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When Overconfident Traders Meet Feedback Traders.

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Abstract

In this paper, we develop a model in which overconfident market participants and rational speculators trade against trend-chasers. We exhibit the unique linear equilibrium and assess the quality of prices according to the proportion of the different types of agents. We highlight how speculative bubbles arise when a large number of traders adopt a trend-chasing behavior. We show that overconfident traders can obtain positive expected profits. In particular, overconfident traders can outperform rational traders. The positive feedback trading enhances the negative correlation between the back-to-back prices changes and the volatility of prices as well. We show that positive feedback traders destabilize prices more than their overconfident opponents. Generally, overconfidence increases the volatility of prices and worsens the market efficiency. But, we show that in the presence of positive feedback trading, overconfidence improves the market efficiency and dampens the excess volatility.

Keywords: Overconfidence, Positive feedback trading, Bubbles, Excess volatility, Market efficiency.

JEL Classification: D43, D82, G14, G24.
Introduction

Most financial studies assume market efficiency and consider that agents behave rationally. In doing so, they fail to explain some properties observed in financial markets such as booms and crashes (Kunieda (2007), Kaizoji (2000)); the underreaction or overreaction of market participants (De Bondt and Thaler (1985, 1987, 1990), Daniel et al. (1998)); the excessive volume traded (Dow and Gorton (1997), Odean (1998)). Behavioral finance has emerged primarily to explain these anomalies. Among all the psychological biases and forms of irrationality, two psychological traits of market participants have been widely investigated: the overconfidence and the trend-chasing behavior.

In this paper we exhibit a model where the bubble crashes emerge from the presence of irrational traders in a dynamic setting. Irrationality is modeled through overconfidence and feedback trading.

However, according to Fama (1965, 1970), there are enough well-informed traders and well-financed arbitrageurs present in the market to guarantee that any potential mispricing induced by behavioral traders is corrected at a certain point. Therefore, the efficiency theory rules out the persistence and even the presence of bubbles. Nevertheless, many bubbles and crashes examples can be found. The most famous one is undoubtedly the Dutch Tulip Mania, which occurred in 1630. This was the first derivatives market crash. Some other famous crashes include the Mississippi Bubble in 1721, the Asian market collapse in 1997, the Russian monetary system bankruptcy in 1998 and more recently the dot-com bubble crash in 2001 as well as the subprime crisis and the “credit crunch”, which began during the summer 2007.

The purpose of this paper is to study the interaction between overconfident and feedback traders (momentum as well as contrarian traders) and how this interaction may trigger the creation of bubbles and their subsequent crash. Indeed, positive feedback or momentum traders buy (sell) securities when prices rise (fall). In doing so, they introduce noise into the market since they lead prices to move away from fundamentals. However, negative feedback or contrarian traders sell (buy) securities when prices rise (fall). This behavior limits the movement of prices.

To study such a situation, we use the dynamic model of De Long et al. (1990). We introduce the presence of informed overconfident traders in their model in order to look at how overconfident traders exploit the presence of feedback traders. When positive feedback traders are present, informed traders stimulate trading by positive feedback investors, by trading, early, large quantities and therefore building up a large position. In the last period, they off-load part of their position by going against their information. For the case of negative feedback trading, informed investors ???(we have to look at the simulations from the price behavior, I think that

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1 As in Germain et al. (2009), we consider that overconfident traders overestimate the mean of the liquidation value of the risky asset (called mean bias) but also misperceives the variance of the liquidation value.
the price should gradually increase, but at a lower speed than if there were no negative feedback).

We show that the growth and the burst of a financial bubble stem mainly from positive feedback trading and that overconfident informed investors are not responsible for its formation. However, we find that the presence of overconfident traders and the risk aversion of the informed speculators only enhance the strength of bubbles (creation and burst). This last result is in contrast to Figlewski (1979), Campbell and Kyle (1988). Indeed, we obtain that the informed traders’ risk aversion (for both the rational and the overconfident traders) does not dampen the fluctuation of the price schedule by keeping informed investors from taking large position.

*This should be completed with the negative feedback if there is anything to say concerning bubbles which I doubt.*

Our analysis, apart from shedding some light on the emergence and the burst of bubbles, will also enable us to answer the following important questions. What is the main determinant of the excess price volatility? How does the link between psychological characteristics of the participants and their trading profits evolve according to different proportions of irrational traders? What is the effect of the traders’ risk aversion on price stabilization? Where do the underreaction and/or overreaction to new information come from?

We first give our results for the case where there is no feedback trading (*Do we need to put that, as there is not much difference with existing results*). In that case, the trading of overconfident investors enhances the volatility of prices, and worsens the quality of prices as well as their expected profits. We obtain that the presence of overconfident implies larger volume traded. In addition, we find that the trading volume increases as the mean bias increases. Statman, Thorley, and Vorkink (2006) use U.S. market level data to test the former effect and argue that after high returns subsequent trading volume will be higher as investment success increases the degree of overconfidence.

Our second set of results looks at the case where feedback traders are present. When there is a sufficient large number of trend-chasing speculators, overconfident traders may earn more profits than their rational opponents. In addition, overconfidence diminishes the volatility of prices (whereas momentum trading always enhances it), and increases market efficiency (momentum trading worsens it). We obtain that asset returns are negatively serially correlated. Daniel, Hirshleifer and Subrahmanyam (1998) consider a situation where investors are overconfident about the precision of their private signals. However, the noisy public information is correctly estimated by all market participants. They then exhibit a positive short-lag autocorrelations (that they call the “overreaction phase”) and a negative correlation between future returns and long-term past stock market (the long-run reversals that they call the “correction phase”). Finally, when positive feedback traders are present we obtain both rational and overconfident traders trade more. Moreover, positive feedback traders earn negative expected profits.
When we look at negative feedback, we find that most of the results obtained with the presence of positive feedback are reversed. Price volatility is reduced whereas price efficiency is enhanced due to the presence of contrarian trading. Price volatility and price efficiency increase with both the number of overconfident traders and with their level of overconfidence. The overall volume traded by rational traders increase with the number of negative feedback whereas the volume traded by overconfident is $U$-shaped with respect to the number of negative feedback. We find the serial correlation of returns to be negative and to decrease with the number of negative feedback. Finally the expected profit of the negative feedback traders decrease with the number of negative feedback traders present in the market and is positive for low enough number of negative feedback traders.

The trend-chasing behavior has been shown to exist both experimentally and empirically. Andreassen and Kraus (1990) and Mardyla and Wada (2009) use experiments to show its existence. Andreassen and Kraus (1990) show that over a period prices exhibit a trend relative to the period variability, subjects chase the trend, buying more when prices rise and selling when prices fall. However, this phenomenon occurs only as a response to significant changes in the price level over a substantial number of observations and not in response to the recent price changes alone. Mardyla and Wada (2009) investigate a virtual stock market to understand the link between public information and short-term investment behavior at the individual decision-making level. They distinguish between three types of public information: prices movements, macroeconomic news and relevant individual-stock information. They find that for 75% of the cases, the subjects adopt positive feedback strategies.

In addition, other analysis have empirically found evidences of trend-chasing behavior in financial markets. Frankel and Froot (1988) observe that in the mid-1980’s the forecasting services were issuing buy recommendations while maintaining that the dollar was overpriced relative to its fundamental value. Lakonishok, Shleifer and Vishny (1994) find evidences that individual investors use positive feedback trading strategies and that this behavior can be attributed to an irrational extrapolation of past growth rates. Several studies have focused on the behavior of institutional investors (Shu (2009), Sias and Starks (1997), Sias, Starks and Titman (2001), Dennis and Strickland (2002) to name but a few). According to most of these papers, institutional investors use positive feedback strategies and therefore destabilize stock prices. Finally, Bohl and Siklos (2005) find the existence of momentum strategies during episodes of stock market crashes. Evidences of contrarian strategies are also present in financial markets. The short-term portfolio composition strategies suggested by Conrad et al. (1994), Cooper (1999), and Gervais et al. (2001) found that US markets showed anomalous, “contrarian” behavior. Moreover, Evans and Lyons (2003) obtain evidence of negative feedback trading, at the daily frequency. This literature concludes that the presence positive (negative) feedback traders leads to negative (positive) serially correlated returns together with an increase (decrease) in volatility.
The overconfidence was firstly analyzed by psychologists. Kahneman and Tversky (1973), and Grether (1980) stress that people overweight salient information. This behavior is well documented in psychology for very diverse situations. Due to the essence of financial markets, overconfidence occurs among market participants. Indeed, the competition between traders leads the most successful ones to survive, leading them to overestimate their own ability.

