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A NOVEL DYNAMIC MODEL OF A REACTION WHEEL ASSEMBLY FOR HIGH ACCURACY POINTING SPACE MISSIONS

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ABSTRACT

This paper proposes a novel dynamic model of a Reaction Wheel Assembly (RWA) based on the Two-Input Two-Output Port framework, already presented by the authors. This method allows the user to study a complex system with a sub-structured approach: each sub-element transfers its dynamic content to the other sub-elements through local attachment points with any set of boundary conditions. An RWA is modelled with this approach and it is then used to study the impact of typical reaction wheel perturbations on a flexible satellite in order to analyze the micro-vibration content for a high accuracy pointing mission. This formulation reveals the impact of any structural design parameter and highlights the need of passive isolators to reduce the micro-vibration issues. The frequency analysis of the transfer between the disturbance sources and the line-of-sight (LOS) jitter highlights the role of the reaction wheel speed on the flexible modes migration and suggests which control strategies can be considered to mitigate the residual micro-vibration content in order to fulfill the mission performances.

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INTRODUCTION

The increasingly stringent requirements in pointing accuracy for observation and scientific Space missions have encouraged researchers to find innovative solutions to the micro-vibration problem for the last three decades. Pointing performances indeed play the main role in high resolution real time imagery and video products. For observation missions several needs have to be satisfied such as: borders and assets surveillance, disaster monitoring, search and rescue. Several scientific missions need high level of precision, like the James Webb Space telescope for the study of light from the first stars and galaxies to understand the origin of the Universe after the Big Bang [1] or the European Euclid mission to map the geometry of the dark Universe [2]. Pointing performances are strictly dependent on the spacecraft capability to reduce vibrations coming from many external or internal sources. Generally the external vibration sources, like the solar pressure, the gravity gradient, the magnetic disturbance (when present), are very low frequency perturbations corrected by the classical Attitude and Orbital Control System (AOCS). However the AOCS actuators, like the Reaction
Wheel Assembly (RWA), are also the main internal source of high frequency micro-vibrations on a spacecraft. Other sources of micro-vibrations can be the Solar Array Drive Mechanism, the cryocoolers (when present), the antenna trimming mechanism. The subsequent degradation of the line-of-sight (LOS) causes a complete loss of resolution at the imagery instrument level which cannot be corrected by on-ground dedicated algorithms like for the low frequency defaults.

For these reasons accurate models of the perturbation sources are needed to conceive hybrid active/passive solutions to counteract the undesirable effects of micro-vibrations at frequencies above the AOCS bandwidth. In this paper a novel and modular approach to model an RWA system is proposed. It extends the Two-Input Two-Output Port (TITOP) method [3] in order to simply assemble a block diagram representing the different elements of an RWA with and without a passive isolator interface. The advantage of this method is to be modular and appropriate for control/structure co-design. The TITOP approach is applied in [4] to study any flexible multi-body system with closed-loop kinematic chains and in [5] to model flexible solar panels considered as bending plates connected to a spacecraft main body.

The TITOP RWA model is then assembled with the dynamic model of a space telescope with two flexible solar arrays and an antenna obtained by the Satellite Dynamics Toolbox (SDT) [6] in order to analyze the possible coupling effects between the spacecraft structural modes and the flexible modes due to the RWA. In this way it is possible to analyze in which range of frequencies a passive solution can act to reduce the problem of micro-vibrations to reach the expected performances. This analysis is conducted according to all structural design parameters.

Moreover the coupled model of the satellite with the RWA proposed in this article is suited for a minimal Linear Fractional Representation (LFR) of the system with any parametric uncertainties linked to the RWA system (inertia properties, stiffness and damping values of bearings, interface plates, isolators, geometric dimensions) or to the spacecraft (inertia properties, frequencies and damping of the modes of the flexible surfaces, appendage attaching positions). In this way, a robust control by the modern $H_{\infty}$ techniques can be synthesized in order to take into account all the system uncertainties.

The present work is inspired by an adapted version of the system presented in [7] with the goal to build a modular model of the RWA in order to be ready to be directly interconnected with the results provided by the SDT.

The main contributions of this paper are: development of a novel TITOP model for an RWA system with passive isolators and analysis of the pointing performances to face the micro-vibrations problem for the modern Space missions.

The construction of the RWA model is firstly outlined. Then the coupled dynamics of the RWA with a spacecraft platform is faced. A pointing performance budget is finally addressed.

