

Detection in the Presence of Surprise or Undernullled Interference

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Abstract—We consider the problem of detecting a signal of interest in the presence of colored noise, in the case of a covariance mismatch between the test cell and the training samples. More precisely, we consider a situation where an interfering signal (e.g., a sidelobe target or an undernullled interference) is present in the test cell and not in the secondary data. We show that the adaptive coherence estimator (ACE) is the generalized likelihood ratio test for such a problem, which may explain the previously observed fact that the ACE has excellent sidelobe rejection capability, at the price of low mainlobe target sensitivity.

Index Terms—Adaptive coherence estimator (ACE), adaptive detection, covariance mismatch, generalized likelihood ratio test (GLRT), sidelobe target.

I. INTRODUCTION

THE PROBLEM of detecting the presence of a signal of interest embedded in noise is a fundamental one in many applications and has been studied extensively in the literature [1]. Given the $N \times 1$ output \mathbf{z} of an array of sensors—referred to as the primary data vector or the test cell—it amounts to deciding whether or not a component with array response vector \mathbf{v} is present, in addition to noise, whose covariance matrix will be denoted as \mathbf{R}_T . In an adaptive detection scenario, L training samples \mathbf{z}_ℓ , which share the same covariance matrix \mathbf{R} , are assumed to be available, and used to infer the noise covariance matrix in the primary data. Assuming that the environment is homogeneous, i.e., $\mathbf{R}_T = \mathbf{R}$, the generalized likelihood ratio test was derived by Kelly [2]. Later, the adaptive matched filter (AMF) was presented in [3]. First, the GLRT for known \mathbf{R}_T is derived; next, the sample covariance matrix obtained from the \mathbf{z}_ℓ 's is substituted for \mathbf{R}_T to yield the AMF. Kelly's GLRT and the AMF are considered as “reference” detectors for this homogeneous environment. Finally the adaptive coherence estimator [4] was introduced in the case of a partially homogeneous environment for which \mathbf{R}_T is only proportional to \mathbf{R} . It turned out that the ACE is the GLRT for this problem [5], as well as the uniformly most powerful invariant test [6].

Both detectors perform well in the homogeneous case but incur a loss of performance in the presence of steering vector mismatch (i.e., the actual array response differs from \mathbf{v}) or covariance matrix mismatch (i.e., $\mathbf{R}_T \neq \mathbf{R}$), see e.g., [7]–[11] for

thorough analyses of these detectors performance in the presence of both types of mismatch. A particular scenario of covariance mismatch which is of interest to us occurs when an interference is present, and not accounted for by the secondary data: this is the case when a sidelobe target is present or with undernullled interference. This unaccounted interference may then trigger a detection, which is an undesirable result as it corresponds to a false alarm. Ideally, one would like a high sensitivity to a target in the mainlobe, and capability to reject mismatched signals. In order to meet these conflicting goals several strategies have been investigated. For instance, a solution is proposed in [12] which consists of a blend of the GLRT and the AMF; a single scalar-sensitivity parameter is used to control the degree to which unwanted signals are rejected. Another solution is to use a 2-stage detection scheme, see, e.g., [13], [14], where two successive tests should be passed before the presence of a target is declared. For example, the adaptive sidelobe blanker (ASB) of [13] consists of an AMF test followed by an ACE test. The principle behind this choice is that the AMF possesses an excellent target sensitivity while the ACE has excellent sidelobe energy rejection capability. Hence, the 2-D ASB allows to trade off target sensitivity versus sidelobe rejection capability. A third alternative is to modify the hypotheses testing problem so as to account for targets with response orthogonal to \mathbf{v} under the null hypothesis. This is the principle of the adaptive beamformer orthogonal rejection test (ABORT) of [15]. When there is no target in the pointing direction but one in another (e.g., sidelobe) direction, the test will more likely incline towards the null hypothesis, which is the desired result. In [16], a similar but more general approach is taken, in which the steering vector is subject to uncertainties.

