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Multiobjective genetic algorithm strategies for electricity production from generation IV nuclear technology

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\textbf{A B S T R A C T}

Development of a technico-economic optimization strategy of cogeneration systems of electricity/hydrogen, consists in finding an optimal efficiency of the generating cycle and heat delivery system, maximizing the energy production and minimizing the production costs. The first part of the paper is related to the development of a multiobjective optimization library (MULTIGEN) to tackle all types of problems arising from cogeneration. After a literature review for identifying the most efficient methods, the MULTIGEN library is described, and the innovative points are listed. A new stopping criterion, based on the stagnation of the Pareto front, may lead to significant decrease of computational times, particularly in the case of problems involving only integer variables. Two practical examples are presented in the last section. The former is devoted to a bicriteria optimization of both exergy destruction and total cost of the plant, for a generating cycle coupled with a Very High Temperature Reactor (VHTR). The second example consists in designing the heat exchanger of the generating turbomachine. Three criteria are optimized: the exchange surface, the exergy destruction and the number of exchange modules.

1. Introduction

Real world engineering problems involve different kinds of variables, from the most common ones as continuous variables (pressure, temperature . . .) to integer type (number of heat exchangers, . . .) or binary variables (choice between alternatives, existence of a given unit into a process, . . .), linked by numerous constraints. Furthermore, several objective functions (cost, efficiency, . . .) have to be optimized simultaneously.

This work is attached to the Gas Turbine – Modular High temperature Reactor (GT-MHR) project developed by the Commissariat à l’Energie Atomique – French governmental research agency for nuclear energy (CEA). The problem under consideration is to study the possibility of coupling a Helium cooled reactor, described by Kiyushin et al. [1], with an innovative direct Brayton cycle (see Fig. 7). This original concept was modified by a new heat source provided by a Very High Temperature Reactor, generation IV nuclear reactor (VHTR) delivering 950 °C to the generating cycle. This heat can be used for the cogeneration of electricity/hydrogen by thermochemical decomposition of water. The cycle sulfur/iodine, using Helium like primary coolant, is the most advanced at the CEA; its production and its investment cost have been quantified.

The general objective is to develop a technico-economic optimization strategy of cogeneration systems of electricity/hydrogen, with a nuclear reactor of 4th generation. The main goal of the study is to find an optimal efficiency of the generating cycles and heat delivery system, maximizing the energy production and minimizing the production costs, which constitutes a multiobjective problem. Of course, all the practical optimization problems cannot be presented in a single paper. That is why this study is restricted to the development of a multiobjective optimization library, in order to tackle all types of problems arising from cogeneration of electricity/hydrogen from nuclear technology, and then two practical problems of the nuclear field are presented and solved. The MULTIGEN library proposes a simple user interface well suited to complex constrained multiobjective optimization problems, on Excel workbooks for compatibility reasons with the simulation tools developed by the CEA.

After a literature analysis for identifying the most efficient numerical methods in multiobjective optimization field, the components of the MULTIGEN library are presented; the innovative points are listed in the corresponding section that ends by three numerical examples. Two examples of multiobjective optimization related to the general nuclear problem under consideration are
2. Solution strategy

2.1. General formulation of a multiobjective constrained problem

2.1.1. Definition of optimality for multiobjective problems

Like many real world examples, the problem under consideration involves several competing measures of performance, or objectives [2]. Using the formulation of multiobjective constrained problems of Fonseca and Fleming [3], a general multiobjective problem is made up a set of $n$ criteria $f_k, k = 1, \ldots, n$ to be minimized or maximized. Each $f_k$ may be nonlinear, but also discontinuous with respect to some components of the general decision variable $x$ in an $m$-dimensional universe $U$.

$$f(x) = (f_1(x), \ldots, f_n(x))$$

This kind of problem has not a unique solution in general, but presents a set of non-dominated solutions named Pareto-optimal set or Pareto-optimal front. The Pareto-dominion concept lies on two basic rules: in the universe $U$ a given vector $u = (u_1, \ldots, u_m)$ dominates another vector $v = (v_1, \ldots, v_m)$, if and only if,

$$\forall i \in \{1, \ldots, n\}: u_i < v_i \land \exists j \in \{1, \ldots, n\}: u_j > v_j$$

For a concrete mathematical problem, Eq. (2) gives the following definition of the Pareto front: for a set of $n$ criteria, a solution $f(x)$, related to a decision variable vector $x = (x_1, \ldots, x_m)$, dominates another solution $f(y)$, related to $y = (y_1, \ldots, y_m)$ when the following condition is checked (for a minimization problem),

$$\forall i \in \{1, \ldots, n\}: f_i(x) \leq f_i(y) \land \exists j \in \{1, \ldots, n\}: f_j(x) < f_j(y)$$

On a given set of solutions, it is possible to distinguish non-dominated sets. This property implies a possible reduction of an $n$-dimension optimization problem, to a one-dimension only, according to Jensen [4]; several algorithms as example NSGA II by Deb et al. [5] use this Pareto based ranking principle. The last definition concerns the Pareto optimality: a solution $x \in U$ is called Pareto-optimal if and only if there is no $x_e \in U$ for which $v = f(x_e) = (v_1, \ldots, v_m)$ dominates $u = f(x) = (u_1, \ldots, u_m)$. These Pareto-optimal non-dominated individuals represent the solutions of the multiobjective problem. In practice, the decision maker has to select a single solution by searching among the whole Pareto front, and it may be difficult to pick one "best" solution out of a large set of alternatives. Branke et al. [6], and Taboada and Coit [7] suggest picking the knees in the Pareto front, that is to say, solutions where a small improvement in one objective function would lead to a large deterioration in at least one other objective.