Many academic papers have looked at the impact of overconfidence on financial markets. Most of the studies predict that overconfident agents trade to their disadvantage (Odean (1998), Gervais and Odean (2001), Caballé and Sákovics (2003), Biais et al. (2004) among others). In contrast, Kyle and Wang (1997), Benos (1998) and Germain et al. (2009) find that overconfident traders can have larger expected profit than their rational counterpart. The other most cited effect of the presence of overconfident traders is a large trading volume (Odean (1998), Barber and Odean (2001) among others).

Hirshleifer, Subrahmanyan and Titman (2006) show that irrational traders can earn positive expected profit in the presence of positive feedback trading if they trade early. Indeed, higher stock prices may attract customers and employees which may reduce the firm’s cost of capital and provide a cheap currency for making acquisition. Also, stock prices increase may initially generate cash flow. This simple mechanism does not require irrational traders to be sophisticated enough to think of the positive feedback trading effect and to realize profits. In contrast to Hirshleifer et al. (2006), we do not consider a direct link between the stock prices and the corresponding firm’s output which might be embodied by the means of positive feedback trading. Such a link is considered through the information which is observed by informed traders.

The outline of this paper is as follows. In section 2, we introduce the general model and characterize the different types of traders. In section 3, we derive the trading equilibrium. In section 4 we analyze the effects of the overconfidence and of the feedback trading on the variances of prices. In section 5, we are interested in understanding the social standpoint. In section 6, we discuss our results in terms of the range of parameters. In section 7, we look at how our results are changed when we replace positive feedback traders by negative feedback tarders. Finally, we conclude in section 7. All proofs are gathered in the Appendix.

1 The Model

We consider a model with a three-period auction. Two assets, namely a riskless and a risky asset, are exchanged at $t = 1, 2, 3$. The riskless interest rate is normalized to zero. The liquidation value of the risky asset, $\tilde{v}$, is assumed to be normally distributed with $\tilde{v} \sim N(\bar{v}, h_v^{-1})$.  

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2 It is straightforward that dot-com economy has benefited from this advantage by attracting the best students from top programs such as Standford University and by raising large funds (see Fortune "MBAs Get Dot.com Fever" August 2 1999).
Consumption takes place at the end of the game, i.e. at $t = 4$.

We consider a trading system where agents are price takers. At each auction $t$, the demands for the risky asset and the riskless asset are $x_t$ and $f_t$ respectively. Three types of investors trade the assets:

- $N_1$ overconfident traders. They receive information about the liquidation value of the risky asset. They believe that their private signals are more accurate than they actually are. Furthermore, they overestimate the expected liquidation value of the risky asset.

- $N_2$ rational traders. These traders receive private information but do not distort it.

- $P$ feedback agents who can either be positive feedback or negative feedback traders. Their order size is proportional to the variation of the price of the asset.

We denote by $P_t$ the price of the risky asset at time $t$ for $t = 1, 2, 3$. At time $t = 4$, the value of the risky asset is publicly revealed, the price is then equal to the realization of $\tilde{v}$.

Trader $i$’s wealth is $W_t = f_t + P_t x_t$ for trading rounds $t = 1, 2, 3$ and $W_4 = f_{3i} + \tilde{v} x_{3i}$ for the last trading round. Let us denote $\bar{x}$ as the per capita supply of the risky asset. It is assumed to be known to all and constant over time.

No information is released before the first trading round. Before each subsequent trading round $t = 2$ and $t = 3$, each rational or overconfident trader receives one of $M$ different signals concerning the liquidation value of the asset. We have that $M < N_1 + N_2$. Each trader receives a private signal $\tilde{y}_t = \tilde{v} + \tilde{\varepsilon}_{tm}$, with $\tilde{\varepsilon}_{tm} \sim N(0, h^{-1}_\varepsilon)$ and $\tilde{\varepsilon}_{t1}, \ldots, \tilde{\varepsilon}_{tm}$ for $\forall t = 1, 2, 3$ are mutually independent. Let $\bar{Y}_t$ be the average private signal at time $t$, we assume that:

$$\bar{Y}_t = \frac{1}{M} \sum_{i=1}^{M} \tilde{y}_t = \frac{N_1}{N_1} \sum_{i=1}^{N_1} \tilde{y}_t + \frac{N_2}{N_2} \sum_{i=1}^{N_2} \tilde{y}_t$$

in other words, the informativeness of the private signals is the same for the two groups.

Overconfident market participants believe that the precision of their two signals, the one received at $t = 2$ and the other received at $t = 3$, is equal to $\kappa h_\varepsilon$ with $\kappa \geq 1$. They also believe that the other signals have a precision equal to $\gamma h_\varepsilon$ with $\gamma \leq 1$. Overconfident traders misperceive the distribution of the asset as well. Indeed, they believe that the average liquidation value equals $\tilde{v} + b$ with $b > 0$ and that the precision of $\tilde{v}$ equals $\eta h_\varepsilon$ ($\eta \leq 1$). This framework is consistent with theoretical and empirical findings. Indeed, traders tend to overestimate their own signals and to correctly evaluate (or at worst to under-weight) public information.

A rational agent correctly estimates both the mean of the liquidation value of the risky asset

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$^3$If the trader is pessimistic we have $b < 0$.  

and her private signal. In other words, a rational investor acts as an optimistic trader with \( \eta = \kappa = \gamma = 1 \) and \( b = 0 \).

For simplification we assume that both \( N_1 \) and \( N_2 \) are multiple of \( M \). This assumption leads to the fact that optimistic traders are, on average, equally informed as their rational counterparts. This setup allows us to exhibit the overconfidence effect without considering informational content bias.

All informed agents are assumed to be risk averse. Their preferences are described by a constant absolute risk aversion (CARA) utility function of the following form

\[
u(W) = -e^{-aW} \]

where \( a \) denotes the coefficient of risk-aversion and \( W \) the wealth.

Each informed trader \( i \) chooses his order at time \( t \), \( x_{ti} \), so that

\[
x_{ti} \in \arg \max E[-e^{-aW_{ti}} | \Phi_{ti}],
\]

where \( \Phi_{ti} \) denotes the information available to them.

As in Odean (1998), informed traders look one period ahead when solving for their optimal strategy.

Finally, at time \( t = 2, 3 \), each feedback positive agent \( i \) submits an order \( x^f_{ti} \), with:

\[
x^f_{ti} = \beta (P_{t-1} - P_{t-2})
\]

2 The equilibrium

To solve their maximization problems, informed traders whether rational or overconfident assume that prices are linear functions of the average signal(s) such that:

\[
P_3 = \alpha_{31} + \alpha_{32}\bar{Y}_2 + \alpha_{33}\bar{Y}_3, \quad (2.1)
\]
\[
P_2 = \alpha_{21} + \alpha_{22}\bar{Y}_2. \quad (2.2)
\]

At each auction, an informed agent determines his demand by considering both his private signal(s) and the price schedule(s). Each informed market participant \( i \) has access to the following information \( \Phi_{2i} = [y_{2i}, P_2]^T \) and \( \Phi_{3i} = [y_{2i}, y_{3i}, P_2, P_3]^T \) for date \( t = 2 \) and \( t = 3 \), respectively. Due to the presence of positive feedback trading, when deciding his demand, an informed trader takes into account that his trade may lead prices away from fundamentals. The positive feedback traders only participate to the last two rounds of trading. We now present the first proposition.
Proposition 2.1 There exists a unique linear equilibrium in the multi-auction market characterized by:

\[
\begin{align*}
\alpha_{31} &= \frac{(N_1\eta + N_2)h_v\bar{v} + N_1\eta \bar{h}_v b - a(N_1 + N_2 + P)x}{(N_1\eta + N_2)h_v + 2(N_1(\kappa + \gamma M - \gamma) + N_2 M)h_v} + \frac{aP\beta}{\alpha_{21}}(\alpha_{21} - P_1) \\
\alpha_{32} &= \frac{\alpha_{33} + \frac{(N_1\eta + N_2)h_v + 2(N_1(\kappa + \gamma M - \gamma) + N_2 M)h_v}{N_1(\kappa + \gamma M - \gamma)h_v + N_2 M h_v}}{\alpha_{22}} \\
\alpha_{33} &= \frac{N_1(\kappa + \gamma M - \gamma)h_v + N_2 M h_v}{N_1(\kappa + \gamma M - \gamma)h_v + N_2 M h_v}
\end{align*}
\]

where \(\alpha_{21}, \alpha_{22}\) and the different agents’ demands over time are given in the Appendix.

Proof: See Appendix.

