Open Archive Toulouse Archive Ouverte (OATAO)

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: [http://oatao.univ-toulouse.fr/](http://oatao.univ-toulouse.fr/)
Eprints ID: 4145


**To cite this version:**

Any correspondence concerning this service should be sent to the repository administrator: [staff-oatao@inp-toulouse.fr](mailto:staff-oatao@inp-toulouse.fr)
Outage Probability of an Optimal Cooperative MAC Protocol in Nakagami-\(m\) Channels

Benoît Escrig
Université de Toulouse
Email: escrig@enseeiht.fr

Abstract—A new cooperative access protocol is presented in the context of IEEE 802.11-based fixed ad hoc networks. The protocol achieves optimal diversity-multiplexing tradeoff thanks to two known functionalities: on-demand cooperation and selection of the best relay. The on-demand approach allows maximization of the spatial multiplexing gain. The selection of the best relay allows maximization of the spatial diversity order. The main contribution of this paper consists in the design of a proactive mechanism in order to select the best relay. The mechanism is centralized at the destination terminal. Destination terminals maintain lists of relays for all possible source terminals by overhearing ongoing transmissions. So when cooperation is needed, a destination terminal just picks the best relay for a specific source terminal in the corresponding table. Hence, collision among relay candidates is now avoided. Moreover, only terminals that can improve the direct transmission are selected. This guarantees the usefulness of relaying. This study focusses on Nakagami-\(m\) wireless channel models in order to encompass a wide variety of fading models.

I. INTRODUCTION

Cooperative communications offer an efficient means to make wireless communications more robust to link failures since they provide more optimization opportunities than a simple retransmission mechanism. Spatial diversity is the main property obtained with cooperative communications since one or several relay terminals are helping the transmission between a source terminal S and a destination terminal D (see Fig. 3). However, this property comes at the price of additional bandwidth (and energy) consumption needed for both relay selection and relay transmission\(^1\). Bandwidth can be expressed in time slots, frequency bands, spreading codes or space time codes. In this context, cooperation techniques are usually compared using the Diversity-Multiplexing Tradeoff (DMT) [1]–[5]. The DMT analysis of a transmission scheme yields the diversity gain \(d(r)\) achievable for a multiplexing gain \(r\) (see Fig. 1). A transmission scheme is said to have a spatial multiplexing gain \(r\) and a diversity gain \(d(r)\) if the spectral efficiency \(R\) scales like \(r \log_2 SNR\), and the outage probability decays like \(1/\text{SNR}^{d(r)}\), where \(\text{SNR}\) denotes the effective signal to noise ratio at the receiver. A protocol achieves an optimal DMT curve when it maximizes both the diversity gain \(d(r)\) and the multiplexing gain \(r\), i.e. when the protocol achieves a target outage probability with both the lowest transmitted power and the lowest bandwidth consumption.

\(^1\)Throughout this paper, we use bandwidth as a general term for resource in a communication network.

When a single relay terminal is involved in a cooperation scenario, an optimal DMT curve can be obtained using on-demand relaying [6]. In an on-demand relaying scenario, the relay terminal is transmitting only when the destination terminal asks for cooperation. In [7], other optimal DMT curves have been established when no feedback to the transmitting terminal is allowed. The optimal DMT curve is given by 

\[ d(r) = 2(1 - r) \]  

for \(0 \leq r \leq 1\) (see Fig. 1). Even if the DMT curve is optimal, the diversity is limited to a value of two. To increase the diversity gain, several relay terminals should be considered.

When \((N - 1)\) relay terminals are involved in a cooperative scenario, a diversity order of \(N\) can be achieved (see Fig. 2). When a repetition scheme is implemented at the relay terminals, relay terminals are just repeating the source message. So the multiplexing gain is limited to the value of \(1/N\). The DMT curve is: 

\[ d(r) = N(1 - Nr) \]  

for \(0 \leq r \leq 1/N\). The value of \(r\) can be increased to \(1/2\) using space-time coding (STC) [3]. The DMT curve is then: 

\[ d(r) = N(1 - 2r) \]  

for \(0 \leq r \leq 1/2\). An alternative solution has been proposed in [8]. The solution is based on the selection of the best relay terminal among the set of \((N - 1)\) relay candidates. When only the best relay terminal is transmitting the source message, the DMT curve is the same as the previous one and less resources are needed to implement the cooperation scheme (allocating space-time codes to the relay terminals). However, even if the spatial diversity order is \(N\), the spatial multiplexing gain \(r\) is still limited to the value of \(1/2\). An optimal DMT curve can be achieved by implementing both on-demand relaying and the selection of the best relay terminal: 

\[ d(r) = N(1 - r) \]  