In this letter, we focus on a situation where the test vector may contain an interference which is not accounted for by the training samples, resulting in $\mathbf{R}_T = \mathbf{R} + \mathbf{q}\mathbf{q}^H$, where \mathbf{q} is unknown. The classical hypotheses testing problem is thus modified to account for this situation. *The main result of this letter is to show that the ACE is the GLRT for this detection problem.* Therefore, this result provides a theoretical explanation of the observed fact that the ACE has excellent sidelobe rejection capabilities.

II. GENERALIZED LIKELIHOOD RATIO TEST

As explained previously, we consider the problem of detecting a signal of interest \mathbf{v} in the presence of: 1) colored noise with covariance matrix \mathbf{R} and 2) an interference associated with an unknown steering vector \mathbf{q} , using training

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samples z_ℓ whose covariance matrix is \mathbf{R} . Formulated mathematically, this amounts to deciding between the following two hypotheses:

$$\begin{aligned} H_0 &: \begin{cases} \mathbf{z} = \mathbf{n}; \\ \mathbf{z}_\ell = \mathbf{n}_\ell; \quad \ell = 1, \dots, L \end{cases} \\ H_1 &: \begin{cases} \mathbf{z} = \alpha \mathbf{v} + \mathbf{n}; \\ \mathbf{z}_\ell = \mathbf{n}_\ell; \quad \ell = 1, \dots, L \end{cases} \end{aligned} \quad (1)$$

with the following statistical assumptions:

- z_ℓ are independent, zero-mean complex-valued Gaussian distributed random vectors with covariance matrix

$$\mathcal{E} \{ \mathbf{z}_\ell \mathbf{z}_\ell^H \} = \mathbf{R}. \quad (2)$$

- \mathbf{n} is a zero-mean complex-valued Gaussian distributed random vector with covariance matrix

$$\mathcal{E} \{ \mathbf{n} \mathbf{n}^H \} = \mathbf{R}_T = \mathbf{R} + \mathbf{q} \mathbf{q}^H. \quad (3)$$

Furthermore, we assume that \mathbf{n} is independent of the z_ℓ 's. Note that we choose to absorb the interference power in \mathbf{q} and do not assume that \mathbf{q} has a given norm.

- \mathbf{v} is the array response for the pointing direction and α denotes the deterministic and unknown amplitude.

We let $\mathbf{Z} = [\mathbf{z}_1 \ \dots \ \mathbf{z}_L]^T$ denote the secondary data matrix. Under the stated assumptions, the joint likelihood function of \mathbf{z} and \mathbf{Z} , under hypothesis H_k , is given by [1]

$$\begin{aligned} f_k(\mathbf{z}, \mathbf{Z}) &= \pi^{-N} |\mathbf{R}_T|^{-1} e^{-(\mathbf{z} - \mu_k \alpha \mathbf{v})^H \mathbf{R}_T^{-1} (\mathbf{z} - \mu_k \alpha \mathbf{v})} \\ &\quad \times \pi^{-NL} |\mathbf{R}|^{-L} e^{-\text{Tr} \{ \mathbf{R}^{-1} \mathbf{S} \}} \end{aligned} \quad (4)$$

where $\mu_k = k$ under H_k , $k = 0, 1$, $\text{Tr} \{ \cdot \}$ stands for the trace of the matrix between braces and

$$\mathbf{S} = \sum_{\ell=1}^L \mathbf{z}_\ell \mathbf{z}_\ell^H \quad (5)$$

is the sample covariance matrix. In order to derive the GLRT, one must maximize the likelihood function with respect to (w.r.t.) all unknown parameters, namely \mathbf{q} , \mathbf{R} under H_0 , and \mathbf{q} , \mathbf{R} , α under H_1 . Let us first observe that

$$|\mathbf{R}_T| = |\mathbf{R}| [1 + \mathbf{q}^H \mathbf{R}^{-1} \mathbf{q}] \quad (6)$$

$$\mathbf{R}_T^{-1} = \mathbf{R}^{-1} - \frac{\mathbf{R}^{-1} \mathbf{q} \mathbf{q}^H \mathbf{R}^{-1}}{1 + \mathbf{q}^H \mathbf{R}^{-1} \mathbf{q}} \quad (7)$$