2.1.2. Constraint handling

Constrained multiobjective optimization is the most common kind of problem in engineering applications. In general, three kinds of constraints are considered: simple inequality ($\leq$), strict inequality ($<$), and equality:

$$g(x) \leq c_1 \quad (\text{constr} 1(x) = c_1 - g(x) > 0)$$
$$r(x) < c_2 \iff (\text{constr} 2(x) = c_2 - r(x) > 0)$$
$$h(x) = c_3 \quad (\text{constr} 3(x) = c_3 - h(x) = 0)$$

where $(g, r, h)$ are real-valued functions of a decision variable $x = (x_1, \ldots, x_m)$ on an $m$-dimensional decisional search space $U$, and $(c_1, c_2, c_3)$ are constant values. In the more general case, these constraints are written as vectors of the type:
where \( n_1, n_2, \) and \( n_3 \) are respectively, the number or inequality, strict inequality and equality constraints. This formulation implies that each constraint value will be negative if and only if this constraint is violated. The conversion of Eq. (4), that is a classical representation of constraints set, to Eq. (3) representation constitutes the first step of a unified formulation of constrained-optimization problems. In practice, due to round-off error on real numbers, the equality constraint \( \text{constr} \) was modified as follows:

\[
\text{constr}^3(x) = (-|c_3 - h(x)|_1 + \varepsilon_1, \ldots, -|c_3 - h(x)|_{n_3} + \varepsilon_{n_3})
\]

\[
\text{constr}^3(x) + \bar{\varepsilon} \quad \bar{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_{n_3}), \quad \forall i \in \{1, \ldots, n_3\}, \quad \varepsilon_i \in R
\]

\( \bar{\varepsilon} \) is called a "precision vector" of the equality vector, and takes low values (less than \( 10^{-6} \) for example). This approximation is not necessary when equality constraint involves only integer or binary variables.

From Eqs. (5) and (6), the constraint satisfaction implies the maximization of violated constraints in vectors \( \text{constr}^1, \text{constr}^2, \) and \( \text{constr}^3 \). According to Fonseca and Fleming [3], the satisfaction of a number of violated inequality constraints is, from Eq. (5), a multiobjective maximization problem. From a theoretical point of view, a constrained multiobjective optimization problem can be formulated as a two-step optimization problem. The first step implies the comparison of constraint satisfaction degrees between two solutions, using the Pareto’s domination definition of Eq. (3), but a more simple solution consists in comparing the sum of values of violated constraints only, as in NSGA II algorithm of Deb et al. [5], which implies there are no priority rules between constraints. This step is performed first, before the second one, which concerns the comparison of the objective function vectors.

2.2. Evolutionary optimization

An evolutionary procedure is a heuristic method for solving a large class of combinatorial problems by combining user-given black-box procedures whose derivatives are not available, with heuristics in the hope of obtaining a good solution for the problem. Some heuristics work on pool of states containing several candidate states. The new states (evolution) are generated by combination or crossover of two or more states of the pool. Since 1975, many evolutionary procedures appear. For example, one can cite genetic algorithms [8], simulated annealing [9], artificial immune systems [10], ant colonies [11], particle swarms [12], differential evolution [13], tabu search [14], constraint propagation [15], artificial bee colonies [16], artificial neural networks [17], and Monte Carlo based-method [18].

All these algorithms can be adapted to the multiobjective case, and as it can be observed in the list of references proposed by Coello Coello [19]. Since 1990, the number of published papers per year is very important (more than 3700). Starting from a quasi null value in 1990, this number continuously increases, reaching 100 in 1995, 200 in 2000, 400 in 2005 and 500 the last year. How can find one’s way again in this jungle of papers? The analysis of the list shows that genetic algorithms are cited in almost 40% of cases, and far behind them they are simulated annealing and particle swarms, followed by tabu search and differential evolution algorithms, then come the ant colonies and artificial neural networks, followed by constraint propagation methods, honeybee colonies, artificial immune systems and Monte-Carlo procedures. The two most popular methods in the chemical engineering field are Multi-Objective Genetic Algorithm (MOGA, see [20]), and MultiObjective Simulated Annealing (MOSA, see [21–23]). None of these two methods is perfect and selecting one depends on the requirements of the particular design situation considered. From the literature survey [24,6,25,26] it appears that MOGA is generally preferred to MOSA.

3. General description of the genetic algorithm library MULTIGEN

3.1. Components of MULTIGEN

Faced to the diversity of mathematical problems, it is recognized that there is no a unique and general algorithm able to solve all the problems perfectly. Actually, method efficiency for a particular example is hardly predictable, and the only certainty we have is expressed by the No Free Lunch theory [27]: there is no method that outdoes all the other ones for any considered problem. This feature generates a common lack of explanation concerning the use of a method for the solution of a particular example, and usually, no relevant justification for its choice is given a priori.