for \(0 \leq r \leq 1\). This optimal DMT curve can be achieved using either a fixed Amplify-and-
In previous approaches, a terminal is always selected as best relay terminal, even if this terminal cannot improve the direct transmission between the source terminal and the destination terminal. The selection of the best relay terminal is the weak point of these optimal protocols since the selection process is a distributed mechanism. Indeed, the selection consists of two steps. During the first step, several terminals may select themselves as best relay terminals. This self-selection is based on channel metrics between the relay and both end terminals. Then, during a second step, the relay terminals try and signal their presence to other terminals [8], [13]. Since there is no coordination among the relay candidates, collisions among relay terminals are not avoided. Collision-free selection algorithms have been proposed in the context of fixed ad-hoc networks [14]. However, these algorithms have not been designed with the purpose of optimizing both the DMT of the cooperative protocol and the signaling needed to select the relays. A last limitation should be pointed out. In previous approaches, a terminal is always selected as best relay terminal, even if this terminal cannot improve the direct transmission between the source terminal and the destination terminal D. Cooperative protocol designs should take this limitation into account. In other words, the relevance and the efficiency of cooperation schemes should be guaranteed.

In order to tackle these issues, we propose a new cooperative MAC (Medium Access Control) protocol. We call this protocol OXIDE: an On-demand Cooperation with a Selection of relays Initiated by the Destination Equipment. The main contribution of this paper consists in the design of a proactive mechanism in order to select the best relay. In our proposed protocol, each destination terminal maintains a table of potential relay terminals that can assist in its decoding by overhearing ongoing transmissions [15]. Each destination terminal maintains a table for each source terminal. When a source terminal sends a message, all the terminals in the range of the source terminal store the data frame and wait for an acknowledgment from the destination terminal. When the destination succeeds in decoding the source message, it transmits a positive acknowledgment and all the terminals in the range of the source terminal discard the source message. When the destination terminal fails in decoding the source message, it looks for a potential relay terminal in the table associated with the source terminal. When a relay terminal is successfully found, the destination terminal sends a negative acknowledgment for the source message with the address of the selected relay in a specific field. All the terminals in the range of the source terminal discard the source message except the selected terminal that retransmits the source message. When there are no relay available, or when the negative acknowledgment is not successfully decoded by the selected relay, the source terminal retransmits its message. This protocol guarantees efficient cooperation. Indeed, destination terminals use the negative acknowledgment frame only when they find terminals that can improve their transmissions. The selection mechanism is centralized at destination terminals. So collision among relay candidates is now avoided [8], [9], [13]. In particular, the problem of hidden groups of relays is avoided. The proposed protocol is optimal in terms of the DMT criterion since it implements the two basic functionalities: on-demand cooperation and selection of a best relay terminal. Moreover, the protocol relies on a selective DF transmission scheme since this scheme is more efficient than the fixed AF scheme in terms of outage probability [9], [10], [12]. The protocol recommends also that the destination terminal discards the source message when D fails in decoding the data transmitted by S. Hence, the destination terminal does not need to allocate resources to the signal/frame combination.

Finally, we consider Nakagami-$m$ fading channels in order to encompass a variety of fading models followed in the literature. Classical Rayleigh fading model corresponds to the case $m = 1$ while the Rice fading model corresponds to the case $m = (\kappa+1)^2/(2\kappa+1) > 1$, where $\kappa$ is the Ricean factor.

This new MAC protocol relies on the IEEE 802.11 standard. Though restricted to the context of fixed ad hoc networks in this paper, this protocol can also be applied to other wireless architectures such as broadcast wireless systems. We believe that our proposal can benefit the delivery of broadband services in several contexts since it provides an efficient transmission scheme in terms of both bandwidth and energy consumption.

The cooperative MAC protocol is described in details in section II. In section III, we show that this new protocol provides an optimal performance in terms of DMT. Simulation results are presented in section IV and we conclude in section V.
II. ON-DEMAND RELAYING WITH SELECTION OF THE BEST RELAY TERMINAL

A. System model

We consider a slow Nakagami - $m$ fading channel model. Our analysis focuses on the case of slow fading, to capture scenarios in which delay constraints are on the order of the channel coherence time. Coherence time is actually a statistical measure of the time duration over which the channel response is essentially invariant. If the inverse frequency band of the transmitted signal is greater than the coherence time, then the channel will change during the transmission of the message, thus causing distortion at the receiver. A half duplex constraint is imposed across each relay terminal, i.e. it cannot transmit and listen simultaneously. Moreover, transmissions are multiplexed in time, they use the same frequency band. Let $h_{ij}$ be the channel gain between a transmitting terminal $i$ and a receiving terminal $j$. We consider scenarios in which each fading coefficient $h_{ij}$ is accurately measured by the receiver $j$, but not known to the transmitter $i$. We also assume that the channel gain $h_{ij}$ is identical to the channel gain $h_{ji}$. This assumption is relevant since both channels are using the same frequency band. Statistically, the channel gain $h_{ij}$ between any two pair of terminals $i$ and $j$ is such that $|h_{ij}|$ is distributed according to a Nakagami-$m$ distribution. In particular, the random variable $|h_{ij}|^2$ is gamma distributed with scale parameter $\theta_{ij}$ ($\theta_{ij} > 0$) and shape parameter $\kappa_{ij}$ ($\kappa_{ij} > 0$). So, the probability density function $f_{|h_{ij}|^2}(x; \kappa_{ij}, \theta_{ij})$ of the random variable $|h_{ij}|^2$ is