The number of positive feedback traders has an impact on the different parameters \(\alpha\) except on \(\alpha_{33}\). Indeed, at the last auction, informed agents cannot trigger feedback trading on the basis of their new information. Nevertheless, all prices are impacted and connected by the presence of trend-chasing traders. More precisely, the link between \(P_3\) and \(\bar{Y}_2\) (captured by \(\alpha_{32}\)) depends on the link between \(P_2\) and \(\bar{Y}_2\) (i.e. \(\alpha_{22}\)). The greater the intensity of feedback trading (\(\beta\) and \(P\)) the stronger is this link. Similarly, the informed traders’ risk aversion, \(a\), strengthens this link.

In our model, as the both types of informed traders are aware of the presence of positive feedback traders, they take that into account when trading. Indeed, upon receiving good news before both \(t = 2\) and \(t = 3\) for instance, informed traders take larger position based on that information at \(t = 2\) in order to drive prices up. This triggers even more buying subsequently from feedback traders which enables them to off load their position at an inflated price resulting in positive expected profit.

Odean (1998) is a particular case of our model and can be retrieved by setting, in our model, the value of \(P\), \(N_2\) and \(b\) to 0. He focuses on the resulting variance of prices, trading volume and market efficiency when overconfident traders are present in the market.

In contrast to us, in Hirshleifer et al. (2006) irrational traders do not anticipate the feedback effect. Moreover, in their model, the rise in price causes stakeholders (for instance workers) to make greater firm-specific investments when they anticipate the growth of the firm. This in turn increases the final payoff of the risky asset. This explains why risk aversion cannot lead price to be negative and has no impact on the existence of a stock market equilibrium. Nevertheless, risk averse traders trade less aggressively and dampen the feedback effect.
3 Volatility, Quality of Prices, Serial Correlation of prices and Trading Volume

In this section, we focus on the influence of the irrational behavior on the volatility of prices, measured as the variance of prices, on the quality of prices at $t$ (the variance of the difference between the price and the liquidation value, $\text{var}(P_t - \tilde{v})$), the serial correlation of prices and on the different market participants’ trading volume.

3.1 Volatility

**Proposition 3.2** The variance of prices, for both trading rounds $t = 2, 3$, increases with the number of positive feedback traders. In the presence of trend-chasing behavior, the overconfidence can diminish the volatility of prices.

**Proof:** See Appendix.

The first part of the proposition implies that in the presence of positive feedback traders, neither rational informed agents nor overconfident informed traders can stabilize stock prices as they exploit the presence of feedback traders. This result is consistent with De long et al. (1990). However, numerous previous studies have pointed out that the excess volatility of asset prices stems from the trading behavior of overconfidence traders (Odean (1998), Caballé and Sákovics (2000), among others). We find that excess volatility depends critically on the number of feedback traders in the market. When there are no positive feedback, we obtain the aforementioned result of Odean (1998) and Caballé and Sákovics (2000). However, when there are positive feedback traders present this result can be reversed. We find that the main source of excess volatility is due to feedback trading rather than the trading from overconfident traders. Hence, overconfident traders can alleviate the effect of the feedback trades and lead to a more stable market. This can be explained as follows, when changing the number of informed traders, the following forces are at work. Increasing the number of informed traders (both rational and overconfident) on the one hand stabilizes prices as it increases the risk bearing capacity of the market. On the other hand it destabilizes prices as more traders anticipate the trend-chasing behaviour. The more positive feedback traders in the market, the larger the latter effect. The combination of the two effects lead to the comparative statics below.

Some other studies, such as Hilton (2000), Glaser, Nöth and Weber (2004), De Bondt (1998), Graham and Harvey (2003), have found a similar result. However, all these studies investigate only one type of irrational trading. In contrast to these papers, we look at different types of irrational traders.
The figures below show the volatility of prices for different parameters of our model.

In figure 1, we obtain that the volatility of prices increases with the number of positive feedback traders and decreases with the number of overconfident agents.

Figure 1: The variance of prices at time $t = 3$ as a function of the number of feedback traders and of the number of overconfident traders.

In figure 2, we observe that the variance of prices increases when overconfident traders underestimate the precision of the liquidation value of the risky asset ($\eta h_v$, with $\eta \leq 1$). The volatility of prices also increases when each overconfident agent believes the other $(2M - 2)$ signals to be $\gamma h_\varepsilon$, with $\gamma \leq 1$. The smaller the two parameters ($\eta$, $\gamma$), the greater is the volatility of prices. This result is consistent with Odean (1998) and shows that underestimating the ability of other market participants causes the price setting to be destabilized.

Figure 2: The variance of prices at time $t = 3$ as a function of the number of $\gamma$ and of $\eta$.

Figure 3 shows that as overconfident traders become more overconfident the volatility of prices decreases when trend chasing traders are present in the market.
Figure 3: *The variance of prices at time t = 3 as a function of the number of feedback traders and of the number of overconfident traders.*

In figure 4, we find that, without trend-chasing behavior, overestimating its own ability enhances the volatility of prices. The last graph shows that for intermediate values of $P$, the number of positive feedback traders, the volatility of prices decreases with the number of overconfident and is non-monotonic with the level of overconfidence.

![Graph](image.png)

**Figure 4:** *The variance of prices at time $t = 3$ as a function of the number of feedback traders and of the number of overconfident traders.*

We now turn to the quality of prices.

### 3.2 Quality of Prices

In that subsection we examine the behavior of the quality of prices.

**Proposition 3.3** *The quality of prices declines as the number of feedback traders increases.*
When the number of positive feedback traders increases, prices move away from fundamentals and therefore become less informative. Moreover, as informed traders anticipate the trend-chasing strategies they purchase ahead of feedback demand. As a consequence, the price quality worsens even more.

In addition to responding to the trend-chasing strategies, overconfident traders alter the quality of prices due to their irrationality.\(^4\) Nevertheless, we can observe that the overconfidence improves market efficiency when there is positive feedback trading (see the figures below). If there is no trend-chasing behavior, the presence of overconfident traders moves prices away from fundamentals and diminishes the market efficiency.

Figures 5 and 6 show that the quality of prices decreases with the number of positive feedback traders. For a fixed number of positive feedback traders, market efficiency increases with the number of overconfident traders and the parameter \(\kappa\).

![Figure 5: The quality of prices at time \(t = 2\) and \(t = 3\) as a function of the number of feedback traders and of the number of overconfident traders.](image)

Figure 7, below, shows the ambiguous impact of the number of feedback traders on the quality of prices. As said before, when \(P = 0\) the quality of prices decreases with both \(N_1\) (number of overconfident traders) and \(\kappa\) (level of overconfidence). When \(P \neq 0\), the quality of prices increases with \(N_1\) and is non-monotonic with \(\kappa\).

In figure 8, we observe that the quality of prices diminishes when each overconfident trader underestimates the ability of the other ones.

\(^4\)Ko and Huang (2007) show that arrogance can be a virtue. Indeed, overconfident investors believe that they can earn extraordinary returns and will consequently invest resources in acquiring information pertaining to financial assets. In our model there is no information-seeking activity which could permit to obtain such a positive externality.
5.1, the number of overconfident agents. $N_2 = 10$; $\eta = \gamma = 0.5$; $\kappa = 2$

$P$, the number of feedback traders. $N_1, N_2 = P = 10$; $\eta = \gamma = 0.5$ $\kappa = 1, 2, 3, 4, 5$

Figure 6: *The quality of prices at time $t = 3$ as a function of the number of overconfident traders and of the parameter $\kappa$.*

Figure 7: *The quality of prices at time $t = 3$ as a function of the number of overconfident traders and of the parameter $\kappa$, for $P = 0$ and $P = 5$.*

### 3.3 Serial Correlation of Prices

**Proposition 3.4** The price changes are more important when the number of positive feedback traders is large.

The serial correlation of prices depends critically on the overconfidence level and on the number of positive feedback traders. *In the presence of positive feedback trading, the serial correlation is generally negative. It implies that positive feedback trading destabilizes the price schedule. Informed traders cannot keep price at fundamentals or dampen the fluctuation of prices.*

The term $\text{cov}(P_3 - P_2, P_2 - P_1)$ describes the correction phase in Daniel, Hirshleifer and Subrahmanyam (1998). They show that overconfident traders begin by overreacting to their private signals. In the second phase, irrational market participants correct their beliefs and
their order as new public information arrives, this is defined as the “correction phase”. In our model, agents update their beliefs concerning their private information. The informed market participants know that their earlier trades move prices away from fundamentals as they try to exploit the presence of feedback trading. The size of the departure of prices from fundamentals increases with the number of feedback traders. At date 3, informed participants abruptly correct their demands after observing their last signal. Odean (1998) has found such negative correlation by considering only overconfident agents. We extend his result, as we show that informed rational traders cannot prevent irrational traders (feedback agents as well optimistic traders) to destabilize prices. In our model, the “correction phase” appears as a bubble crash.