so that $f_k(\mathbf{z}, \mathbf{Z})$ can be rewritten as

$$f_k(\mathbf{z}, \mathbf{Z}) = C_k [1 + \mathbf{q}^H \mathbf{R}^{-1} \mathbf{q}]^{-1} \exp \left\{ \frac{|(\mathbf{z} - \mu_k \alpha \mathbf{v})^H \mathbf{R}^{-1} \mathbf{q}|^2}{1 + \mathbf{q}^H \mathbf{R}^{-1} \mathbf{q}} \right\} \quad (8)$$

where

$$\begin{aligned} C_k &= \pi^{-N(L+1)} |\mathbf{R}|^{-(L+1)} \\ &\quad \times e^{-(\mathbf{z} - \mu_k \alpha \mathbf{v})^H \mathbf{R}^{-1} (\mathbf{z} - \mu_k \alpha \mathbf{v})} e^{-\text{Tr} \{ \mathbf{R}^{-1} \mathbf{S} \}} \end{aligned} \quad (9)$$

does only depend on \mathbf{R} (and possibly α). In order to maximize the likelihood function w.r.t. \mathbf{q} , we need to maximize

$$J_k = [1 + \mathbf{q}^H \mathbf{R}^{-1} \mathbf{q}]^{-1} \exp \left\{ \frac{|\mathbf{y}_k^H \mathbf{R}^{-1} \mathbf{q}|^2}{1 + \mathbf{q}^H \mathbf{R}^{-1} \mathbf{q}} \right\} \quad (10)$$

where $\mathbf{y}_k = \mathbf{z} - \mu_k \alpha \mathbf{v}$. For the sake of convenience, let us introduce the quantities $\tilde{\mathbf{y}}_k = \mathbf{R}^{-1/2} \mathbf{y}_k$, $\tilde{\mathbf{v}} = \mathbf{R}^{-1/2} \mathbf{v}$, and $\tilde{\mathbf{q}} = \mathbf{R}^{-1/2} \mathbf{q}$, so that the logarithm of J_k can be rewritten as

$$\ln J_k = -\ln [1 + \tilde{\mathbf{q}}^H \tilde{\mathbf{q}}] + \frac{|\tilde{\mathbf{y}}_k^H \tilde{\mathbf{q}}|^2}{1 + \tilde{\mathbf{q}}^H \tilde{\mathbf{q}}}. \quad (11)$$

Differentiating $\ln J_k$ w.r.t. $\tilde{\mathbf{q}}$ yields

$$\frac{\partial \ln J_k}{\partial \tilde{\mathbf{q}}} = -\frac{\tilde{\mathbf{q}}}{1 + \tilde{\mathbf{q}}^H \tilde{\mathbf{q}}} + \frac{(\tilde{\mathbf{y}}_k^H \tilde{\mathbf{q}}) \tilde{\mathbf{y}}_k}{1 + \tilde{\mathbf{q}}^H \tilde{\mathbf{q}}} - \frac{|\tilde{\mathbf{y}}_k^H \tilde{\mathbf{q}}|^2}{(1 + \tilde{\mathbf{q}}^H \tilde{\mathbf{q}})^2} \tilde{\mathbf{q}}. \quad (12)$$

Setting this derivative to zero implies that $\tilde{\mathbf{q}}$ is proportional to $\tilde{\mathbf{y}}_k$, i.e., $\tilde{\mathbf{q}} = \gamma \tilde{\mathbf{y}}_k$. In order to obtain γ , let us first observe that, with $\tilde{\mathbf{q}} = \gamma \tilde{\mathbf{y}}_k$, $\tilde{\mathbf{q}}^H \tilde{\mathbf{q}} = |\gamma|^2 \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k$ and $\tilde{\mathbf{y}}_k^H \tilde{\mathbf{q}} = \gamma \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k$. Hence, $\ln J_k$ in (11) can be rewritten as

$$g(\gamma) = -\ln [1 + |\gamma|^2 \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k] + \frac{|\gamma|^2 (\tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k)^2}{1 + |\gamma|^2 \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k} \quad (13)$$

where we emphasize the dependence towards γ . Differentiating the previous equation w.r.t. γ yields

$$\frac{\partial g(\gamma)}{\partial \gamma} = \frac{\gamma \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k}{(1 + |\gamma|^2 \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k)^2} [-|\gamma|^2 \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k + \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k - 1]. \quad (14)$$