$$f_{|h_{ij}|^2}(x; \kappa_{ij}, \theta_{ij}) = \frac{x^{\kappa_{ij} - 1}}{\theta_{ij}^{\kappa_{ij}} \Gamma(\kappa_{ij})} \exp\left(-\frac{x}{\theta_{ij}}\right)$$

where $\Gamma(y)$ denotes the complete gamma function

$$\Gamma(y) = \int_0^\infty t^{y-1} \exp(-t) dt$$

Let $P$ be the power transmitted by each terminal and $\sigma_w^2$ be the variance of the Additive White Gaussian Noise (AWGN) in the wireless channel. We define $SNR = P/\sigma_w^2$ to be the effective signal-to-noise ratio.

We also restrict our study to a single source-destination pair. We assume that there are $(N-1)$ terminals situated in the range of both the source and the destination terminals (see Fig. 3).

B. Protocol Description

We model the protocol by four main tasks: activation of the cooperative transmission mode, collection of cooperation information (CoI), relay selection, and notification of the terminals [16]. In the following, we review these four tasks.

1) Cooperation Mode Activation: We assume that all the terminals in the network are implementing the new cooperative MAC protocol. The inter-operability issue will be addressed in future work. We also assume that the cooperation functionality has been turned on. When the source terminal $S$ sends its message, each terminal in the range of $S$ can overhear the transmission. This event triggers the storage of the source message at the relay candidates.

2) CoI Collection: Each terminal maintains a table of terminals, referred to as the Relay-Table. The stored terminals can be used as relays when cooperation is needed. One Relay-Table is maintained for each possible source address. The maintenance of a Relay-Table involves two tasks [15]: creation and updating. The two tasks are done by overhearing all ongoing transmissions. This assumes that there is always information to be sent between the terminals. This assumption is consistent since signaling packets must be transmitted repeatedly in order to create and update routes between the terminals. As soon as terminal $D$ overhears a transmission between terminal $S$ and terminal $T$, terminal $T$ is considered as a potential relay for the transmission between $S$ and $D$. Hence, whenever terminal $T$ sends a message, terminal $D$ stores the channel state information (CSI) related to the channel between $T$ and $D$. This CSI can be a channel metric such as $SNR|h_{TD}|^2$, where $h_{TD}$ is the channel gain between terminal $T$ and terminal $D$, and $SNR$ is the signal to noise ratio at the receiver. Since all the terminals use the same frequency band for transmission and reception, the channel between any two terminals is symmetric. So, we assume that $h_{TD}$ equals $h_{DT}$. The fields contained in the Relay-Table are shown in Fig. 4. Entries are ordered by the timestamp values, based on the last time a frame from that terminal is overheard. The first column in Fig. 4, namely the ID field, stores the MAC address of a potential relay terminal $T$ learned from the data frames transmitted by terminal $T$. The Time field stores the time of the last frame transmission heard from terminal $T$. The CSI field stores the channel metric $SNR|h_{TD}|^2$ from terminal $T$ to the destination terminal. The last field in the table, NbFailures tracks the number of sequential failures associated with the particular terminal $T$. When this number exceeds a predefined threshold value$^2$, the corresponding entry is removed from the Relay-Table. The value of NbFailures is incremented after every failed transmission attempt through terminal $T$, and this value is reset to zero after a successful transmission through

$^2$The recommended value in [15] is 3.
terminal T. Each of these entries is updated to reflect the current channel conditions and status.

<table>
<thead>
<tr>
<th>ID</th>
<th>Time</th>
<th>CSI</th>
<th>NrFailures</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 bytes</td>
<td>1 byte</td>
<td>4 bytes</td>
<td>1 byte</td>
</tr>
</tbody>
</table>

- MAC address of terminal T
- The rate of the last frame transmission between terminal T and terminal D
- The channel state information recorded from terminal T to terminal D
- The number of sequential failures associated with terminal T

Fig. 4. Format of the Relay-Table for terminal D.

3) Relay Selection Algorithm: When terminal D succeeds in decoding the data frame, terminal D sends a positive acknowledgment frame (ACK). Otherwise, D discards the data frame. This allows processing optimization at the destination terminal without sacrificing the optimality of the DMT. Terminal D searches for a relay terminal in the Relay-Table dedicated to terminal S. In order to select useful relay terminals, a terminal D should select a terminal T in its Relay-Table if terminal T satisfies

\[ I_{TD} > R \]

where \( I_{TD} \) denotes the mutual information of cooperation transmission through the channel between T and D and R denotes the spectral efficiency of the direct transmission between S and D. Note that D can compute the spectral efficiency \( R \) by knowing the frequency band of the transmission, the duration field in the data frame, and other physical layer parameters, such as the modulation type. Thus, only terminals that can improve a transmission will be selected as relay terminals in a Relay-Table. When selective DF is considered, the mutual information of the cooperative transmission, \( I_{TD} \), is defined as

\[ I_{TD} = \frac{1}{2} \log_2 \left(1 + \frac{SNR|h_{TD}|^2}{1} \right) \]