A possible solution is to develop several algorithms, distinguishing them by their structure and by their type of variables (continuous, integer, binary) and collect them into a database: MULTIGEN lies on this principle, and currently, six different algorithms are available (Table 1). The aim was to treat multiobjective constrained-optimization problems involving mixed variables (bolean, integer, real) and some of these problems can be structural optimization ones. This library must be compatible with the tools developed by the CEA (COPERNIC, CYCLOP, SEMER, see Haubenbass et al. [28]) which are written in VBA So an Excel interface was created. Different types of variable coding (real, bolean, integer) are provided. Constraints as well as Pareto domination principles must be handled by the algorithms. In that way, procedures based on independent objectives to carry out the selection (like VEGA, Schaffer [29]) are not adapted to the considered problems. Procedures based of the niche notion (NPGA – Horn et al. [30], MOGA – Fonseca and Fleming [31]) cannot guarantee a correct convergence of the Pareto front, due to the low diversity of generated populations. On the contrary, methods like SPEA [32] and NSGA II [5] favor not dominated isolated individuals. In SPEA, the probability of selection is a function of the individual isolation, which is quite difficult to implement. In NSGA, individuals from the most crowded zones are eliminated according to a crowding sorting. Taking into account all the previous items, NSGA II was chosen as a basis of development of the MULTIGEN library, summarized in the following Table.

3.1.1. Non-Sorted Genetic Algorithm II (NSGA II)

The first algorithm coded in the MULTIGEN database is NSGA II by Deb et al. [5]. This elitist algorithm is based on a ranking procedure, where the rank of each solution is defined as the rank of the Pareto front to which it belongs. The diversity of non-dominated solutions is guaranteed by using a crowding distance measurement, which is an estimation of the size of the largest cuboid enclosing a given solution without including any other. This
crowding sorting avoids the use of the sharing parameter used in the previous version of the NSGA algorithm.

3.1.2. Non-Sorted Genetic Algorithm II (NSGAIiII): SBX crossover modified version

NSGA II SBX-modified algorithm uses a different SBX crossover operator than the one of the first NSGA II version of Deb and Agrawal [33]. This new operator differs from the classical one by the crossover probability allocation for each gene. The “SBX Modified Crossover” has a global probability of crossover per gene higher than the NSGA II version. Consequently, the SBX modified version carries out a more efficient gene mixing (see Gomez [34]).

3.1.3. NSGAIIb: new genetic operators for clone creation limiting

NSGA IIb implements the same algorithm than NSGA II, with corrections on crossover operator to avoid the creation of clones inherent of SBX original version. When the generated random number used to perform the crossover is greater than a given crossover probability, the crossover may produce two children identical to the parents: SBX crossover coded in NSGA IIb includes a forced mutation of children when this event occurs. The objective is to avoid unnecessary calculations for clones of existing solutions: all solutions generated by the reproduction procedure are statistically different.

3.1.4. NSGA II Mixed Integer (MI)

NSGA II MI, is identical to NSGA II, with the same crossover and mutation operators for continuous and integer variables. The crossover and mutation operation use the Simulated Binary Crossover and the parametric mutation described by Deb and Agrawal [33], considering each integer variable as a continuous value. Using the space change method proposed by Shopova and Bancheva [35] after the mutation and crossover in the continuous space, the continuous variable \( u^k \) related to integer variable \( i \) of the decision variable \( u = (u_1, \ldots, u_n) \) is reduced to \( \tilde{u}^i \) in the continuous range \([0, 1] \) by using Eq. (7). The decision variable \( u^k \) recovers an integer value with Eq. (8).

\[
\tilde{u}^i = \frac{u_i - u_i^{\min}}{u_i^{\max} - u_i^{\min}}
\]
\[
u_i = \begin{cases} 
0 \leq \tilde{u}^i < 1, & u_i^\text{min} + \tilde{u}^i \times (u_i^{\max} - u_i^{\min} + 1) \\
\text{if } \tilde{u}^i = 1, & u_i^{\max} 
\end{cases}
\]

3.1.5. NSGA II MIB

NSGA II MIB implements another version of NSGA II MI algorithm for problems involving also binary variables, by adding binary crossover and mutation rules to the existing MI operators, as represented respectively on Figs. 1 and 2. If a random number, generated in the range \([0, 1] \) for performing the crossover, is greater than a given probability, the crossover consists of a simple permutation of binary values, and the mutation by a classic change from 0 to 1 or from 1 to 0.