In the next two figures we have simulated the price changes as a function of the number of positive feedback traders. Figure 9 shows that the returns reversal increases with the number of feedback traders. This result is reinforced by the level of overconfidence in the market. Figure 10 confirms the previous result, however it shows that the price change is less important when each irrational informed trader underestimates the precision of the other speciﬁc market participants’ signals.

3.4 Trading Volume

Proposition 3.5 The trading volume increases with the number of feedback positive traders in the market.

In the presence of positive feedback trading strategies, informed agents trade more aggressively. They anticipate that the initial price increase will stimulate buying by feedback traders at the subsequent auctions. In doing so, they drive prices up higher than fundamentals. Consequently, feedback positive traders respond by trading even more. We find that the feedback trading as
well as the overconfidence enhances the trading volume. Our result is consistent with empirical findings. For instance, Glaser and Weber (2007) (could not find reference) have shown that investors who think that they are above average in terms of investment skills or past performance trade more.

In figure 11, we show that the volume from overconfident traders is a non-monotonic function of $\kappa$. For large values of $P$, this volume decreases with $\kappa$.

In figure 12, we show that the volume originating from positive feedback traders increases with the number of feedback agents. It decreases with the level of overconfidence (measured by $\kappa$) and it increases with the underestimation the precision of the other overconfident agents’ signal.

In figure 13, we compare the volume from overconfident traders and from rational traders with no feedback traders. As expected, we see that overconfident traders trade more aggressively than...
Figure 11: The total individual informed orders as a function of the number of feedback traders, for different values of the parameter $\kappa$.

Figure 12: The total individual feedback trading volume as a function of the number of feedback traders, for different values of the parameters $\kappa$ and $\eta$.

their rational counterparts. The volume from overconfident investors increases with $\kappa$, whereas the volume from rational investor’s order decreases with $\kappa$.

4 Profits

We now look at expected profits.

Proposition 4.6 Gains from trade:

- Positive feedback traders always earn negative profits.
- If there are enough positive feedback traders, both overconfident and rational traders can earn positive profits.
• When the number of trend chasers is large, overconfident traders earn more than rational speculators. Nevertheless, the profit earned by overconfident decreases with $b$. When $b$ is large, they realize losses.

Positive feedback traders act as pure noise traders since they are uninformed. Such trading behavior leads these agents to always lose money. Informed agents anticipate their strategy. In doing so, they introduce at the earlier auctions noise which is enhanced by the positive feedback traders themselves at the last rounds.

When there are only rational and overconfident traders in the market, we find that overconfident traders earn less profit than their rational counterparts. This result is consistent with, among others, Odean (1998), and Gervais and Odean (2001). When the 3 types of traders are present (overconfident, rational and positive feedback agents) overconfident agent may outperform rational traders. This result confirms Benos (1998) and Germain et al. (2009). Indeed in Benos (1998) and Germain et al. (2009), pure noise traders trading for liquidity reasons are present. However, in our model, overconfident traders earn more profits than their rational counterparts as they trade less aggressively. They believe that the main reason for prices to be different from fundamentals is the presence of trend-chasers. Moreover, Benos (1998) analyzes a static market and overconfident market participants cannot correct their earlier trades in the last auctions.

We now look at the different simulations we have performed regarding the expected profits for the different traders present in the market.

Firstly, we obtain, as anticipated, that positive feedback traders always realize losses, their losses increase with the number of positive feedback traders (see figure 14).

We look now at the influence of the different overconfidence parameters on the positive feedback trader’s profit.
The figure below (figure 15) shows that positive feedback trader’s losses are less important when each overconfident trader treats his own private information as being better than it is, as captured by the parameter $\kappa$. It also shows that the higher the overconfidence parameters $\eta$ and $\gamma$, the more severe the positive feedback traders’ losses are. These results stem from the fact that the trend-chasers’ trading volume decreases with $\kappa$ and increases with $\eta$ and $\gamma$.

Figure 14: The total individual positive feedback trader’s profit as a function of the number of overconfident agents and of the number of positive feedback traders, for different values of the parameter $\kappa$.

Figure 15: The total individual positive feedback trader’s profit as a function of the number of positive feedback traders, for different values of the parameters $\eta$, $\gamma$.

In figure 16, we can observe that the individual expected profits of overconfident traders decrease with $b$. Nevertheless, when the mean bias is not too high, overconfident agents earn positive expected profit when there are enough positive feedback traders. Similarly to Germain et al. (2009), we find that the mean bias affects the investors’ trading volumes and therefore their profits. Indeed, when the mean bias $b$ increases, overconfident agents trade more aggressively in the wrong direction, as a result they incur large losses. These losses may be reduced if a large number of positive feedback traders are present in the market. Finally, in figure 16, we observe
that for an overconfident trader a misinterpretation about the precision of the fundamental value $(h_v)$ is worse than an error of judgment on his specific own private information $(h_e)$.

![Graph and Diagram](image-url)

Figure 16: The total individual overconfident trader’s profit as a function of the number of positive feedback traders, for different values of the mean bias $b$ of the overconfidence. And the total individual overconfident trader’s profit as a function of the parameters $\kappa$ and $\eta$. 

These graphs illustrate the impact of various parameters on the profit of an overconfident trader in different scenarios.
5 Discussion

In this part, we are interested in understanding how price bubbles may arise in a financial market. We have shown that positive feedback trading induces an price volatility and worsens market efficiency more than overconfident trading. When positive feedback agents are introduced into the market, overconfident investors may make more profit than rational speculators. However, positive feedback traders can suffer very important losses if they are too many in the market.

On the other hand, a financial bubble can be characterized by an important gap between market price and fundamentals. Since overconfident agents receive private signals about fundamentals, they trade on private information and exploit the presence of feedback traders. Only, feedback trading can trigger a bubble. As a result overconfident traders may worsen the bubble phenomenon as well as the collapse of the bubble.\(^5\)

To see the bubble phenomenon, we have simulated prices over the time according to the number of positive feedback traders, the number of overconfident traders and the mean bias of the overconfident agents. Moreover, we also focus on the effect of the risk aversion coefficient \(\alpha\).

Figure 17 confirms that the bubble phenomenon depends mainly on the number of feedback traders. Feedback trading is based on the previous prices movement and it may appears as a lagged trading. Abreu and Brunnermeier (2003) have shown that the synchronization problem can be a reason for the persistence of bubbles. In their model, rational arbitrageurs face a lack of synchronization because there are different optimal timing trading strategies. The bubble can grow and last as long as there is a sufficient number of rational traders to sell out. In our model, when the mass of informed traders is very large in comparison with the feedback trading, bubbles cannot exist.

In figure 18, we observe that the volatility of price and the price change increase with the risk aversion coefficient. Informed investors react abruptly to new information and provoke prices to be more volatile. This result emphasizes that the risk aversion of the market participants can explain part of the excess volatility observed in the market.

Figure 19 shows that the price changes is less extreme when the market participants are less risk averse. Moreover, the difference between the overconfident investors ’demand and the rational one is more depend on the strength of the positive feedback trading than the risk aversion of the speculators.

The presence of bubbles and crashes is mainly explained by two factors: the positive feedback trading and the risk aversion of the investors. The overconfidence has an important impact on the volatility of prices, on the trading volume and on the wealth of the informed traders without

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\(^5\) Feedback trading is often cited as the reason of the bubble (see Zhou and Sornette (2006, 2009), Cajuueiro, Tabak and Werneck (2009), and Johansen, Ledoit, and Sornette (2003)).
Figure 17: Prices over time for different values of $P$ (the number of positive feedback traders) and for different values of $N_1$ (the number of overconfident agents).

Figure 18: The volatility of prices at date $t = 3$ and the $\text{cov}(P_3 - P_2, P_2 - P_1)$, for different values of the parameter $a$.

provoking the growth and the burst of bubbles. Nevertheless, in the presence of feedback trading, risk averse overconfident investors can enhance the volatility of prices and both the bubble and crash phenomenon too.

For the sake of exposition we dedicate the next section to contrarian trading, i.e. feedback traders selling (buying) when prices increase (decrease).

6 On Contrarian Trading

In this section we look at the case of contrarian trading.