This expression differs from the one that is usually given

\[ \frac{1}{2} \log_2 \left(1 + SNR|h_{SD}|^2 + SNR|h_{TD}|^2 \right) \]

This is due to the fact that the destination terminal discards the source message in case of a decoding failure. Note that the 1/2 factor comes from the fact that cooperation uses twice the bandwidth. When the Relay-Table has more than one entry, terminal D selects the terminal that maximizes the mutual information of the cooperative transmission \( I_{TD} \). Let’s denote this terminal, terminal B (B for Best terminal).

4) Relay Notification: Terminal D notifies the result of the relay selection by sending a negative acknowledgment on the data frame transmitted by terminal S, denoted CFC frame for Claim For Cooperation following [17], [18]. The CFC frame includes the MAC address of terminal B. When terminal B has successfully decoded both the source message and the negative acknowledgment frame from the destination terminal, it retransmits the source message it overheard. When terminal T fails in decoding either the source message or the negative acknowledgment, terminal B remains silent (selective DF forwarding scheme) and S re-transmits its data frame. Note that an extra time-slot is added to the waiting time of terminal S in order to avoid collision between its retransmission and the forwarding of terminal B\(^3\). This time slot equals a SIFS (Short Interframe Space). When D succeeds in decoding the data frame transmitted by either B or S, D sends a positive ACK frame (see Fig. 5). Otherwise, D remains silent and the timeout at the source terminal triggers another retransmission. When the CFC frame is lost, the protocol implements a classical error recovery mechanism. Terminals that have stored the source message, wait for either a positive ACK frame or a CFC frame. If any of these two frames is not received within a given time-slot, the source message is discarded. Terminals that just receive either a positive ACK frame or a CFC frame ignore the signaling frame.

5) Comments on the Protocol Design: We give here some additional comments on the protocol design:

- RTS/CTS optional access method: Several cooperative MAC protocols rely on the exchange of modified Request-to-Send (RTS) and Clear-to-Send (CTS) signaling frames [13], [14], [19]. If CTS frames transmitted by the destination terminal D can be modified, we can infer that channel state information is available at the transmitter. Hence, the source can actually choose not to transmit when it cannot support a given spectral efficiency \( R \). This gives rise to new cooperative protocols, the study of which is left for future work.

- NAV modification: The Network Allocation Vector (NAV) values at each terminal should be increased according to the new frame scheduling. In particular, an additional time-slot should be considered when terminal D transmits its CFC frame and waits for a response from either the best relay B or the source terminal S. This should avoid unnecessary soundings by neighboring terminals.

- The negative acknowledgement approach: the negative acknowledgement approach assumes that the CRC (Cyclic Redundancy Check) of the source message is wrong whereas the destination address is correct. We guarantee the detection of this case by appending an additional CRC to the control field of the source message. This additional CRC is only dedicated to the detection of erroneous destination addresses.

- Error recovery mechanism: A timeout is used at the source terminal to avoid blocking states. In particular, as soon as a frame is missing or when the set of relay is empty, the protocol returns to its starting point according to a given timeout.

\(^3\)Note that the source terminal does not need to overhear the CFC frame from D. Its retransmission is triggered by a given timeout.
terminal, B is the best relay terminal, and IEEE 802.11 access method (S is the source terminal, D is the destination terminal, B is the best relay terminal, and T is a relay candidate). The top figure represents the case when D succeeds in decoding the data frame from S. The middle figure represents the case when D asks for cooperation and B is relaying. The bottom figure represents the case when the cooperation fails and S must retransmit its frame.

1) Source Terminal S:
   - S waits for data to send. As soon as S has data to send, S enters the subsequent step.
   - S sends the data frame and triggers a timeout.
   - When no acknowledgment frame (positive or negative) has been successfully received by S before the timeout, S goes to the previous step. Otherwise, S proceeds to the next step.
   - S tests whether it has received a positive acknowledgment frame ACK or a negative acknowledgment frame CFC.
     - When S receives an ACK frame, it goes back to the first step.
     - When S receives a CFC frame, it adjusts its timer according to a new timeout in order to take into account the transmission of the best relay. Then, S waits for a positive acknowledgment frame ACK from D.
   - When S successfully receives a positive acknowledgment frame ACK from D, it proceeds to the first step. Otherwise, it retransmits the data frame.