3.1.6. MIB MOGA structural (MMS)

The MMS algorithm, developed for further design of complex plants, uses the MIB operators with a particular rule taking into account structural links between binary and continuous or integer variables. The problem arises when coding a process structure, for example the existence or not of a component in a flowsheet with a binary value and a continuous variable (operating condition of the component), or an integer one (number of process stages in the component). For the sake of illustration, when designing a classical chemical process, an alternative between a plug-flow reactor and a series of three continuous stirred tank reactors may exist (see Fig. 3). The choice is performed according to the binary variables \( y_1 \) and \( y_2 \) where:

\[
y_1 + y_2 = 1
\]

The links between the binary variables and the continuous ones representing the volumes are:

If \( y_1 = 0 \) then \( V_{pfr} = 0 \)
If \( y_2 = 0 \) then \( V_1 = V_2 = V_3 = 0 \)

An example of this particular crossover operator is presented on Fig. 4, where there is one link between binary and continuous variables. The crossover operation includes two steps: first, crossing

Table 1
Algorithms in the MULTIGEN database.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Continuous variable</th>
<th>Integer variable</th>
<th>Binary variable</th>
<th>Continuous problem</th>
<th>MI problem</th>
<th>MIB problem</th>
<th>MIB structural problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSGA II</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSGA II SBX modified</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSGA IIb</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSGA II MI</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>NSGA II MIB</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>MIB MOGA Structural (MMS)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

![Fig. 1. MIB crossover rule for binary variables.](image1)

![Fig. 2. MIB mutation rule for binary variables.](image2)

![Fig. 3. Choice between two solutions.](image3)
binary variables of parents $a$ and $b$ with an automatic modification of related continuous or integer variables of children $a'$ and $b'$ (if a binary variable is null, the linked variables (integer or continuous) with it, are also zero). Then the crossing of $a'$ and $a''$ is performed on integer and continuous variables taking into account the existing links, for producing the final children $a'$ and $b''$. The mutation is carried out according to the same strategy.

As in the NSGA II of Deb et al. [5], the crossover operator generates a population of size $2N$. The survival selection in order to reduce the population size to $N$ individuals is new (see Gomez [34]). The procedure lies on unfeasibility of configurations with regard to the constraints and on the stagnation of a configuration (a configuration stagnates when its first Pareto front, i.e. first non-dominated solutions, does not change during a given number of generations, 50 for example). In a first phase, configurations are cancelled according unfeasibility and stagnation criteria, and in a second phase, if necessary, the population size is reduced to $N$ individuals by comparing the Pareto front domination ranks.

### 3.2. Main differences with NSGA II [5]

Compared with the classical NSGA II algorithm, this library involves the innovative following points:

- In NSGA II SBX, a new SBX crossover operator carries out a more efficient gene mixing.
- The SBX crossover coded in NSGA IIb includes a forced mutation of children when they are identical to the parents (clone limiting strategy).
- In NSGA II MI, the same crossover and mutation operators for continuous and integer variables are used. For integer variables, the strategy proposed by Shopova and Bancheva [35] is implemented.
- NSGA II MIB implements another version of NSGA II MI algorithm for problems involving also binary variables, by adding binary crossover and mutation rules to the existing MI operators.
- The MMS algorithm, developed for further design of complex plants, uses the MIB operators with a particular rule taking into account structural links between binary and continuous or integer variables. The problem arises when coding a process structure, where some continuous or integer variables can exist only if a binary variable is not zero. Furthermore, the crossover operator generates a population of size $2N$, and the survival selection in order to reduce the population size to $N$ individuals is new [34]. The procedure lies on unfeasibility of configurations with respect to the constraints and on the stagnation of configurations.
- The initial population may be generated according to a meshing strategy of the variable definition domains. Two options are provided in the MULTIGEN library for computing the initial population. The classical random generation of the initial population, may provide over-crowded or under-crowded zones. Another solution consists in meshing the definition domain of variables and randomly generating the same number of points into each cuboid of the mesh, in order to ensure a uniform overlapping of the entire domain.

### 3.3. Stopping criterion

The implementation of an efficient stopping criterion is a basic point for any iterative method. Classically genetic algorithms stop when a given maximum number of generations is reached. By observing the evolution of solutions, it can be noted that the number of generations necessary to reach the optimum is generally much larger than this maximum number. Therefore, a more efficient stopping criterion may lead to big savings in computational times. However, despite the real impact of stopping criteria, no reliable bibliographical study is available, particularly in multiobjective optimization.

In mono-objective optimization, a convergence threshold based on the stagnation of some statistical items (mean value, standard deviation) computed from the objective function values on the current population, can be used. Such a threshold of convergence can not be defined in the frame of multiobjective optimization. The stopping criterion implemented in MULTIGEN (in addition to the maximum number of generations) consists in comparing the Pareto fronts associated with not dominated solutions for populations $n$ and $n + p$, where the period $p \in [10, 20, 30, 40, 50]$ for example. If the union of the two fronts provides a single not dominated front, the procedure stops; else the iterations continue.
However, this stopping condition is theoretically not valid because the populations can stagnate during several generations, if no new individual gives an improvement, and then evolve again. Nevertheless, numerical experiments show that this stopping criterion leads in the great majority of cases, to the same solution as the one obtained after the maximum number of generations.

The implementation of this stopping criterion must be carried out considering the following points:

- The probability of stopping decreases when the size of the population, the number of variables and the number of objectives increases: each one of these parameters has a direct impact on the diversity of populations. For a great diversity, convergence is slow because the number of explored solutions is important, implying an increase of the number of generations necessary to reach an acceptable Pareto front.
- In the period of control is too small, the probability of stopping may decrease because the populations can stagnate on a few generations, before evolving again; so a too weak frequency can lead to a premature convergence.
- This stopping criterion is more suitable for problems involving only integer or binary variables, because the probability of founding again the same Pareto front is much higher than the one associated with continuous variables, where an infinite space of potential solutions is explored. Numerical practice shows that this stopping criterion is inefficient for problems involving continuous variables in a great number of cases. So it will not be implemented for this type of problem.