**TITOP MODEL OF AN RWA SYSTEM**

The TITOP model is a multi-input multi-output transfer with two channels corresponding to the two connection points of the sub-structure: a clamped node $P$ which links the element under study to the parent sub-structure and a free node $C$ which links the element to the child sub-structure. This clamped-free model can then be adapted to any complex boundary conditions by a simple inversion of the interested channels as it has been shown in [4].

Let us consider the flexible appendage $A$ in Fig. 1. It is connected to a parent sub-structure $P$ at point $P$ and to a child sub-structure $C$ at point $C$. The TITOP model [3] $D_{TITOP}(s)$ associated to the appendage $A$ is the $2 \times 2$ transfer shown in the block diagram of Fig. 2 that links the forces and the accelerations of $A$ at the points $P$ and $C$:

$$
\begin{bmatrix}
\dot{q}_C(s) \\
F_{A/P,P}(s)
\end{bmatrix} = D_{TITOP}(s) \begin{bmatrix}
F_{C/A,C}(s) \\
\dot{q}_P(s)
\end{bmatrix}
$$

(1)

with:

- $F_{A/P,P}(s)$ is the 6 degrees of freedom (DOFs) vector (3 translations, 3 rotations) of forces and torques applied by $A$ to the parent sub-structure $P$ (s is the LAPLACE variable),
- $\dot{q}_P(s)$ is the 6 DOFs inertial acceleration vector at the connecting point between $A$ and the parent sub-structure $P$,
- $F_{C/A,C}(s)$ is the 6 DOFs vector of forces and torques applied by the child sub-structure $C$ to $A$,
- $\dot{q}_C(s)$ is the 6 DOFs inertial acceleration vector at the connecting point between $A$ and the child sub-structure $P$.

![Figure 1. FLEXIBLE BODY $A$ CONNECTED TO THE SUB-STRUCTURE $P$ AT NODE $P$ AND TO THE CHILD SUB-STRUCTURE $C$ AT NODE $C$.

![Figure 2. TITOP MODEL OF THE APPENDAGE $A$.](image-url)
Let us now consider the RWA shown in Fig. 3, adapted configuration of the scheme presented in [7]. The assembly is composed of a reaction wheel activated by an electric motor. Flexible bearings are employed to permit the rotation of the wheel. An interface platform finally links the RWA to the spacecraft main hub. In order to simplify, the attachment point is considered corresponding to the center of mass (CoM) \( O_p \) of the interface plate.

The reaction wheel is considered as a rigid body. A fixed reference frame \((O_w; x, y, z)\), centred at the wheel center of mass, is considered to describe the motion of the wheel w.r.t the initial position. The wheel motion is described at each instant by the relative displacements \( x(t), y(t), \) and \( z(t) \) w.r.t. the inertial frame and the Euler angles \( \theta_x(t), \theta_y(t), \) and \( \theta_z(t) \), which results in the 6 DOFs vector \( \mathbf{q}_w(t) = [x(t) \ y(t) \ z(t) \ \theta_x(t) \ \theta_y(t) \ \theta_z(t)]^T \).

Similarly a fixed reference frame \((O_p; a, b, c)\) is defined for the interface plate and attached to its center of mass. The displacements and the rotations of the interface plate are described by the vector \( \mathbf{q}_p(t) = [a(t) \ b(t) \ c(t) \ \theta_a(t) \ \theta_b(t) \ \theta_c(t)]^T \).

In Fig. 3 the static and dynamic reaction wheel imbalances are respectively represented by the mass \( m_w \) on the perpendicular axis to the rotation one and the pair of masses \( m_d \) positioned on opposite sides of the rotational axis with a lever arm \( h_{rw} \). At the point \( O_p \) the spacecraft (SC) transmits to the RWA the forces/torques vector \( \mathbf{F}_{RW A/SC} \) produced by the RWA.

The described system can be easily decomposed in three elementary sub-systems in a TITOP framework:

- A TITOP model of a rigid body to describe the dynamics of the interface platform,
- A mass-independent model of a 6 degrees of freedom (DOFs) spring-damper system to simulate the kinematic and viscous properties of the bearing system,
- A simple port model to represent the dynamic of the isolated reaction wheel.

The motor torque \( T_m \) is parallel and acts in opposite directions on the reaction wheel and on the interface plate.

**Double Port model for a Rigid Body appendage**

Let us consider the rigid body in Fig. 4 clamped at node \( P \) and free at node \( C \).