2) Destination Terminal D:
   - A flag bit F is defined and set to 0.
     - When set to 0, the flag indicates that D is waiting a data frame from a source terminal.
     - When set to 1, the flag indicates that D waits for the same data frame transmitted by the best relay terminal.
   - Whenever D receives a data frame from a source terminal S, it checks the CRC.
     - When the data frame passes the CRC check, the flag F is set to 0 and the data frame is sent to upper layers. Otherwise, the flag F is checked.
       - When the flag is 1, a CFC frame has already been sent. So, D remains silent and waits for a retransmission of the data frame.
       - When the flag is 0, D checks the additional CRC on the destination address field. When the additional CRC is correct, D sends a negative acknowledgment frame CFC. Otherwise, D remains silent and goes back to a waiting state.

3) Potential Relay T:
   - Whenever T successfully overhears and decodes a data frame from a terminal S, it stores the data frame and waits for an acknowledgment frame from D.
   - As soon as T successfully decodes an acknowledgment frame from D before a timeout, it checks whether it received a positive acknowledgment frame ACK or a negative acknowledgment frame CFC. Otherwise, it discards the data frame and goes back to a waiting state.
   - When T receives an ACK frame, it discards the data frame from S and goes back to a waiting state.
   - When T receives a CFC frame, it sends the data frame from S, then discards the data frame and goes back to a waiting state.

III. PERFORMANCE ANALYSIS OF THE OXIDE PROTOCOL

A. DMT Analysis of the Cooperative Protocol

In this section, we study the DMT curve of the OXIDE protocol. We characterize our channel models using the system...
model described in the previous section, and a time-division notation; frequency-division counterparts to this model are straightforward. We use a base-band-equivalent, discrete-time channel model for the continuous-time channel. Three discrete time received signals are defined in the following. Here, \( y_{ij}(n) \) denotes the signal received by terminal \( j \) and transmitted by terminal \( i \). During a first time-slot, \( D \) and the best relay \( B \) are receiving signals from \( S \)

\[
\begin{align*}
    y_{SD}(n) &= h_{SD}x(n) + w_{SD}(n) \\
    y_{SB}(n) &= h_{SB}x(n) + w_{SB}(n)
\end{align*}
\]

for \( n = 1, 2, \ldots, T_M/2 \), where \( T_M \) denotes the duration of time-slots reserved for each message. When terminal \( D \) succeeds in decoding the data frame from \( S \), no signal is transmitted by the best relay terminal \( B \). Otherwise, the best relay terminal sends a new signal using a selective DF scheme, i.e. if and only if it has been able to decode the source message. So we consider that the estimation of signal \( x(n) \), denoted \( \hat{x}(n) \), is error free. Hence, during the second time slot, \( D \) is receiving a signal from \( B \)

\[
y_{BD}(n) = \begin{cases} 
    h_{BD}x(n) + w_{BD}(n), & \text{if } I_{SB} > R \\
    0, & \text{if } I_{SB} \leq R
\end{cases}
\]

for \( n = T_M/2 + 1, \ldots, T_M \), where the mutual information \( I_{SB} \) is given by

\[
I_{SB} = \log_2(1 + SNR|h_{SB}|^2)
\]

The noise \( w_{ij}(n) \) between transmitting terminal \( i \) and receiving terminal \( j \) are all assumed to be i.i.d. circularly symmetric complex Gaussian with zero mean and variance \( \sigma^2_{w_{ij}} \). Symbols transmitted by the source terminal \( S \) are denoted \( x(n) \). For simplicity, we impose the same power constraint at both the source and the relay: \( E[|x(n)|^2] \leq P \). We assume that the source and the relay each transmit orthogonally on half of the time-slots. We also consider that a perfect synchronization is provided at the block, carrier, and symbol level.

We define the diversity gain \( d_{OXIDE}(r) \) of the OXIDE protocol by

\[
d_{OXIDE}(r) = \lim_{SNR \to \infty} -\frac{\log p_{out}^{OXIDE}(SNR, r)}{\log(SNR)}
\]

The probability \( p_{out}^{OXIDE}(SNR, r) \) is the outage probability for a signal to noise ratio \( SNR \) and a spatial multiplexing gain \( r \) defined by

\[
r = \lim_{SNR \to \infty} \frac{R}{\log_2(SNR)}
\]

where \( R \) is the spectral efficiency of the direct transmission (in b/s/Hz). For high \( SNR \) values, we use

\[
R = \frac{r}{\log_2 SNR} \tag{1}
\]

The event that the relay has successfully decoded the data transmitted by \( S \) with a spectral efficiency \( R \) is equivalent to the event that the mutual information of the channel between \( S \) and the best relay \( B \), \( I_{SB} \), lies above the spectral efficiency \( R \) [3], [8]. When \( (N-1) \) terminals are available, the OXIDE protocol is in outage if all the \( (N-1) \) relay candidates fail in improving the direct transmission

\[
p_{out}^{OXIDE}(SNR, r) = \Pr[I_{SD} \leq R] \ \times \ \Pr[\bigcup_{i=1}^{N-1} (I_{OXIDE}^{(i)} \leq \frac{R}{2}) | I_{SD} \leq R] \tag{2}
\]
where $I_{OXIDE}^{(i)}$ is the mutual information of the relayed transmission using selective DF cooperation scheme at terminal $R_i$ and implementing frame dropping at the destination terminal (the source message is discarded at the destination terminal when cooperation is needed)