### 3.4. Numerical examples

For each numerical example, the computations were made on a Pentium Core 2 Duo 1.86 GHz PC, with 2 GB RAM. Insofar as all these test problem

#### 3.4.1. Continuous problem TNK

This bicriteria problem (two continuous variables and two inequality constraints) was first proposed par Tanaka [36], and involves two discontinuities in the Pareto front. The same Pareto front as Deb et al. [5] and Chafekar et al. [37] was obtained after about the 50th generation by NSGA II, NSGA II SBX modified and NSGA IIb. The computation time is in the order of magnitude of 15 s for one run.

#### 3.4.2. Mixed continuous-Integer problem

This other bicriteria problem, involving three continuous, three integer variables and nine inequality constraints, was presented and solved by a parametric MINLP [38]. It is solved here by NSGA II MI and NSGA II MIB in order to show that specific genetic operators developed for handling binary variables (in NSGA II MIB), give better results than genetic operators implemented for integer ones (NSGA II MI). NSGA II MI provides only one Pareto front for \((y_1, y_2, y_3) = (0, 0, 0)\), while NSGA II MIB leads to two Pareto fronts for \((y_1, y_2, y_3) = (0, 0, 0)\) and \((y_1, y_2, y_3) = (0, 0, 1)\); so a greater diversity for integer solutions is obtained.

#### 3.4.3. Structural optimization

This last numerical example concerns the structural optimization of the process shown in Fig. 5. This problem, defined by the system (10), consisting in simultaneously minimizing the production cost of product \(C\) and the demand \(D\), was solved by Kocis and Grossmann [39] and Acevedo and Pistikopoulos [40]. Each integer variable \(y_i\) is related to the existence of process \(P_i\) in the solution.

\[
\begin{align*}
\text{Min Cost} & = 250A + 300B + (5A_1 + 15A_2 + 5A_3 + 15B) \\
& - 550C + 80y_1 + 130y_2 + 150y_3 + 100
\end{align*}
\]

Min \(D\)

\[

g_1 = 18 \ln(1 + A_1/20) - B_1 \geq 0
\]

\[

g_2 = 20 \ln(1 + A_2/21) - B_2 \geq 0
\]

\[

g_3 = 15 \ln(1 + A_3/26) - B_3 \geq 0
\]

\[
	h_1 = B_1 + B_2 + B_3 - B \geq 0
\]

\[

g_4 = 15 - B_1 \geq 0
\]

\[

g_5 = D - C \geq 0
\]

\[

g_6 = 25y_1 - A_1 \geq 0
\]

\[

g_7 = 20y_2 - A_2 \geq 0
\]

\[

g_8 = 20y_3 - A_3 \geq 0
\]

\(17 < D < 25\)

MMS retrieves the two solutions previously given by Papalexandri and Dimkou [38]: \((y_1, y_2, y_3) = (1, 0, 1)\) for \(D \in [17, 17.54]\) and \((y_1, y_2, y_3) = (1, 1, 0)\) for \(D \in [17.55, 25]\). The corresponding Pareto front is shown in Fig. 6. When the values of binary variables are fixed, the problem is reduced to a continuous one, and can be solved by using for example NSGA IIb for avoiding the creation of clones. The two Pareto fronts obtained by NSGA IIb for \((y_1, y_2, y_3) = (1, 0, 1)\) and \((y_1, y_2, y_3) = (1, 1, 0)\) are also reported in Fig. 6, where it can be noted that the Pareto front provided by MMS dominates the Pareto front obtained from NSGA IIb for \((y_1, y_2, y_3) = (1, 1, 0)\), and is slightly dominated by the front given by NSGA IIb for \((y_1, y_2, y_3) = (1, 1, 0)\). However, this last front does not cover all the range \([17, 25]\) for the demand \(D\) (it ends when \(D\) is about 22).

### 4. Examples of design problems in the nuclear field

#### 4.1. Continuous problem: technico-economic optimization of the generating cycle coupled with VHTR Generation IV nuclear reactor

The GT-MHR (Gas Turbine – Modular High temperature Reactor) project studies the possibility of coupling a Helium cooled reactor, described by Kiryushin et al. [1], with an output temperature level at about 850 °C, with an innovating generating Brayton cycle (Fig. 7) with a heat recovery, two compressors (Low and High Pressure) and coolers. This section presents a new technico-economic optimization case based on VHTR heat source, delivering 950 °C to the generating cycle. Two criteria are considered: an energetic one, based on the exergy theory, and a second one related to the total cost of power plant during its lifetime.
4.1.1. Problem variables
The mechanical and isentropic efficiencies were fixed for the turbines and compressors, as well as the efficiency and pressure losses for the exchangers, except for the heat recuperator. From the analysis of degrees of freedom, the following optimization variables were selected: the turbine pressure ratio, the low pressure compressor ratio, and the regenerative heat exchanger efficiency.