Therefore, the derivative in (14) is null when $\gamma = \gamma_0$ such that

$$1 + |\gamma_0|^2 \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k = \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k. \quad (15)$$

With γ_0 given by (15), it can be readily verified that

$$\left. \frac{\partial^2 g(\gamma)}{\partial \gamma \partial \gamma^*} \right|_{\gamma=\gamma_0} = -\frac{(\tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k)^2 |\gamma_0|^2}{(1 + |\gamma_0|^2 \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k)^2} < 0 \quad (16)$$

and therefore γ_0 in (15) corresponds to a maximum of $g(\gamma)$. It follows that the maximizer of J_k is $\tilde{\mathbf{q}}_0 = \gamma_0 \tilde{\mathbf{y}}_k$. Reporting this value in (10), and using (15), it ensues that

$$\max_{\mathbf{q}} J_k = (\tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k)^{-1} \exp \{ \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k - 1 \} \quad (17)$$

which implies that

$$\begin{aligned} \max_{\mathbf{q}} f_k(\mathbf{z}, \mathbf{Z}) &= C_k (\tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k)^{-1} \exp \{ \tilde{\mathbf{y}}_k^H \tilde{\mathbf{y}}_k - 1 \} \\ &= \pi^{-N(L+1)} |\mathbf{R}|^{-(L+1)} e^{-1} (\mathbf{y}_k^H \mathbf{R}^{-1} \mathbf{y}_k)^{-1} e^{-\text{Tr} \{ \mathbf{R}^{-1} \mathbf{S} \}}. \end{aligned} \quad (18)$$

In order to obtain \mathbf{R} , we need now to maximize

$$J'_k(\mathbf{R}) = |\mathbf{R}|^{-(L+1)} (\mathbf{y}_k^H \mathbf{R}^{-1} \mathbf{y}_k)^{-1} e^{-\text{Tr} \{ \mathbf{R}^{-1} \mathbf{S} \}} \quad (19)$$

with respect to \mathbf{R} . It is straightforward to show that

$$\frac{\partial \ln J'_k(\mathbf{R})}{\partial \mathbf{R}} = -(L+1)\mathbf{R}^{-1} + \frac{\mathbf{R}^{-1}\mathbf{y}_k\mathbf{y}_k^H\mathbf{R}^{-1}}{\mathbf{y}_k^H\mathbf{R}^{-1}\mathbf{y}_k} + \mathbf{R}^{-1}\mathbf{S}\mathbf{R}^{-1}. \quad (20)$$

Setting the previous derivative to zero, it follows that the maximizer of $J'_k(\mathbf{R})$ must satisfy

$$(L+1)\mathbf{R} = \mathbf{S} + \frac{\mathbf{y}_k\mathbf{y}_k^H}{\mathbf{y}_k^H\mathbf{R}^{-1}\mathbf{y}_k}. \quad (21)$$

Therefore, the solution is necessarily of the form

$$\mathbf{R}(\beta) = (L+1)^{-1} [\mathbf{S} + \beta\mathbf{y}_k\mathbf{y}_k^H] \quad (22)$$

where β should be chosen so that (21) is fulfilled. However, with $\mathbf{R}(\beta)$ of the form (22), it can readily be shown that

$$\mathbf{y}_k^H\mathbf{R}^{-1}(\beta)\mathbf{y}_k = (L+1) \frac{\mathbf{y}_k^H\mathbf{S}^{-1}\mathbf{y}_k}{1 + \beta\mathbf{y}_k^H\mathbf{S}^{-1}\mathbf{y}_k}. \quad (23)$$