$$I_{OXIDE}^{(i)} = \left\{ \begin{array}{ll} \frac{1}{2} \log_2(1 + SNR|h_{SD}|^2), & \text{if } I_{SR_i} \leq R \\ \frac{1}{2} \log_2(1 + SNR|h_{RD}|^2), & \text{if } I_{SR_i} > R \end{array} \right.$$ where the mutual information $I_{SR_i}$ is defined by

$$I_{SR_i} = \log_2(1 + SNR|h_{SR_i}|^2)$$

and the mutual information $I_{SD}$ is defined by

$$I_{SD} = \log_2(1 + SNR|h_{SD}|^2)$$

The probability $p_{out}^{OXIDE}(SNR,r)$ can be expressed as the sum of $2^{(N-1)}$ terms

$$p_{out}^{OXIDE}(SNR,r) = \sum_{j=1}^{2^{(N-1)}} P_j$$

where $P_j$ is given by

$$P_j = P_j^E \prod_{i=1}^{N-1} \Pr[\epsilon_{(i)}]$$

where

$$P_j^E = \Pr[I_{D} \leq R] \times \Pr\left\{ \bigcup_{i=1}^{N-1} I_{OXIDE}^{(i)} \leq \frac{R}{2} \bigg| (\epsilon_{(i)}, I_{SD} \leq R) \right\}$$

The event $\epsilon_{(i)}$ equals the event $I_{SR_i} \leq R$ or $I_{SR_i} > R$ according to the value of index $j$, $1 \leq j \leq 2^{(N-1)}$. The probability $P_j$ in (3) is constituted with $N$ components. The first component $P_j^E$ is the probability denoted in (2) where each value of $I_{OXIDE}^{(i)}$ is conditioned to the value of $I_{SR_i}$. The $(N-1)$ last terms in the product exhibit the probabilities that the $I_{SR_i}$ are above or beyond the threshold $R$, for $1 \leq i \leq (N-1)$. We assume that there are $N_j$ passive relay terminals such that $\epsilon_{(i)} = [I_{SR_k} \leq R]$ in $P_j$. We define the set $K_j$ such that

$$K_j = \{ k / \epsilon_{(k)} = [I_{SR_k} \leq R], 1 \leq k \leq (N-1) \}$$

with cardinality $|K_j| = N_j$. Thus, there are $(N-1) - N_j$ active relay terminals such that $\epsilon_{(i)} = [I_{SR_i} > R]$ in $P_j$. We define the set $L_j$ such that

$$L_j = \{ l / \epsilon_{(l)} = [I_{SR_l} > R], 1 \leq l \leq (N-1) \}$$

with cardinality $|L_j| = (N-1) - N_j$. Note also that $0 \leq N_j \leq (N-1)$. So, we have that

$$P_j^E = \Pr[I_{D} \leq R] \times \Pr\left\{ \bigcup_{k \in K_j} I_{OXIDE}^{(k)} \leq \frac{R}{2} \bigg| (\epsilon_{(i)}), I_{SD} \leq R \right\} \times \Pr\left\{ \bigcup_{l \in L_j} I_{OXIDE}^{(l)} \leq \frac{R}{2} \bigg| (\epsilon_{(l)}), I_{SD} \leq R \right\}$$

For the $N_j$ passive relay terminals, we have that

$$I_{OXIDE}^{(k)} \leq \frac{R}{2} \epsilon_{(k)} \iff I_{OXIDE}^{(l)} \leq \frac{R}{2} \epsilon_{(l)} \leq R \iff I_{SD} \leq R$$

So, we have that

$$\Pr[I_{OXIDE}^{(k)} \leq \frac{R}{2} \epsilon_{(k)} \leq R] = \Pr[I_{SR_k} \leq R] = \Pr[\log_2(1 + SNR|h_{SR_k}|^2) \leq R]$$

Moreover, we have that

$$\Pr[\epsilon_{(l)}] = \Pr[I_{SR_l} > R] = \Pr[\log_2(1 + SNR|h_{SR_l}|^2) \leq R]$$

For the $(N-1) - N_j$ active relay terminals, we have that

$$I_{OXIDE}^{(l)} \leq \frac{R}{2} \epsilon_{(l)} \iff I_{OXIDE}^{(l)} \leq \frac{R}{2} \epsilon_{(l)} \leq R \iff \log_2(1 + SNR|h_{RD}|^2) \leq R$$

and

$$\Pr[\epsilon_{(l)}] = \Pr[I_{SR_l} > R]$$

For high SNR values, we have a simpler expression for (4)

$$P_j^E = \Pr[\log_2(1 + SNR|h_{SD}|^2) \leq R] \times \Pr\left\{ \bigcup_{l \in L_j} \log_2(1 + SNR|h_{RD}|^2) \leq R \right\}$$