4.1.2. Energetic objective function: exergy loss minimization
The theory of exergy, first sensed by Carnot, and concretely described by Gouy [41], represents the workable part of energy. Particularly Borel [42] and Bejan [43] admit the Eq. (11) formulation, as an exergy balance for conversion systems.

\[
\left(1 - \frac{T_0}{T_{as}}\right) + W_{mech} + \sum_{a} Q_a = \sum m_i \frac{m_{\text{out}}}{m_i} \frac{e_{\text{out}}}{S_{\text{ref}}} - T_0 \cdot S_{\text{ref}}
\]

According to Gouy [41], the difference between the maximum energy that can be used (term “c”), and the energy produced (terms “a” and “b”) has to be minimized. The atmosphere is the energy potential reference. The term “e” represents the sum of component’s internal irreversibility production and “d” is the heat released by cooling systems to the atmosphere. Consequently, the maximization of the “a + b” quantity is equivalent to minimize the “d + e” term representing the “exergetic losses” or the “lost available work”. Concretely, the final expression (Eq. (12)) of the exergetic loss, to be minimized is:

\[
E_{X_{\text{loss}}} = \left(\sum m_i \frac{m_{\text{out}}}{m_i} \frac{e_{\text{out}}}{S_{\text{ref}}} - T_0 \cdot S_{\text{ref}}\right)
\]

4.1.3. Economic objective function: Lifetime cost minimization
The second objective consists of the lifetime cost of the system, represented by Eq. (13).

\[
TC = \sum_{i=1}^{T_{\text{tot}}} \left(\left[C_{\text{oper}}\right]_{i} + \left[C_{\text{main}}\right]_{i} \times [1 + \tau]^{-1}\right)
\]

Three terms constitute this function: investment concerning all components, internal pipes and external systems, as cooling water installation, and operational and maintenance costs related to components as a percentage of the investment. These costs models are not reported here for confidentiality reasons.

4.1.4. Results
For solving this problem, the CEA tools COPERNIC (equipment design), CYCLOP (Brayton cycle calculation) and SEMER (cost computations) (see Haubensack et al. [28]) were used within MULTIGEN via the Excel interface, as shown in Fig. 8. For limiting the number of clones, NSGA IIb was implemented. The average computation time is about 5 h (which represents 35 s per generation) for populations...
of 1000 individuals during 500 generations. Note that the greatest part of the computational time is due to the CEA modules. The crossover probability is 90\% and the mutation probability is 50\%.

For confidentiality reasons, on the Pareto fronts, anyone actual value of cost is reported. The Pareto fronts are plotted in Figs. 9 and 10 with a relative scale from 0 to 100. The electricity production curve (Fig. 10) shows a clear discontinuity at 166 MW of exergy loss. This rupture represents in fact a structure variation on the gas turbomachine (Fig. 7): for exergy losses greater than 166 MW, the gas turbine involves one sequence of cooler – compressor, but for lower losses, it is necessary to add another cooler-compressor sequence in order to increase the efficiency. At 166 MW, the curve indicates an abrupt increase of electricity production cost, due to the cooler-compressor sequence adding, but the new configuration generated gives finally, about 123 MW of exergy losses. This best solution is reached for a turbine pressure ratio of 3.49, low pressure compression ratio of 1.89 and recuperator efficiency at 95\%.

This optimal design point gives the operating conditions for each components of the turbomachine, in particular for the heat recuperator: these conditions are given in Table 2, and used for the following geometrical design problem presented in the next section.

### 4.2. Integer problem: design of the heat recuperator exchanger

#### 4.2.1. Mathematical formulation

The previous technico-economic study gives optimal operational conditions for the Power Conversion System (PCS) recuperator (Table 2). The recuperator is made up of a plate fin with offset strips technology (Fig. 11), for obtaining a large surface area/volume ratio, and a substantial heat transfer value under laminar conditions. This technology is particularly required for gas/gas heat exchange, as in the PCS module. The characteristics of channels are reported in Table 3.

The PCS recuperator is a multi-modular heat exchanger and each module uses plate fin offset strips technology. Modules are located into two different annular compartments with the same volume and height (Appendix, Fig. 15). Each module consists of a

#### Table 2

<table>
<thead>
<tr>
<th>Operating condition</th>
<th>Inlet value</th>
<th>Outlet value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C) – hot side</td>
<td>498.3</td>
<td>151.2</td>
</tr>
<tr>
<td>Temperature (°C) – cold side</td>
<td>132.5</td>
<td>480.0</td>
</tr>
<tr>
<td>Exchanger efficiency (%)</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>Pressure (bar) – hot side</td>
<td>19.7</td>
<td>?</td>
</tr>
<tr>
<td>Pressure (bar) – cold side</td>
<td>71.4</td>
<td>?</td>
</tr>
<tr>
<td>He flow (kg/s)</td>
<td>244.8</td>
<td></td>
</tr>
<tr>
<td>Heat Power (MW)</td>
<td>441</td>
<td></td>
</tr>
</tbody>
</table>
A stack of channels located between two plates (Fig. 11) with several arrangements possibilities. Therefore, the variables for PCS recuperator problem are listed below:

- The number of modules \( N_{\text{Module}} \).
- The number of channels \( N_{\text{Channel}} \).
- The number of fin channels \( N_{\text{Fin Channels}} \).