From the Newton principle, the dynamics of the elementary rigid body can be described at the center of mass \( G \) by the following equation:

\[
- \tau_{PC}^{T} \mathbf{F}_{A/P,P} + \tau_{PC}^{T} \mathbf{F}_{C/A,C} = D_G^A \dot{\mathbf{q}}_G \tag{2}
\]

where \( D_G^A \) is the direct dynamic model of the rigid appendage \( A \):

\[
D_G^A = \begin{bmatrix} m_A^G & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{J}_G^A \end{bmatrix} \tag{3}
\]

with \( m_A^G \) and \( \mathbf{J}_G^A \) respectively mass and inertia of the appendage \( A \) w.r.t. the center of mass \( G \). \( \tau_{PG} \) is the kinematic model between points \( P \) and \( G \):

\[
\tau_{PG} = \begin{bmatrix} \mathbf{I}_3 & [PG]^x \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_3 \end{bmatrix} \tag{4}
\]

with \([PG]^x\) skew symmetric matrix associated with the vector \( PG \). The same notation convention is valid for the kinematic
model \( \tau_{CG} \). Thanks to the kinematic models, the accelerations at points \( P \) and \( C \) result:

\[
\ddot{q}_P = \tau_{PG} \ddot{q}_G \\
\ddot{q}_C = \tau_{CG} \ddot{q}_G
\]  

Equations (2), (5) can be represented by the block diagram depicted in Fig. 5.

The state-space representation of the TITOP model for a rigid body is thus reduced to the static model \( D_{PC}^A \):

\[
\begin{bmatrix}
\ddot{q}_C \\
F_{A/P,P}
\end{bmatrix} = \begin{bmatrix} 0_{6 \times 6} & \tau_{CP} \\ \tau_{CG} - \tau_{GP} D_{GP}^A \tau_{GP} & \tau_{PC} \end{bmatrix} \begin{bmatrix} F_{C/A,C} \\ \dot{q}_P \end{bmatrix} = D_{PC}^A \begin{bmatrix} F_{C/A,C} \\ \ddot{q}_P \end{bmatrix}
\]  

Note that, for the TITOP model of the rigid body, the first channel is not invertible. Nonetheless the inverse global exists and it can be written as:

\[
\begin{bmatrix}
F_{C/A,C} \\
\dot{q}_P
\end{bmatrix} = D_{PC}^A^{-1} \begin{bmatrix} \tau_{CG} D_{CG}^A \tau_{CG} \\ \tau_{PC} - \tau_{GP} D_{GP}^A \tau_{GP} \end{bmatrix} \begin{bmatrix} \ddot{q}_C \\
\dot{q}_P
\end{bmatrix} = D_{PC}^A^{-1} \begin{bmatrix} \ddot{q}_C \\
\dot{q}_P
\end{bmatrix}
\]  

Note also that the second channel of the direct model in Eqn. (6) is invertible. Thus the TITOP free-free model can be written as:

\[
\begin{bmatrix}
\dot{q}_C \\
\dot{q}_P
\end{bmatrix} = D_{PC}^A^{-1} \begin{bmatrix} \tau_{CG} \\ \tau_{PG} \end{bmatrix} \begin{bmatrix} \tau_{CG} - \tau_{PG} D_{GP}^A \tau_{PG} \end{bmatrix} \begin{bmatrix} F_{C/A,C} \\ \dot{q}_P \end{bmatrix} = D_{PC}^A^{-1} \begin{bmatrix} \ddot{q}_C \\
\dot{q}_P
\end{bmatrix}
\]  

where \( D_{PC}^A^{-1} \) is the direct model \( D_{PC}^A \) where all the six channels (6 DOFs) corresponding to the second port have been inverted.

Mass-independent model of a spring-damper system

Let us consider the spring-damper system in Fig. 6. Since this is a mass/inertia-lacking system, a TITOP model cannot be derived. However the free-free model \( D_{PC}^A \) can be obtained thanks to the 6 DOFs equality:

\[
\tau_{CG}^T F_{C/A,C} = \tau_{PG}^T F_{A/P,P} = \\
= K_G^A \tau_{CG} \ddot{q}_C + C_G^A \tau_{PG} \ddot{q}_P = \\
= K_G^A \Delta \ddot{q}_G + C_G^A \Delta \ddot{q}_G
\]

with \( K_G^A \) and \( C_G^A \) respectively kinetic and damping \( [6 \times 6] \) matrices associated to the appendage \( A \) at the point \( G \).

