up a fund to manage the portfolio of the customer and is compensated on the basis of the profits of this fund, while simultaneously trading on her own account. The compensation schedule gives the informed agent incentives to add some noise to trades. This reduces the overall content of the order flow, thus maximizing the profitability of the overall informed trades, equal to the sum of the fund’s trades and the proprietary trades of the informed agent. The optimal contract can be implemented with a transfer schedule consistent with stylized facts on the compensation of hedge funds managers. Note however that our analysis is carried in the context of a single risky asset, and under the assumption of perfect private information. With non-linear optimal contracts relaxing these two assumptions would greatly complicate the analysis. We leave this extension for further research.

Finally, our analysis illustrates that, in situations where agents lack commitment power, optimal contracting with a third party can enhance their ability to commit, and thus enable them to obtain larger gains. In further research it could be interesting to analyze this point more generally in other contexts.
References


APPENDIX

Proof of Proposition 2.3:

As in the no-commitment case, in equilibrium the informed agent never trades any other amount than $-L, 0$ or $L$. This is supported by the out of equilibrium belief that the probability of $u$ is 1 conditional on observing any other purchase than $L$ and 0 conditional on observing any other sale than $-L$.

In state $u$ the insider trades 0 with probability $\alpha$ and $L$ with probability $1-\alpha$ while in state $d$ she trades 0 with probability $\alpha$ and $-L$ with probability $1-\alpha$, and in state $m$ she plays 0. It is straightforward to show that it is not optimal to trade $-L$ in state $u$ or $+L$ in state $d$.

In this context, the updated probabilities for the market makers are the following:

\[
p(u \mid L, L) = 1 \\
p(u \mid 0, 0) = \frac{1}{2} \\
p(u \mid 0, L) = \frac{2+3\delta}{4+2\alpha+3\delta(\alpha-1)} \\
p(u \mid 0, -L) = \frac{\alpha(2+3\delta)}{4+2\alpha+3\delta(\alpha-1)} \\
p(m \mid L, 0) = \frac{2-6\delta}{4+2\alpha+3(\alpha-1)\delta}
\]

The expected profit of the informed agent if she could commit ex-ante to a given mixing probability $\alpha$ is:

\[
\frac{2L}{3} \left( \frac{1}{3} + \frac{\delta}{2} \right)^3 \frac{2 + 2\alpha + 3\delta(\alpha-1)}{4 + 2\alpha + 3\delta(\alpha-1)} (1 - \alpha) \epsilon. 
\] (2)
Maximizing (2) is equivalent to finding the root of the following quadratic:

\[-\alpha^2(2 + 3\delta)^2 + \alpha(18\delta^2 - 12\delta - 16) - 9\delta^2 + 24\delta - 4.\]  

(3)

The discriminant of this polynomial is: \(\Delta = 3(2 + 3\delta)^24^2\) and there is a unique \(\alpha \in [0, 1]\) maximizing (2) iff:

\[\delta > \frac{4 - 2\sqrt{3}}{3} \equiv \delta_1.\]  

(4)

To complete the proof it is easy to check that for the informed agent in state \(m\) there is no profitable trade.

QED

**Computation of the value of \(\alpha^*\)**

In the two-point distribution case, adapting the expressions from the proof of Proposition 1, the expected profit of the informed agent is proportional to:

\[(1 - \alpha)^{1 + 3\alpha}\]  

\[\frac{1 + 3\alpha}{1 + \alpha}.\]  

(5)

The first order condition is:

\[\alpha^* = -1 + 2/3\sqrt{3}.\]  

QED

**Proof of Proposition 2 :**
First, consider the incentive compatibility conditions under which the informed agent weakly prefers to report \( y \) than \(-y\) as the fund’s trade, when the state is \( u \) (the situation in state \( d \) is symmetric). When, overall, the institution trades \( L \) the condition is:

\[
(L - y)(u - p) + t(y(u - p)) \geq (L + y)(u - p) + t(-y(u - p)), \forall p,
\]

while when, overall, the institution trades 0, the condition is:

\[
-y(u - p) + t(y(u - p)) \geq +y(u - p) + t(-y(u - p)), \forall p,
\]

These conditions simplify to:

\[
t(y(u - p)) - t(-y(u - p)) \geq 2y(u - p), \forall p.
\] (6)

Second consider the condition under which the informed agent is indifferent between 0 and \( L \). The informed agent randomizes in state \( u \) if she earns the same expected profit when i) buying \( L - y \) for herself and \( y \) for the fund, and ii) trading \(-y\) for herself and \( y \) for the fund. Equalizing the expected profits he earns in these two cases:

\[
t(0) + (L - y)\epsilon + t(y\epsilon) + (L - y)\frac{2\alpha}{1 + \alpha}\epsilon + t(y\frac{2\alpha}{1 + \alpha}\epsilon) =
\]

\[
-\frac{y\epsilon}{1 + \alpha} + t(y\frac{2\alpha}{1 + \alpha}\epsilon) - y\epsilon + t(y\epsilon) - \frac{2y\epsilon}{1 + \alpha} + t(y\frac{2\alpha}{1 + \alpha}\epsilon).
\]