Three objective functions to be minimized can be selected for this design problem:

- **Heat exchange area**: this objective is linked to the cost of the regenerator.
- **Exergy loss**: for given operating conditions (Table 2), optimization acts on pressure loss limitation, and finally on entropy generation minimization.
- **The module number**: flow distribution pipes, proportional to the module number induce a higher complexity, maintenance cost and pressure losses.

According to the exergy theory, an exergy loss minimization for the recuperation (term “\( \varepsilon \)” of Eq. (11)) induces a parallel increase of electric production. Design procedures were carried out by means of COPERNIC sheets [28]. Finally, the optimization problem is expressed as:

---

**Table 3**
Characteristics of fin channels.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Value (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fin length ( l )</td>
<td>( 3.175 \times 10^{-1} )</td>
</tr>
<tr>
<td>Tall ( h )</td>
<td>( 2.0 \times 10^{-3} )</td>
</tr>
<tr>
<td>Transverse spacing pitch ( s )</td>
<td>( 1.24 \times 10^{-1} )</td>
</tr>
<tr>
<td>Fin Thickness ( t )</td>
<td>( 2.0 \times 10^{-4} )</td>
</tr>
<tr>
<td>Plate thickness ( w )</td>
<td>( 8.0 \times 10^{-4} )</td>
</tr>
</tbody>
</table>
In the original concept of the GT-MHR, the pressure losses for cold and hot streams could not exceed 0.4 bar (Eqs. (17) and (18)) and the Reynolds number must be under 2300 (Eqs. (19) and (20)), to remain into the laminar flow domain. The plate fin with offset strips technology is taken into account, by means of Eqs. (22) and (23), limiting to 0.6 m large and high values for each module. The last three equality constraints indicate that the number of rings will be an even number (Eq. (25)), the number of module per ring will be a constant (Eq. (26)), and Eq. (27) permits to check that the defined modular arrangement presented in the Appendix (Eqs. (A.1)–(A.4)) gives the correct module number value.

4.2.2 Results

This combinatorial problem is first solved with a population of 1000 individuals during 1000 generations using NSGA II MI. As in the previous case, the crossover probability is 90% and the mutation probability is 50%. The average computation time, is about 6 h 33 min. As in the previous example, it can be observed that the greatest part of the computational effort is due to the CEA module COPERNIC.

The optimization gives the feasibility domain for this geometrical problem. In Fig. 12 (on the right upper side), a correlation between the heat exchanger surface and the total exergy losses of the recuperator, can be observed. Exergy losses increase when the heat exchange surface decreases. From Table 4, it can be noted that five possible configurations for the number of module rings are obtained: an even number varying between 6 and 14. For each configuration, Table 4 gives the variation domain for exergy losses, the number of modules and the heat exchange surface, and the geometric range. The arrows indicate the evolution of the corresponding term. Whatever the number of rings, the decrease of exergy losses implies an increase of module number and exchange surface, as a consequence. Exergy losses are due to the reduction of pressure discharge at both sides of the heat exchanger, caused by a reduction of the module length. An arrow oriented from the bad values of exergy losses to the best represents this evolution in Table 5.

Table 5 gives two extreme solutions. The worst solution is represented by the higher value for exergy losses (10,676 kW), with the higher pressure discharge (0.36 bar for the hot side) and a minimal exchange surface (12,082 m²). The low exergy loss solution (8498 kW) presents a decrease of 75% for the pressure discharge (0.09 bar) for the hot side and 78% for cold side of the module. The exchange surface increases at about 27% with 15,345 m² for the best exergy solution. In Fig. 13, where the modules dimensions are represented (hₜₘ, lₑ, and lₜ), five different sets of solutions, corresponding to the five possible numbers of rings described in Table

Table 5

<table>
<thead>
<tr>
<th>Exergy Loss (kW)</th>
<th>Modules</th>
<th>Surface (m²)</th>
<th>Rings</th>
<th>hₜₘ (m)</th>
<th>Canals</th>
<th>lₑ (m)</th>
<th>Transverse spacing</th>
<th>ðₜₘ (m)</th>
<th>ðₑ (m/²K)</th>
<th>ΔPₑ (bar)</th>
<th>ΔPₜₘ (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,676</td>
<td>198</td>
<td>12,082</td>
<td>6</td>
<td>0.41</td>
<td>74</td>
<td>0.37</td>
<td>300</td>
<td>0.09</td>
<td>1975</td>
<td>0.36</td>
<td>0.09</td>
</tr>
<tr>
<td>8498</td>
<td>192</td>
<td>15,345</td>
<td>8</td>
<td>0.56</td>
<td>100</td>
<td>0.59</td>
<td>483</td>
<td>0.69</td>
<td>1556</td>
<td>0.09</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Fig. 13. Variation domain of module dimensions.
4, can be observed. The decrease of all the dimensions of modules can be noted when exergy efficiency increases.