By considering the differential displacements/rotations \( \Delta \ddot{q}_G \) and translational/rotational velocities \( \Delta \dot{q}_G \) as states, the wanted model is written in state-space form as follows:

\[
\begin{bmatrix}
\Delta \ddot{q}_G \\
\Delta \dot{q}_G
\end{bmatrix} = \begin{bmatrix} 0_{6 \times 6} & \mathbf{I}_6 \\ \tau_{CG} & \tau_{PG} \end{bmatrix} \begin{bmatrix} \dot{\Delta \ddot{q}_G} \\
\dot{\Delta \dot{q}_G}
\end{bmatrix} = \begin{bmatrix} \Delta \ddot{q}_G \\
\Delta \dot{q}_G
\end{bmatrix}
\]  

Simple port model of a reaction wheel

Let us consider the reaction wheel in Fig. 7 rotating around the spinning z-axis at the angular speed \( \Omega \). Under the assumptions:

- \( \Omega \) and z-axis are constant in the wheel inertial frame,
- the wheel is balanced,
the Newton law of the system can be written as follows (for a detailed derivation see [8]):

$$F_{P/A,G} = M^A_G(s)q_G$$ (11)

where

$$M^A_G(s) = \begin{bmatrix} m^A_G \mathbf{I}_3 & 0_{3 \times 3} \\ 0_{3 \times 3} & J^A_G \end{bmatrix} + \begin{bmatrix} 0_{3 \times 3} \\ 0_{3 \times 3} \end{bmatrix} - \frac{1}{s} J_2 \Omega \mathbf{z}_x$$ (12)

with $m^A_G$ reaction wheel mass, $J^A_G = \text{diag}[J_r, J_r, J_z]$ inertia matrix with $J_r$ radial inertia and $J_z$ inertia around the spinning axis, and $\mathbf{z}_x$ skew symmetric matrix associated to the unitary axis vector $\mathbf{z}$:

$$\mathbf{z}_x = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ (13)

![Figure 7. REACTION WHEEL.](image)

From Eqn. (11) a minimal state-space model can be derived by considering the two state variables proportional to the angular speeds around the two axes perpendicular to the spinning axis:

$$x_1 = J_r \Omega \dot{\theta}_G,$$

$$x_2 = -J_z \Omega \dot{\theta}_G.$$ (14)

With the previous considerations the simple direct port model can be represented in state-space form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ F_{P/A,G} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m^A_G \mathbf{I}_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ q_G \end{bmatrix}$$ (15)

The inverse model $[M^A_G]^{-1}$ is thus:

$$\begin{bmatrix} \frac{1}{m^A_G} \mathbf{I}_3 \\ 0_{3 \times 3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_{P/A,G} \\ 0 \end{bmatrix}$$ (16)

**Assembled model of an RWA**

The complete TITOP model of the RWA system in Fig. 3 can be now obtained by assembling the three models described in the previous sections by considering the scheme in Fig. 8. Note that the motor responsible of the reaction wheel rotation can be modelled as a torque $T_m$ parallel to the wheel spin axis. $T_m$ is applied at the point $G_3$ and at the point $F_1$ as well with opposite orientation according to the action/reaction principle. The stiffness and damping of the spring-damper element, corresponding to the rotation axis, have to be imposed null in order to guarantee the free rotation of the wheel. Moreover it is also possible to introduce the reaction wheel disturbances $d_{bd}$ expressed as forces and torques acting in the wheel rocking frame centred at $G_3$:

$$d_{bd,G_3} = \begin{bmatrix} dF_1(\Omega) \\ dF_2(\Omega) \\ dF_3(\Omega) \\ dT_1(\Omega) \\ dT_2(\Omega) \end{bmatrix}^T$$ (17)

The main disturbance produced by the operating wheel is of harmonic nature centred on the wheel speed $\Omega$ and it is due to the static and dynamic mass imbalances. The sub/higher harmonics are caused by various disturbance sources such as bearing irregularities, dynamic lubrication behaviour or motor disturbances [9]. As proposed by [7], all the non-fundamental harmonics can be lumped into a $L_2$ bounded broadband disturbance $v = [v_{F_1}, v_{F_2}, v_{F_3}, v_{T_1}, v_{T_2}]^T$. With the previous considerations, the disturbances in Eqn. (17) can be written as:

$$dF_1(\Omega) = -U_1 \Omega^2 \sin(\Omega t + \phi') + v_{F_1},$$

$$dF_2(\Omega) = -U_2 \Omega^2 \sin(\Omega t + \phi') + v_{F_2},$$

$$dF_3(\Omega) = U_3 \Omega^2 \cos(\Omega t + \phi') + v_{F_3},$$

$$dT_1(\Omega) = -U_4 \Omega^2 \sin(\Omega t + \phi') + v_{T_1},$$

$$dF_2(\Omega) = U_5 \Omega^2 \cos(\Omega t + \phi') + v_{F_2},$$

where $U_1 = m_r r_w$ and $U_2 = m_d r_w \omega_w$, are respectively the static and dynamic imbalances, $\phi'$, $\phi'$ and $\phi^2$ are initial phase
angles and $\alpha_c = 0.2$ is a factor to characterize the axial harmonic disturbance caused by $U_x$.