Using the previous equation along with (21)–(22), it ensues that the value of β for which (21) holds true is

$$\beta_0 = [L(\mathbf{y}_k^H\mathbf{S}^{-1}\mathbf{y}_k)]^{-1} \quad (24)$$

and the corresponding covariance matrix is

$$\mathbf{R}_0 = (L+1)^{-1} \left[\mathbf{S} + \frac{\mathbf{y}_k\mathbf{y}_k^H}{L(\mathbf{y}_k^H\mathbf{S}^{-1}\mathbf{y}_k)} \right]. \quad (25)$$

It can readily be verified that for any β , $J'_k(\mathbf{R}(\beta)) \leq J'_k(\mathbf{R}_0)$ and, hence, \mathbf{R}_0 in (25) is the maximizer of $J'_k(\mathbf{R})$. Additionally, with the matrix \mathbf{R}_0 of (25), one has $\mathbf{y}_k^H\mathbf{R}_0^{-1}\mathbf{y}_k = L\mathbf{y}_k^H\mathbf{S}^{-1}\mathbf{y}_k$, and $|\mathbf{R}_0|$ and $\text{Tr}\{\mathbf{R}_0^{-1}\mathbf{S}\}$ are constants which do not depend on \mathbf{y}_k . Gathering these results, we finally obtain

$$\max_{\mathbf{q}, \mathbf{R}} f_k(\mathbf{z}, \mathbf{Z}) = C' [\mathbf{y}_k^H\mathbf{S}^{-1}\mathbf{y}_k]^{-1}. \quad (26)$$

Under H_0 , the likelihood function has been maximized under all unknown parameters. Under H_1 , it remains to minimize $(\mathbf{z} - \alpha\mathbf{v})^H\mathbf{S}^{-1}(\mathbf{z} - \alpha\mathbf{v})$. However, it is well known [2] that

$$\min_{\alpha} (\mathbf{z} - \alpha\mathbf{v})^H\mathbf{S}^{-1}(\mathbf{z} - \alpha\mathbf{v}) = \mathbf{z}^H\mathbf{S}^{-1}\mathbf{z} - \frac{|\mathbf{z}^H\mathbf{S}^{-1}\mathbf{v}|^2}{\mathbf{v}^H\mathbf{S}^{-1}\mathbf{v}}. \quad (27)$$

Consequently, the GLR is

$$GLR = \frac{\mathbf{z}^H\mathbf{S}^{-1}\mathbf{z}}{\mathbf{z}^H\mathbf{S}^{-1}\mathbf{z} - \frac{|\mathbf{z}^H\mathbf{S}^{-1}\mathbf{v}|^2}{\mathbf{v}^H\mathbf{S}^{-1}\mathbf{v}}} \quad (28)$$

and the GLRT is equivalent to

$$\frac{|\mathbf{z}^H\mathbf{S}^{-1}\mathbf{v}|^2}{(\mathbf{v}^H\mathbf{S}^{-1}\mathbf{v})(\mathbf{z}^H\mathbf{S}^{-1}\mathbf{z})} \underset{H_0}{\overset{H_1}{\geq}} \eta \quad (29)$$

which is exactly the ACE!

Theoretical and numerical performance analysis of the ACE detector for arbitrary covariance mismatch can be found in [10]. When \mathbf{R}_T and \mathbf{R} satisfy the so-called generalized eigenrelation, i.e., when $\mathbf{R}^{-1}\mathbf{v} \propto \mathbf{R}_T^{-1}\mathbf{v}$ —which, in the case $\mathbf{R}_T = \mathbf{R} + \mathbf{q}\mathbf{q}^H$ amounts to $\mathbf{q}^H\mathbf{R}^{-1}\mathbf{v} = 0$ —, closed-form expressions for the distribution of the ACE test statistic are derived in [9]. These two references also provide analyses of and comparisons with the AMF and the GLRT.

III. CONCLUSIONS

In this letter, we considered the adaptive detection of a signal when there exists a covariance mismatch between the test cell and the training samples, due to a surprise or undernullified interference. We showed that the ACE is the GLRT for such a problem. This result thus shades a new light on the already observed fact that the ACE has excellent sidelobe rejection capability.

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