Finally, this study provides without any assumption on configurations, an efficient design procedure. The results constitute a panel of choices for engineers, covering cost (e.g., surface) and energetic efficiency (exergy loss + pressure losses limitation) with a real possibility to select an efficient compromise solution.

4.2.3. Use of the stopping criterion

As indicated above, the computational time for this example is quite high (6 h 33 min). The problem involving only integer variables, the stopping procedure described in Section 3.3 is now used. To promote the convergence of the Pareto fronts, the population size has to be reduced. However, this reduction may lead to a poor scanning of the variable definition domains. The previous optimization problem uses a population of 1000 individuals; a decrease of the population size to 333 individuals would increase the probability of convergence with regard to the Pareto fronts. To explore the “same” domain as in the case of 1000 individuals per population, three optimizations with 333 individuals are carried out instead of an optimization with 1000 individuals per population. The convergence testing period is fixed at 50 generations.

The cumulated computational times of cases 2–4 is reduced by a factor 4.3 compared with the first case (stopping at 1000 generations). The different Pareto fronts obtained from each experience have to be compared; Fig. 14 shows that they are perfectly superposed. This result demonstrates that all optimizations converge to the same Pareto front.

5. Conclusions

In the actual energetic and environmental context, nuclear technology appears to be an efficient solution for the cogeneration of electricity/hydrogen. So the CEA has devoted an important research effort to these systems, based on the one hand on the VHTR (generation IV of nuclear reactors), and on the other hand of the cycle sulfur/iodine for thermochemical decomposition of water. The prospective present study is carried out in the frame of collaboration between the CEA (Cadarache, France) and the LGC (Toulouse, France), and aims at proposing technico-economic optimization approaches, for identifying the most promising strategies. The main goal is maximizing the energy production while minimizing the production costs, which obviously constitutes a multiobjective optimization problem, having several possible formulations according to the considered objectives.

<table>
<thead>
<tr>
<th>Case</th>
<th>Population size</th>
<th>Maximum number of generation</th>
<th>Stop at generation number</th>
<th>Computation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>23,623</td>
</tr>
<tr>
<td>2</td>
<td>333</td>
<td>1000</td>
<td>400</td>
<td>1960</td>
</tr>
<tr>
<td>3</td>
<td>333</td>
<td>1000</td>
<td>250</td>
<td>1880</td>
</tr>
<tr>
<td>4</td>
<td>333</td>
<td>1000</td>
<td>150</td>
<td>1704</td>
</tr>
</tbody>
</table>

Fig. 14. Pareto fronts for experiences 1–4.

Fig. 15. Distribution map of recuperator modules on PCS vessel.
The MULTIGEN library described in this paper, consists of several multiobjective genetic algorithms. After an intensive study of the literature on multiobjective optimization, the procedure NSGA II [5] was chosen for the basis of the MULTIGEN implementation. Compared to the initial algorithm, several innovative points related to genetic operators, variable coding, clone limiting strategy, initial population generation, are implemented. A new stopping criterion based on the repetition of Pareto fronts on a given number of generations, particularly suitable for problems involving only binary or integer variables, is presented. The various algorithms of the MULTIGEN library were validated on several numerical examples [34], and three of them are reported in this paper.

Concerning applications in the nuclear field, two engineering examples of MULTIGEN use are reported. The first one involves only continuous variables for the bicriteria optimization (cost of produced electricity, exergy losses) of the VHTR generating cycle. The minimum cost of electricity production gives the operating conditions for a key component of the system: the heat recuperator, whose design is studied in the second example. This new problem involves only integer variables and the purpose was to define the compromise solutions between exchange surface, related to cost, and pressure losses, which have a significant impact on the generating cycle efficiency. From the set of solutions, two different designs can be proposed: a low exchange surface with important pressure losses, and the opposite solution. The new stopping criterion efficiency is shown on this problem, where the computational time is divided by a factor 4.3 compared with the classical stopping procedure based on a maximum number of generations.

These two optimization cases represent the two main stages in design strategy: process and operating conditions definition, and followed by the design of system components.

Acknowledgement

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Appendix A. Multi-modular geometric representation for the recuperator technology

The PCS recuperator is composed of $N_{modulus}$ modules, arranged as shown in Fig. 15. COPERNIC design sheet only gives the dimensions of the module, but not the distribution in the annular space, between the turbo-machine zone and the PCS vessel. Each module occupies an angle $\hat{\theta}_M$, calculated by the following expression, taking into account the turbomachine maximum diameter, the flow distribution devoted space and the wide value of modules:

$$\hat{\theta}_M = 2 \times \arctan \left( \frac{h_M/2}{d_M/2 + r} \right)$$  (A.1)

The number of modules per ring ($N_{modules/Ring}$) is given by:

$$N_{Module/Ring} = \left\lfloor \frac{2\pi}{\hat{\theta}_M} \right\rfloor$$  (A.2)

Finally, the number of rings ($N_{Rings}$) and the total high of the recuperator ($H_{Total}$) are calculated by the two formulas:

$$N_{Nappes} = \left\lfloor \frac{N_{modules}}{N_{modules/Ring}} \right\rfloor$$  (A.3)

$$H_{Total} = N_{Nappes} \times h_M$$  (A.4)

This equation set permits to code the modular disposition represented in Fig. 11.

References