The $|h_{RD}|^2$ random variables being mutually independent, we have that

$$P_j^E = \Pr[|h_{SD}|^2 \leq SNR^{-1}] \times \prod_{l \in L_j} \Pr[|h_{RD}|^2 \leq SNR^{-1}]$$

using (1) The random variables $|h_{SR_k}|^2$ and $|h_{RD}|^2$ being mutually independent, we have that

$$\prod_{l \in L_j} \Pr[\epsilon_{(l)}] \leq \prod_{k \in K_j} \Pr[|h_{SR_k}|^2 \leq SNR^{-1}]$$

So, using (5), we have that

$$P_j \leq \Pr[|h_{SD}|^2 \leq SNR^{-1}] \times \prod_{l \in L_j} \Pr[|h_{RD}|^2 \leq SNR^{-1}] \times \prod_{k \in K_j} \Pr[|h_{SR_k}|^2 \leq SNR^{-1}]$$

The random variables $|h_{SD}|^2$, $|h_{RD}|^2$ for $l \in L_j$, and $|h_{SR_k}|^2$ for $k \in K_j$ are all gamma distributed variables. Let $|h|^2$ be one of these random variables, with shape parameter $\kappa$. From Lemma 2 in [9], we have that

$$\lim_{SNR \rightarrow \infty} \log\{\Pr[|h|^2 \leq SNR^{-1}]\} \log(SNR) = \kappa(r - 1)$$

So, using (7) in (6), we have that

$$\lim_{SNR \rightarrow \infty} \log\{P_j\} = (\kappa_{SD} + \sum_{k \in K_j} \kappa_{SR_k} + \sum_{l \in L_j} \kappa_{RD})(r - 1)$$
for every $j$, $1 \leq j \leq 2^{(N-1)}$, where $\kappa_{ij}$ is the shape parameter or the random variable $|h_{ij}|^2$, where $i$ (resp. $j$) denotes the transmitting (resp. receiving) terminal. For $1 \leq i \leq (N-1)$, we define $\kappa_i = \min\{\kappa_{SR_i}, \kappa_{RD_i}\}$. So, we have that

$$\lim_{SNR \to \infty} \frac{\log[P_{out}]}{\log(SNR)} \geq (\kappa_{SD} + \sum_{i=1}^{N-1} \kappa_i)(1-r)$$

So, we have that

$$\lim_{SNR \to \infty} -\frac{\log[P_{out}(SNR,r)]}{\log(SNR)} \geq (\kappa_{SD} + \sum_{i=1}^{N-1} \kappa_i)(1-r)$$

Hence, the diversity curve $d_{OXIDE}(r)$, i.e. the DMT of the OXIDE protocol, is lowerbounded by the following expression

$$d_{OXIDE}(r) \geq (\kappa_{SD} + \sum_{i=1}^{N-1} \kappa_i)(1-r) \quad (8)$$

For the special case of Rayleigh fading, i.e $\kappa_{SD} = \kappa_i = 1$ for $1 \leq i \leq (N-1)$, we have that

$$d_{OXIDE}(r) = N(1-r)$$

So, when there are $(N-1)$ potential relay terminals, the OXIDE protocol achieves the optimal DMT curve reaching the two extremes points $d(0) = N$ and $d(1) = 0$ (see Fig. 2).

### B. Spectral Efficiency Analysis

We have studied the performance of the proposed protocol in terms of outage probability. We have shown that the OXIDE protocol achieves optimal performance in terms of DMT. We now compute the spectral efficiency of the OXIDE protocol and compare it to one of other existing protocols. The spectral efficiency of the protocol is the ratio between the data rate and the bandwidth used by the protocol in order to achieve the data rate. Let $R$ denote the spectral efficiency of the direct transmission between terminals S and D. The effective spectral efficiency of the OXIDE protocol is $\bar{R}$

$$\bar{R} = R \times \Pr[I_{SD} > R] + \frac{R}{2} \times \Pr[I_{SD} \leq R] \quad (9)$$

where the mutual information of the direct transmission, $I_{SD}$, is defined in (3). Equation (9) follows from the fact that the OXIDE protocol operates at spectral efficiency $R$ when the direct transmission is successful, and operates at spectral efficiency $R/2$ when a cooperative transmission is needed [6].

Using the expression of $I_{SD}$, we have that

$$\Pr[I_{SD} \leq R] = \Pr[\log(1 + SNR|h_{SD}|^2) \leq R] = \Pr[|h_{SD}|^2 \leq \frac{2R - 1}{SNR}]$$

For Nakagami-$m$ fading with $m = 1$ (Rayleigh fading), $|h_{SD}|^2$ is exponentially distributed with parameter $\sigma^2$. So, we have that

$$\Pr[I_{SD} \leq R] = 1 - \exp(-\frac{1}{\sigma^2} \frac{2R - 1}{SNR})$$

Using the above expression in (9), we have that

$$\bar{R} = \frac{R}{2} [1 + \exp(-\frac{1}{\sigma^2} \frac{2R - 1}{SNR})]$$

So the effective spectral efficiency $\bar{R}$ of the OXIDE protocol tends toward $R$ when the effective signal-to-noise ratio $SNR$ tends towards infinity. Fig. 11 shows several curves of $\bar{R}$ as a function of the signal-to-noise ratio $SNR$ for a given spectral efficiency ($R = 1$) and several values of parameter $\sigma^2$. We note that a given spectral efficiency $\bar{R}$ can arise from several possible spectral efficiency $R$, depending upon the value of the signal-to-noise ratio $SNR$. In that case, the lowest spectral efficiency $R$ should be considered (see Fig. 10).