First of all, it should be noticed that due to their trading behaviour, the negative feedback traders limit the movement of prices. Informed traders upon receiving some information will
always follow their information and trade larger quantities as they know that due to negative feedback trading subsequent prices will not fully reflect their information (I think! This should be checked with simulations). This alters some of the comparative statics we found earlier for the case of positive feedback trading. The reader can refer to the figures in the Appendix, at the end of the paper, for the different comparative statics.

We find the obvious result that the volatility of prices decreases with the number of negative feedback. This volatility of prices increases with the number of overconfident.

The quality of prices in both periods decreases with the number of negative feedback whereas the quality of prices in the third period increases with the number of overconfident and with $\kappa$.

The serial correlation is negative and decreases with the number of negative feedback.

The overall volume traded by rational investors increases with the number of negative feedback and decreases with the level of overconfidence in the market, $\kappa$. An interesting result arises, when we look at the overall volume traded by overconfident traders. For a low number of feedback traders, the volume traded by overconfident decreases with $P$ whereas for a large number of feedback traders it increases with $P$. The range for which it decreases with $P$ increases as the level of overconfidence, $\kappa$, increases.

Finally, we find that the expected profit of the negative feedback traders decreases with the number of negative feedback present in the market and that they can derive positive expected profits for a low number of negative feedback traders. In that case the second round gains compensate the third round losses. However, as overconfident traders become more overconfident, measured by an increase of $\kappa$, the expected profit decreases.
Conclusion

In this paper, we have analyzed the effects of overconfidence and trend-chasing behavior on financial markets.

According to our model, when there are only overconfident agents in the market, no significant bubbles arise. However, overconfidence noticeably affects trading volume and profit. Overconfident agents tend to trade very aggressively as they overreact to information. Overreaction to information is the main reason why overconfident agents increase volatility and decrease the quality of prices.

When overconfident agents are competing with rational traders, they tend to have the same behavior as described previously. They trade very aggressively and tend to increase the price volatility and to decrease the quality of the prices in the market. The higher the proportion of overconfident traders, the more these agents trade and as a reaction the less rational participants trade. Rational traders always earn larger expected profit than overconfident traders. In such a market, bubbles do not appear.

Eventually, when overconfident, rational and feedback positive traders participate in the market, the situation tends to be a little different from the two previous ones. Positive feedback traders increase price volatility. Their trade is based on past prices which are determined by the informed trading in earlier stages. This causes a temporary miscoordination between traders. This phenomenon is amplified by the fact that both overconfident and rational traders anticipate the behavior of feedback-positive agents. As a result feedback traders never generate profits. When there are few feedback traders in the market, both overconfident and rational traders are unable to earn positive expected profit. When feedback traders are very numerous, both overconfident and rational investors earn positive expected profit. In both situations, overconfident traders are always better off than rational traders.

7 References


Appendix

Proof of Proposition 2.1: Equilibrium

This proposition is proved by backward induction, we then start with the last period, i.e. \( t = 3 \).

7.0.1 Third round \( t = 3 \)

At time \( t = 3 \), each trader, noted \( i \), has private information \( \Phi_{3i} \) which has a multivariate distribution. The information available for trader \( i \) is: \( \Phi_{3i} = [y_{2i}, y_{3i}, P_{2}, P_{3}]^{T} \).

An overconfident trader infers the mean of this distribution, \( E_{b}(\Phi_{3i}) \), and the variance-covariance matrix, \( \Psi_{b} \), as follows:

\[
\Psi_{b} = \begin{bmatrix}
\frac{1}{\eta h_{v}} + \frac{1}{\kappa h_{e}} & \frac{1}{\eta h_{v}} + \frac{1}{\kappa h_{e}} & \frac{\alpha_{22}}{\eta h_{v}} + \frac{\alpha_{22}}{M_{\kappa h_{e}}} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_{v}} + \frac{\alpha_{32}+\alpha_{33}}{M_{\kappa h_{e}}} \\
\frac{\alpha_{22}}{\eta h_{v}} + \frac{\alpha_{22}}{M_{\kappa h_{e}}} & \frac{\alpha_{22}}{\eta h_{v}} + \frac{\alpha_{22}}{M_{\kappa h_{e}}} & \frac{\alpha_{32}}{\eta h_{v}} + \frac{\alpha_{32}}{M_{\kappa h_{e}}} & C_{1} \\
\frac{\alpha_{32}+\alpha_{33}}{\eta h_{v}} + \frac{\alpha_{33}}{M_{\kappa h_{e}}} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_{v}} + \frac{\alpha_{33}}{M_{\kappa h_{e}}} & \frac{\alpha_{33}}{\eta h_{v}} + \frac{\alpha_{33}}{M_{\kappa h_{e}}} & C_{2}
\end{bmatrix},
\]

with \( C_{1} = \frac{\alpha_{22}+\alpha_{33}}{\eta h_{v}} + \frac{\alpha_{22}+\alpha_{33}}{M_{\kappa h_{e}}} \left( \frac{\gamma+M_{\kappa}}{\kappa h_{v}} \right) \), and \( C_{2} = \frac{(\alpha_{22}+\alpha_{33})^{2}}{\eta h_{v}} + (\alpha_{32}+\alpha_{33})^{2} \left( \frac{\gamma+M_{\kappa}}{M_{\kappa h_{e}}} \right) \).

The mean, \( E_{r}(\Phi_{3i}) \), and the variance-covariance matrix \( \Psi_{r} \) for the rational agent are obtained by setting \( b = 0, \eta = \kappa = \gamma = 1 \) in \( E_{b}(\Phi_{3i}) \) and \( \Psi_{b} \).

By solving the mean-variance problem, we obtain the ith insider’s orders:

\[
x_{3i}^{b} = \frac{E_{b}(\tilde{v}|\Phi_{3i}) - P_{3}}{\text{avar}_{b}(\tilde{v}|\Phi_{3i})},
\]

\[
x_{3i}^{r} = \frac{E_{r}(\tilde{v}|\Phi_{3i}) - P_{3}}{\text{avar}_{r}(\tilde{v}|\Phi_{3i})}.
\]
On the other hand, we know that each feedback trader $i$ determines her order by considering the trend of prices as follows:

$$x_{3i}^f = \beta(P_2 - P_1).$$

Using the projection theorem, we obtain the following

$$E_b(\tilde{v}|\Phi_{3i}) = \frac{(y_{2i} + y_{3i})(\kappa - \gamma)h_v + (\tilde{Y}_2 + \tilde{Y}_3)\gamma h_v M + \eta h_v(\bar{v} + b)}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\varepsilon},$$

$$\text{var}_b(\tilde{v}|\Phi_{3i}) = \frac{1}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\varepsilon},$$

$$E_r(\tilde{v}|\Phi_{3i}) = \frac{(\tilde{Y}_2 + \tilde{Y}_3)h_v M + h_v \bar{v} - P_3(h_v + 2Mh_\varepsilon)}{h_v + 2Mh_\varepsilon},$$

$$\text{var}_r(\tilde{v}|\Phi_{3i}) = \frac{1}{h_v + 2Mh_\varepsilon}.$$ 

Replacing the above into the expressions of the different orders for the different types of traders we obtain

$$x_{3i}^b = \frac{1}{\Lambda}[(y_{2i} + y_{3i})(\kappa - \gamma)h_v + (\tilde{Y}_2 + \tilde{Y}_3)\gamma h_v M + \eta h_v(\bar{v} + b) - P_3(\eta h_v + 2(\kappa + \gamma M - \gamma)h_\varepsilon)],$$

$$x_{3i}^r = \frac{1}{\Lambda}[(\tilde{Y}_2 + \tilde{Y}_3)h_v M + h_v \bar{v} - P_3(h_v + 2Mh_\varepsilon)],$$

$$x_{3i}^f = \beta(P_2 - P_1).$$

In equilibrium the total demand must be equal to the exogenous total supply, this is given by

$$\sum_{i=1}^{N_1} x_{3i}^b + \sum_{i=1}^{N_2} x_{3i}^r + \sum_{i=1}^{P} x_{3i}^f = (N_1 + N_2 + P)\bar{x}. \quad (7.3)$$

From (7.3), the price $P_3$ can be obtained as a function of $\tilde{Y}_2$, $\tilde{Y}_3$, $P_2$ and $P_1$

$$P_3 = \frac{1}{\Lambda}[(N_1\gamma + N_2)Mh_\varepsilon + (\kappa - \gamma)N_1h_v) (\tilde{Y}_2 + \tilde{Y}_3) + (N_1\eta + N_2)h_v \bar{v} + N_1\eta h_v b - a(N_1 + N_2 + P)\bar{x} + aP\beta(P_2 - P_1)],$$

with $\Lambda = (N_1\eta + N_2)h_v + 2(N_1(\kappa + \gamma M - \gamma) + N_2 M)h_\varepsilon$, $\tilde{Y}_2 = \tilde{v} + \frac{\varepsilon_2}{M}$, $\tilde{Y}_3 = \tilde{v} + \frac{\varepsilon_3}{M}$ and $\bar{Y}_3 = \tilde{v} + \frac{\varepsilon_3}{M}$.