After simplification this becomes:

\[
t(0) + (L - y)\epsilon + \frac{L\epsilon}{1 + \alpha} = t(y\epsilon\frac{2}{1 + \alpha}) - y\epsilon - \frac{2y\epsilon}{1 + \alpha}.
\]
which yields:

\[ y = \frac{1}{2\varepsilon}(1 + \alpha)[-t(0) + t(y\varepsilon \frac{2}{1 + \alpha}) - \frac{1 + 3\alpha L}{1 + \alpha}L\varepsilon] \quad (7) \]

Defining \( K \) and \( C \) by: \( K = 3k \), and \( C = 3c \), the individual rationality condition is given by:

\[ y(\varepsilon \frac{1 + 5\alpha}{1 + \alpha} - t(\frac{2\alpha y\varepsilon}{1 + \alpha}) - t(y) - t(2\varepsilon \frac{1 + 3\alpha}{1 + \alpha}) - t(0) \geq K + C. \quad (8) \]

The set of conditions characterized above can be summarized as:

\[ t(y(u - p)) - t(-y(u - p)) \geq 2y(u - p), \forall p. \quad (9) \]

\[ y = \frac{1}{2\varepsilon}(1 + \alpha^*)[-t(0) + t(\frac{2y\varepsilon}{1 + \alpha^*}) - \frac{1 + 3\alpha^*}{1 + \alpha^*}L\varepsilon], \quad (10) \]

\[ y^e \frac{1 + 5\alpha^*}{1 + \alpha^*} - t(\frac{2\alpha y\varepsilon^*}{1 + \alpha^*}) - t(y) + \alpha(t(0) - t(2\varepsilon \frac{1 + 3\alpha^*}{1 + \alpha^*}) - t(0) \geq K + C. \quad (11) \]

To establish the proposition we need to show that there exists a vector:

\[(c, y, t(-2\varepsilon \frac{1 + \alpha^*}{1 + \alpha^*}), t(\frac{2\alpha y\varepsilon}{1 + \alpha^*}), t(\frac{2y\varepsilon}{1 + \alpha^*}), t(0), t(\frac{2y\varepsilon}{1 + \alpha^*}), t(\frac{2\alpha y\varepsilon}{1 + \alpha^*}))\]

for which these conditions hold.

To establish that there exists at least one solution, we prove that the equilibrium conditions hold with the following contract:

\[ c > \frac{(1 - \alpha)(1 + 3\alpha)}{(1 + \alpha)}L\varepsilon - K, \]

\[ t(\pi_f) = \gamma \pi_f, \forall \pi_f > 0, \]
\[ t(\pi_f) < (2 - \gamma)\pi_f, \forall \pi_f < 0, \]
\[ t(0) = - [\frac{2y}{1 + \alpha} + \frac{1 + 3\alpha}{1 + \alpha} L\epsilon + \gamma \frac{2y}{1 + \alpha} \epsilon], \]
\[ \gamma = 1 - \frac{K + c - \frac{(1 - \alpha)(1 + 3\alpha)}{(1 + \alpha)} L\epsilon}{3y\epsilon} < 1, \]

which specifies that positive fund profits are split between the manager (who obtains a fraction \( \gamma \)) and the investor (who obtains fraction \( 1 - \gamma \)), while non-positive fund profits give rise to penalties paid by the fund manager to the customer.\(^{20}\) (Note that since we have a large number of degrees of freedom, we don’t need to give the exact value of \( y \). Any value below \( L \) will do.)

Obviously, the transfers specified above for (out of equilibrium) negative profits \( t(\pi_f) < (2 - \gamma)\pi_f, \forall \pi_f < 0, \) imply that the incentive compatibility conditions, \( t(y(u-p)) - t(-y(u-p))) \geq 2y(u-p), \forall p, \) hold. Substituting in the mixing condition the transfers specified above for strictly positive profits gives \( t(0) \) as a function of \( \gamma \) and \( y \):

\[ t(0) = - [\frac{2y}{1 + \alpha} + \frac{1 + 3\alpha}{1 + \alpha} L\epsilon + \gamma \frac{2y}{1 + \alpha} \epsilon]. \]

Substituting this value of \( t(0) \) in the participation constraint of the customer, we obtain, after straightforward manipulations:

\[ \gamma \leq 1 - \frac{K + c - \frac{(1 - \alpha)(1 + 3\alpha)}{(1 + \alpha)} L\epsilon}{3y\epsilon}. \]

\(^{20}\)Note that strictly negative fund profits, and the associated large penalties paid by the informed agent, are out of the equilibrium path.
Note that, for the value of $c$ specified above, this implies: $\gamma < 1$. Saturating this participation constraint, we obtain the value of $\gamma$.

QED

Table 1:
Trading profits in state $u$ with sales of information when the financial institution submits $L$ or $0$.

<table>
<thead>
<tr>
<th>Fund trade</th>
<th>Proprietary trade</th>
<th>Liquidity trade</th>
<th>Fund profit</th>
<th>Proprietary profit</th>
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<td>$L - y$</td>
<td>$L$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$y$</td>
<td>$L - y$</td>
<td>$0$</td>
<td>$y\epsilon \frac{2\alpha}{1+\alpha}$</td>
<td>$(L - y)\epsilon \frac{2\alpha}{1+\alpha}$</td>
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<tr>
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