### IV. Simulation Results

For simulation purposes, we consider a single source-destination pair. We assume that there are $(N-1)$ terminals available for cooperation in the range of both source and destination terminals, i.e. they all implement the cooperation functionality. We assume slow fading Nakagami-$m$ channels with $m = 1$ (Rayleigh fading) between each pair of terminals, with equal variance: $\sigma^2 = 1$. The channel gains are assumed to be known at the receiver side. Each channel gain has a constant value during the transmission of a frame and this value may change from one frame to another. Fig. 12 shows the simulation results for several values of the signal-to-noise ratio $SNR$ and for several values of $N$, the number of transmitting
transmitting terminals: Moreover, the slope of each curve equals the number of transmission scheme increases with the number of relays.

To Noise Ratio SNR

Fig. 10. Effective Spectral Efficiency \( R \) of the OXIDE protocol vs Spectral Efficiency \( R \) for several values of the signal-to-noise ratio \( SNR \).

10

3

outage events occurred. For large terminals. Each simulation has been run until a hundred of

10

3

in (9). Note that when a target outage probability is set to result is consistent with the expression of the DMT curve that the OXIDE protocol achieves full diversity order. This

2

1

cooperation scheme is considered.

2

3

relay candidates \( N \). As Fig. 13

1

2

indicates, all the presented protocols achieve full diversity, the direct transmission achieves a full diversity order or 1 since there is only one transmitter. The OAF protocols achieves a diversity order of 2 because there are exactly two transmitting terminals: the source and the relay. The OAFB and the OXIDE protocols use the same number of relay candidates \( (N - 1) \). As Fig. 13 indicates, all the presented protocols achieve full diversity. the direct transmission achieves a full diversity order or 1 since there is only one transmitter. The OAF protocols achieves a diversity order of 2 because there are exactly two transmitting terminals: the source and the relay. The OAFB and the OXIDE protocols achieve full spatial diversity of order \( N \), the number of cooperative terminals, for sufficiently large \( SNR \). In other words, the outage probability of these protocols scales like \( 1/SNR^N \) where \( N \) is the number of transmitting terminals.

Moreover, the new OXIDE protocol achieves a better outage probability because it uses a selective DF transmission scheme instead of a fixed AF one.

V. CONCLUSION

A new cooperative MAC protocol, the OXIDE protocol, has been proposed in the context of IEEE 802.11-based fixed ad hoc networks. This protocol implements an on-demand cooperative transmission between a source terminal S and a destination terminal D. When the destination terminal fails in decoding the source message, the destination terminals asks for cooperation using a signaling frame. The frame includes the MAC address of the best relay that can help the transmission between S and D. Indeed, each destination terminal maintains a table of potential relays, the Relay-Table, one table for each source address. The maintenance of a Relay-Table is done by overhearing ongoing transmissions. In particular, information
about terminal T is gathered by terminal D in two steps: overhearing transmissions between S and T, and receiving frames from T. Hence, when cooperation is needed and the RelayTable related to S is not empty, a relay is successfully found in the cooperation table. The MAC address of the relay is included in the signaling frame asking for cooperation. When the best relay has successfully received the source message, it retransmits the data toward D according to a selective DF scheme. The main contribution of this paper consists in the implementation of a DMT optimal MAC protocol using a collision-free mechanism to select the best relay. Moreover, only terminals that can improve transmissions between S and D are considered. This property guarantees efficient and useful cooperation. Hence, when \( N - 1 \) terminals are situated in the range of both a source terminal S and a destination terminal D, a spatial diversity gain of \( N \) is provided while a spatial multiplexing gain of one is achieved. Thus, the protocol implements a DMT optimal transmission scheme.

The impact and the performance of cooperative transmissions have been evaluated at the level of a single link between a source terminal and a destination terminal. The impact of cooperative communications should now be assessed at the network level. Indeed, as cooperative communications involve more than two terminals (the source terminal and the destination terminal), the contention area is increased when one or several terminals must forward the source message. So the overall throughput of the network may be affected by the implementation of the OXIDE protocol. A detailed analysis of this point is currently in progress. Moreover, the traffic load generated by cooperative transmissions at a relay terminal depends on the actual position of the terminal. When a terminal is often relaying because it has a central location in the network, energy consumption and billing issues must then be addressed. These issues ask for cross-layer designs, the study of which is left for future work.

REFERENCES