Using equation (2.1) and identifying the parameters $\alpha_{i,j}$ we obtain:
\[\alpha_{31} = \frac{(N_1 \eta + N_2) h_v \theta + N_1 \eta h_v b - a(N_1 + N_2 + P) \tilde{\varphi}}{\Lambda} + \frac{a P \beta}{\Lambda} (\alpha_{21} - P_1),\]
\[\alpha_{32} = \frac{N_1 (\gamma + \gamma M - \gamma) h_v + N_2 M h_v}{\Lambda} + a \beta P \alpha_{22},\]
\[\alpha_{33} = \frac{N_1 (\kappa + \gamma M - \gamma) h_v + N_2 M h_v}{\Lambda}.
\]

### 7.0.2 Second round \( t = 2 \)

Using the third round’s results, we can obtain the second round’s parameters.

Let us introduce the following notations where \( b \) stands for the overconfident trader whereas \( r \) stands for the rational one:

\[B^T_b = \text{cov}_b(P_3, \Phi_{2i}) = [\text{cov}_b(y_{2i}, P_3), \text{cov}_b(P_2, P_3)],\]
\[B^T_r = \text{cov}_r(P_3, \Phi_{2i}) = [\text{cov}_r(y_{2i}, P_3), \text{cov}_r(P_2, P_3)].\]

Using the projection theorem, we get

\[E_j(P_3|\Phi_{2i}) = E_j(P_3) + B^T_j \text{var}_j(\Phi_{2i})^{-1}(\Phi_{2i} - E_j(\Phi_{2i})),\]

where \( j = b, r \) and \( \text{var}_j \) denote the fact that they are computed following trader \( j \)’s beliefs.

We obtain

\[E_j(P_3|\Phi_{2i}) = (\alpha_{32} + \alpha_{33}) \bar{v}_j + \alpha_{31} + \frac{1}{\alpha_{22}}((\text{cov}_j(y_{2i}, P_3)D^j_1 + \text{cov}_j(P_2, P_3) \frac{D^j_3}{\alpha_{22}})(y_{2i} - \bar{v}) + \text{cov}_j(y_{2i}, P_3)D^j_3 + \text{cov}_j(P_2, P_3) \frac{D^j_3}{\alpha_{22}}(\bar{Y}_2 - \bar{v})),\]

\[\text{var}_j(P_3|\Phi_{2i}) = \text{var}_j(P_3) - \frac{1}{\alpha_{22}}(D^j_1 \text{cov}_j(y_{2i}, P_3)^2 + 2 \frac{D^j_3}{\alpha_{22}} \text{cov}_j(y_{2i}, P_3) \text{cov}_j(P_2, P_3)
+ \frac{D^j_3}{\alpha_{22}} \text{cov}_j(P_2, P_3)^2),\]

with

\[\bar{v}_j = \begin{cases} \bar{v} & \text{if } j = r \\ \bar{v} + b & \text{if } j = b \end{cases},\]
\[D^b_1 = \text{var}_b(\bar{Y}_2) = \frac{1}{\eta_{hv}} + \frac{(\gamma + M \kappa - \kappa)}{\eta_{hv} \epsilon},\]
\[D^b_2 = \text{var}_b(y_{2i}) = \frac{1}{\eta_{hv}} + \frac{1}{\eta_{hv} \epsilon},\]
\[D^b_3 = -\text{cov}_b(y_{2i}, \bar{Y}_2) = -\left(\frac{1}{\eta_{hv}} + \frac{1}{\eta_{hv} \epsilon}\right),\]
\[L_b = \text{var}_b(y_{2i}) \text{var}_b(\bar{Y}_2) - \text{cov}_b(y_{2i}, \bar{Y}_2)^2 = \frac{(M-1)((M-1)\gamma + \kappa) \eta_{hv} + \eta_{hv}}{\Lambda^2} + \frac{2}{\Lambda^2} \eta_{hv} \epsilon \epsilon,\]

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\( D_1', D_1', D_3' \), and \( L_r \) can be obtained by setting \( \kappa = \gamma = \eta = 1 \) in \( D_1^b, D_1^b, D_3^b \), and \( L_b \).

At the second round, informed trader \( i \)’s order is:

\[
\begin{align*}
  x_{2i}^b &= \frac{E_b(P_3|\Phi_{2i}) - P_2}{avar_b(P_3|\Phi_{2i})}, \\
  x_{2i}^r &= \frac{E_r(P_3|\Phi_{2i}) - P_2}{avar_r(P_3|\Phi_{2i})}.
\end{align*}
\]

The \( i \)th feedback agent’s order is:

\[ x_{2i}^f = \beta(P_1 - P_0). \]

By equalizing exogenous supply and demand, we have:

\[ (N_1 + N_2 + P)\bar{x} = \sum_{i=1}^{N_1} \frac{E_b(P_3|\Phi_{2i}) - P_2}{avar_b(P_3|\Phi_{2i})} + \sum_{i=1}^{N_2} \frac{E_r(P_3|\Phi_{2i}) - P_2}{avar_r(P_3|\Phi_{2i})} + P\beta(P_1 - P_0). \]

### 7.0.3 First round \( t = 1 \)

At the first round, none of the traders participating to the market are informed. The different agents’ orders are:

\[
\begin{align*}
  x_{1i}^r &= \frac{E_r(P_2) - P_1}{avar_r(P_2)} = \frac{\alpha_{21} + \alpha_{22}\bar{v} - P_1}{\alpha_{22}^2 \text{var}_r(\bar{Y}_2)}, \\
  x_{1i}^b &= \frac{E_b(P_2) - P_1}{avar_b(P_2)} = \frac{\alpha_{21} + \alpha_{22}(\bar{v} + b) - P_1}{\alpha_{22}^2 \text{var}_b(\bar{Y}_2)},
\end{align*}
\]

There are no feedback traders in the first round. However, the agents who will become informed subsequently anticipate the presence of such behavior for the next two rounds.

Again, the no-excess supply equation leads to

\[ (N_1 + N_2)\bar{x} = \sum_{i=1}^{N_1} x_{1i}^b + \sum_{i=1}^{N_2} x_{1i}^r. \]

The price \( P_1 \) can then be derived:

\[
P_1 = \alpha_{21} + \alpha_{22}\bar{v} + \frac{-\alpha_{22}^2\bar{x}(N_1 + N_2)\text{var}_r(\bar{Y}_2)\text{var}_b(\bar{Y}_2) + N_1\alpha_{22}\text{var}_r(\bar{Y}_2)}{N_1\text{var}_r(\bar{Y}_2) + N_2\text{var}_b(\bar{Y}_2)}.
\]
From the above expression the parameters $\alpha_{21}$ and $\alpha_{22}$ can be identified.

After some computations, one can show that $\alpha_{22} = \frac{N}{\alpha_{21}P + d}$ where $N$ is independent of $P$, with

$$c = -[Z_1(z h_v + \eta h_v) + Z_2(M h_v + h_v)],$$
$$d = \Lambda \eta h_v M^2 \kappa \gamma h_v^2 L_b L_r (N_1 \text{var}_r(P_3|\Phi_{2i}) + N_2 \text{var}_b(P_3|\Phi_{2i})),
N = Z_1 Z_3 h_v (\eta h_v + 2 h_v j) + Z_2 Z_3 h_v (h_v + 2 M h_v),$$

$$Z_1 = N_1 \text{var}_r(P_3|\Phi_{2i}) L_r (M - 1),$$
$$Z_2 = N_2 \text{var}_b(P_3|\Phi_{2i}) L_b (M - 1) \eta \kappa \gamma,$$
$$Z_3 = N_1 z + N_2 M,$$
$$z = \kappa + \gamma M - \gamma,$$
$$\text{var}_b(P_3|\Phi_{2i}) = \frac{\alpha_{23}^2 \left[ \eta \left( \kappa (M - 1) + \gamma \right) h_v + \left( (M - 1) (\kappa - \gamma)^2 + 2 M \kappa \gamma \right) h_v \right]}{M^2 \gamma \kappa h_v (z h_v + \eta h_v)},$$
$$L_b = \frac{M^2 \gamma + M (\kappa + \eta - 2 \gamma) + \gamma - \kappa - \eta}{\eta \gamma \kappa M^2 h_v h_v},$$

where $\text{var}_r(P_3|\Phi_{2i})$ and $L_r$ can be obtained by setting $\gamma = \kappa = \eta = 1$ in the expressions of $\text{var}_b(P_3|\Phi_{2i})$ and $L_b$.

It can be shown after some tedious computations that $c < 0$, $d > 0$ and $N > 0$.

Once $\alpha_{22}$ has been determined, the other parameters can be derived.

**Proof of Proposition 3.2**

We now derive the variance of prices with respect to $P$.

$t = 2$

The variance is given by $\text{var}(P_2) = \alpha_{22}^2 \text{var}(Y_2)$ with $\alpha_{22} = \frac{N}{\alpha_{21}P + d}$. After some tedious computations it can be shown that $c < 0$, $d > 0$ and $N > 0$.

The derivative of $\text{var}(P_2)$ with respect to $P$ is then equal to

$$\frac{\partial \text{var}(P_2)}{\partial P} = \frac{\partial \alpha_{22}^2}{\partial P} \text{var}(Y_2) = 2 \alpha_{22} \frac{\partial \alpha_{22}}{\partial P} \text{var}(Y_2) = 2 \alpha_{22} \left( -c \frac{N}{(cP + d)^2} \right) \text{var}(Y_2).$$

Given that $\text{var}(Y_2) > 0$, $\alpha_{22} > 0$ and $-c \frac{N}{(cP + d)^2} > 0$ since $c < 0$ and $N > 0$, it can be established that $\frac{\partial \text{var}(P_2)}{\partial P} > 0$. 

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The variance for \( t = 3 \) prices is given by

\[
var(P_3) = \alpha_{32}^2 \var(Y_2) + \alpha_{33}^2 \var(Y_3) + 2 \alpha_{32} \alpha_{33} \cov(Y_2, Y_3),
\]

\[
= (\alpha_{32}^2 + \alpha_{33}^2) \left( \frac{1}{h_v} + \frac{1}{M h_\epsilon} \right) + \frac{2 \alpha_{32} \alpha_{33}}{h_v}.
\]

Using the fact that \( \alpha_{32} = \alpha_{33} + \frac{a P_\beta}{\Lambda} \alpha_{22} \), we can rewrite the variance of prices as follows

\[
var(P_3) = (\alpha_{33} + \frac{a P_\beta}{\Lambda} \alpha_{22}) [\left( \frac{1}{h_v} + \frac{1}{M h_\epsilon} \right) + \frac{2 \alpha_{33}}{h_v} + \frac{1}{M h_\epsilon}] + \alpha_{33}^2 \left( \frac{1}{h_v} + \frac{1}{M h_\epsilon} \right).
\]

The derivative is then given by the following expression

\[
\frac{\partial \var(P_3)}{\partial P} = \frac{2 a P_\beta \alpha_{22}}{\Lambda} \left( \frac{\alpha_{33} + \frac{a P_\beta}{\Lambda} \alpha_{22}}{h_v} \right) (\frac{1}{h_v} + \frac{1}{M h_\epsilon}) > 0.
\]

**Proof of Proposition 3.3**

We now look at the quality of prices. We first start with \( t = 2 \) and then turn to \( t = 3 \).

The quality of prices for \( t = 2 \) is given by \( var(P_2 - \bar{v}) \) which is given by the following expression

\[
var(P_2 - \bar{v}) = \frac{\alpha_{22}^2}{h_v} + \frac{\alpha_{22}^2}{M h_\epsilon} + \frac{1}{h_v} - 2 \frac{\alpha_{22}}{h_v} = \frac{(\alpha_{22} - 1)^2}{h_v} + \frac{\alpha_{22}^2}{M h_\epsilon}.
\]

The derivative with respect to \( P \) is positive and given by

\[
\frac{\partial var(P_2 - \bar{v})}{\partial P} = \frac{2(\alpha_{22} - 1)}{h_v} \frac{\partial \alpha_{22}}{\partial P} + \frac{2 \alpha_{22}}{M h_\epsilon} \frac{\partial \alpha_{22}}{\partial P} = 2 \frac{\partial \alpha_{22}}{\partial P} \left( \frac{\alpha_{22} - 1}{h_v} + \frac{\alpha_{22}}{M h_\epsilon} \right) > 0.
\]

For the quality of prices for \( t = 3 \), we proceed as for \( t = 2 \). The quality of prices is equal to

\[
var(P_3 - \bar{v}) = \frac{(\alpha_{32} + \alpha_{33} - 1)^2}{h_v} + \frac{\alpha_{32}^2 + \alpha_{33}^2}{M h_\epsilon}.
\]

The derivative is then

\[
\frac{\partial var(P_3 - \bar{v})}{\partial P} = \frac{\partial \alpha_{32}}{\partial P} \left( \frac{2(\alpha_{32} + \alpha_{33} - 1)}{h_v} + \frac{2 \alpha_{32}}{M h_\epsilon} \right) > 0.
\]

**Proof of Proposition 3.4**

We now look at the serial correlation of prices. It is given by \( \cov(P_3 - P_2, P_2 - P_1) \).
We have
\[
cov(P_3 - P_2, P_2 - P_1) = cov(P_3 - P_2, P_2),
= cov(P_3, P_2) - cov(P_2, P_2).
\]

After some manipulations, it can be rewritten as
\[
cov(P_3 - P_2, P_2 - P_1) = \alpha_{22} \left( \var(Y_2 \mid \alpha_{32} - \alpha_{22}) + \alpha_{32} \text{cov} \left( \bar{Y}_2, \bar{Y}_3 \right) \right).
\]

Using the fact that \( \alpha_{32} = \frac{a \beta P}{\Lambda} \alpha_{22}, \text{cov} \left( \bar{Y}_2, \bar{Y}_3 \right) = \frac{1}{h_v} \) and that \( \var(Y_2) = \frac{1}{h_v + M h_z} \), we obtain
\[
cov(P_3 - P_2, P_2 - P_1) = \alpha_{22} \left[ \left( \alpha_{33} + \alpha_{22} \left( \frac{a \beta P}{\Lambda} - 1 \right) \right) \frac{1}{h_v + M h_z} + \frac{\alpha_{33}}{h_v} \right].
\]

The derivative with respect to \( P \) is then given by
\[
\frac{\partial \text{cov}(P_3 - P_2, P_2 - P_1)}{\partial P} = \frac{\partial \alpha_{22}}{\partial P} \alpha_{33} \left( \frac{2}{h_v} + \frac{1}{M h_z} \right) + \left( \frac{a \beta P}{\Lambda} - 1 \right) \left( \alpha_{32} \frac{\partial \alpha_{22}}{\partial P} + \alpha_{22} \right) \frac{1}{h_v + M h_z}.
\]

**Proof of Proposition 4.6**

We now compute the expected profits for all type of traders for each trading round.

**Third round profits**

The optimistic traders’ expected profit, \( \Pi_{b_{3i}}^b \), is given by
\[
\Pi_{b_{3i}}^b = E(x_{3i}^b (\bar{v} - P_3)) = E(x_{3i}^b \bar{v}) - E(x_{3i}^b P_3),
\]
where the demand, \( x_{3i}^b \), is
\[
x_{3i}^b = \alpha^* (y_{2i} + y_{3i}) + \beta^* (Y_2 + Y_3) - \gamma^* P_3 + \delta^*,
\]
with \( \alpha^* = \frac{(\kappa - \gamma) h_v}{a}, \beta^* = \frac{\gamma M h_z}{a}, \gamma^* = \frac{\eta h_v + 2(\kappa + \gamma)(M - 1) h_z}{a} \) and \( \delta^* = \frac{\eta h_v (\bar{v} + b)}{a} \).

After having computed the two elements of the expected profit and after some simplifications we obtain
\[ \Pi_{3i}^b = (\alpha^* + \beta^*) \left[ \frac{1}{h_v} + 2\tilde{v}^2 - 2\alpha_{31}\tilde{v} - (\alpha_{32} + \alpha_{33})\left( \frac{2}{h_v} + 2\tilde{v}^2 + \frac{1}{Mh_e} \right) \right] - \gamma^*[\alpha_{31}(1 - 2(\alpha_{32} + \alpha_{33}))\tilde{v} - \alpha_{31}^2 + \\
\left( \alpha_{32} + \alpha_{33} \right)(1 - (\alpha_{32} + \alpha_{33}))\left( \frac{1}{h_v} + \tilde{v}^2 \right) - \frac{\alpha_{32}^2 + \alpha_{33}^2}{Mh_e} - \alpha_{31}^2] \\
+ \delta^*[\tilde{v}(1 - (\alpha_{32} + \alpha_{33})) - \alpha_{31}]. \tag{7.4} \]

We now focus on the rational traders’ expected profit, \( \Pi_{3i}^r \). In order to compute \( \Pi_{3i}^r \) the same steps as for calculating the expected profit of the optimistic traders can be followed. The demand for the rational trader, \( x_{3i}^r \), admits the same linear form as the demand for the optimistic trader. The coefficients are given by \( \alpha^* = 0, \beta^* = \frac{Mh_e}{a}, \gamma^* = \frac{h_v + 2Mh_e}{a} \) and \( \delta^* = \frac{h_v}{a} \).

The rational traders’ expected profit, \( \Pi_{3i}^r \), can be obtained from (7.4) by replacing \( \kappa = \gamma = \eta = 1 \) and \( b = 0 \).

The feedback traders’ expected profit \( \Pi_{3i}^f \) depends on the first and second round prices \( P_1 \) and \( P_2 \) through their demand \( x_{3i}^f = \beta(P_2 - P_1) \). It can be computed by computing the following expression

\[ \Pi_{3i}^f = E(x_{3i}^f(\tilde{v} - P_3)) = E(x_{3i}^f\tilde{v}) - E(x_{3i}^fP_3). \]

After some computations and simplifications, it is given

\[ \Pi_{3i}^f = \beta \left[ (\alpha_{21} - P_1)(\alpha_{31} + (1 - (\alpha_{32} + \alpha_{33}))\tilde{v}) - \alpha_{22}\alpha_{31}\tilde{v} \right] + \beta \left[ \left( \frac{1}{h_v} + \tilde{v}^2 \right)(\alpha_{22}(1 - \alpha_{32} - \alpha_{33})) - \alpha_{22}\alpha_{32}\frac{1}{Mh_e} \right]. \]

**Second round profit**

Similarly to the third round, we have already calculated the price at the second round \( P_2 \) and the terms \( \alpha_{21} \) and \( \alpha_{22} \). We can now determine the optimistic traders’ expected profit \( \Pi_{2i}^o \). As before the expected profit is given by

\[ \Pi_{2i}^o = E(x_{2i}^o(\tilde{v} - P_2)) = E(x_{2i}^o\tilde{v}) - E(x_{2i}^oP_2), \]

where \( x_{2i}^o = \frac{E_b[P_b/\Phi_2i][-P_2]}{\alpha\vartheta a[P_b/\Phi_2i]} \).

After some computations, we obtain

\[ \Pi_{2i}^o = \left( S_b + T_b - \alpha_{22} \right) \left( \frac{1}{h_v} + \tilde{v}^2 - \alpha_{21}\tilde{v} - \alpha_{22} \left( \frac{1}{h_v} + \tilde{v}^2 + \frac{1}{Mh_e} \right) \right), \]

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with

\[
A_b = \alpha_{31} + (\alpha_{33} + \alpha_{22})(\bar{v} + b) - (\text{cov}_b(\tilde{y}_{2i}, P_3))(D_1^b + D_3^b)
+ \left(\frac{\text{cov}_b(P_2, P_3)(D_2^b + D_3^b)}{\alpha_{22} L_b}\right)\frac{\bar{v} + b}{L_b},
\]

\[
S_b = \frac{\text{cov}_b(\tilde{y}_{2i}, P_3)}{L_b} D_1^b + \frac{\text{cov}_b(P_2, P_3)}{\alpha_{22} L_b} D_3^b,
\]

\[
T_b = \frac{\text{cov}_b(\tilde{y}_{2i}, P_3)}{L_b} D_3^b + \frac{\text{cov}_b(P_2, P_3)}{\alpha_{22} L_b} D_2^b.
\]

The expected profit of the rational traders can be computed following the same steps, we then obtain that

\[
\Pi_r^{2i} = \frac{1}{\text{a. v. r. } (P_3/\Phi_{2i})} \left((A_r - \alpha_{21}) (-\alpha_{21} + (1 - \alpha_{22})\bar{v})
+ (\text{cov}_r(P_2, P_3)(D_0^r + D_1^r))\frac{\bar{v}}{L_r}\right),
\]

with

\[
A_r = \alpha_{31} + (\alpha_{33} + \alpha_{22})(\bar{v}) - (\text{cov}_r(\tilde{y}_{2i}, P_3))(D_1^r + D_3^r)
+ \left(\frac{\text{cov}_r(P_2, P_3)(D_2^r + D_3^r)}{\alpha_{22} L_r}\right)\frac{\bar{v}}{L_r}.
\]

\[
S_r = \frac{\text{cov}_r(\tilde{y}_{2i}, P_3)}{L_r} D_1^r + \frac{\text{cov}_r(P_2, P_3)}{\alpha_{22} L_r} D_3^r,
\]

\[
T_r = \frac{\text{cov}_r(\tilde{y}_{2i}, P_3)}{L_r} D_3^r + \frac{\text{cov}_r(P_2, P_3)}{\alpha_{22} L_r} D_2^r.
\]

The expected profit of the feedback traders is given by:

\[
\Pi_f^{2i} = E(x_{2i}^f(\tilde{v} - P_2)) = E(x_{2i}^f\tilde{v}) - E(x_{2i}^f P_2),
\]

with \(x_{2i}^f = \beta(P_1 - P_0)\).

The prices \(P_1\) and \(P_0\) are known before trading, the expected profit can be written as

\[
\Pi_f^{2i} = \beta(P_1 - P_0)(\tilde{v}(1 - \alpha_{22}) - \alpha_{21}).
\]

**First Round profit**

At the first round only the rational and the optimistic agents are trading. However, they are not informed at \(t = 1\). As a consequence, the price at the first round \(P_1\) is known before any trade is done.
The expected profit for the optimistic traders is given by
\[
\Pi_{b1}^i = E[x_{1i}^b(\bar{v} - P_1)],
\]
with \( x_{1i}^b = \frac{\alpha_{21} + \alpha_{22}(\bar{v} + b)}{a \alpha_{22}^2 \var_{r}(Y_2)} - P_1 \). It is straightforward to find that
\[
\Pi_{b1}^i = \frac{\alpha_{21} + \alpha_{22}(\bar{v} + b) - P_1}{a \alpha_{22}^2 \var_{r}(Y_2)} (\bar{v} - P_1).
\]
The expected profit for the rational traders’ profit is given by
\[
\Pi_{r1}^i = \frac{\alpha_{21} + \alpha_{22}(\bar{v} - P_1)}{a \alpha_{22}^2 \var_{r}(Y_2)} (\bar{v} - P_1).
\]

7.1 Graphs on Contrarian trading

7.1.1 Volatility of prices

![Graph showing volatility of prices](image)

Figure 20: The variance of prices at time \( t = 3 \) as a function of the number of feedback traders and of the number of overconfident traders.

7.1.2 Quality of Prices

![Graph showing quality of prices](image)
Figure 21: The variance of prices at time $t = 3$ as a function of the number of feedback traders and of $\gamma$ and $\eta$.

Figure 22: The variance of prices at time $t = 3$ as a function of the number of feedback traders and of the number of overconfident traders for different values of $\kappa$. 

\[ \text{var}(P^3) = f(P) \text{ for several values of } \gamma, \eta, \kappa. \]
\[ \text{var}(P_3 - V) = f(N_1) \] for several values of \( \kappa \)

1. \( \kappa = 1 \)
2. \( \kappa = 2 \)
3. \( \kappa = 3 \)
4. \( \kappa = 4 \)
5. \( \kappa = 5 \)

\[ \text{var}(P_3 - V) = f(N_1) \] for several values of \( \eta \)

1. \( \eta = 0.2 \)
2. \( \eta = 0.4 \)
3. \( \eta = 0.6 \)
4. \( \eta = 0.8 \)
5. \( \eta = 1 \)

\[ \text{var}(P_3 - V) = f(N_1) \] for several values of \( \gamma \)

1. \( \gamma = 0.2 \)
2. \( \gamma = 0.4 \)
3. \( \gamma = 0.6 \)
4. \( \gamma = 0.8 \)
5. \( \gamma = 1 \)
7.1.3 Serial Correlation

![Graph showing serial correlation for different values of kappa](image)

7.1.4 Volume

![Graph showing volume for different values of kappa](image)
7.1.5 Expected Profits
$\eta = \gamma = 0.5$

- Aggregate profit feedback $= f(P)$ for several values of $\kappa$
- Rational trading volume $= f(P)$ for several values of $\kappa$