With the previous considerations the RWA dynamic model can be represented by the block diagram in Fig. 9 where $D_{B_1C_1}(s)$, $D_{B_2C_2}^{-1}[1.6](s)$ and $M_{G_1}^{-1}(s)$ are respectively computed using Eqn. (6), (10), and (16).

**RWA model with passive isolator**

A passive isolator is now introduced between the interface plate and the spacecraft attachment point $O_k$ as in the schematic view in Fig. 10. The problem is quite similar to the previous one. Under the assumption that the passive isolators behave like a 6-DOFs viscoelastic joint with linear stiffness and damping, this element can be modelled as a spring-damper element.

**DYNAMICS OF A SPACECRAFT WITH AN RWA SYSTEM**

In this section the RWA model is coupled with the dynamics of a flexible spacecraft. In this way a preliminary analysis of the critical frequencies can be conducted. The used approach is modular: the models of the RWA and of the satellite are separately derived and then simply connected in a block diagram. This approach is straightforward because it allows the user to test different configurations and equipments in a puzzle-like way.

Let us consider a spacecraft with two flexible solar panels and a flexible antenna as in Fig. 12. An RWA system is clamped at a certain point of the spacecraft structure through the point $O_p$ of the RWA. In Fig. 12 the SC reference system $(O_{SC}, x_{SC}, y_{SC}, z_{SC})$ is centred in the center of mass of the satellite (which takes into account the mass properties of the RWA).
The dynamic model of the spacecraft without the RWA dynamic contribution is obtained thanks to the Satellite Dynamics Toolbox (SDT) [6].

The frequencies of the modes of the solar panels and the antenna are resumed in Table 1.

The SDT produces the inverse dynamic model of the spacecraft that is the transfer between the external forces/torques acting on the satellite at its center of mass and the accelerations produced in the SC reference frame.

If now the RWA dynamics is coupled with the spacecraft, a block diagram as the one of Fig. 13 has to be considered. As shown in the figure a rotation matrix $\mathbf{R}$ and a kinematic model $\tau_{O_{\text{p}}O_{\text{SC}}}$ project the dynamic information from the RWA reference frame centred in $O_{\text{p}}$ to the SC reference frame. $\mathbf{R}$ is in fact the rotation matrix between the two reference frames and $\tau_{O_{\text{p}}O_{\text{SC}}}$ is defined as:

$$
\tau_{O_{\text{p}}O_{\text{SC}}} = \begin{bmatrix}
1_{3} \\
\mathbf{r}_{O_{\text{p}}O_{\text{SC}}} \\
0_{3}
\end{bmatrix}_{\times}
$$

where $\mathbf{r}_{O_{\text{p}}O_{\text{SC}}}$ is the vector from $O_{\text{p}}$ to $O_{\text{SC}}$.

Let us consider the simple case where $O_{\text{p}}$ is exactly centred at the SC center of mass $O_{\text{SC}}$ and the RWA is used to control the axis $z_{\text{SC}}$.

Let now analyze the transfer between the RWA external disturbances $\mathbf{d}$ and the accelerations of the spacecraft $\mathbf{q}_{\text{SC}}$. The singular values of this transfer are shown in Fig. 14 for the configuration without isolator and in Fig. 15 for the configuration with isolator. The transfer is evaluated for the range $\Omega = [10 : 50]$ Hz of the reaction wheel speed. The coupling between the spacecraft structure and the RWA gives birth to gyroscopic modes at low frequencies. Note how the passive isolator acts like a low-pass filter by strongly reducing the resonance peaks around 100 Hz and 200 Hz due to the bearing modes. This role is highlighted in Fig. 16 where the magnitude of the transfer from the disturbance $dT_{m}$ to the angular accelerations $\dot{\omega}_{\text{SC}}$ for the configuration with and without passive isolators (with $\Omega = 30$ Hz) is shown. The bearing modes at 89.9 Hz, 111 Hz, 217.2 Hz, 231.9 Hz are highly damped with the configuration with the passive isolator.

Let better analyze the migration of the modes as function of the wheel speed $\Omega$. Fig. 17 and Fig. 18 show a zoom of the singular values of the transfer $\mathbf{d} \rightarrow \mathbf{q}_{\text{SC}}$ respectively for the configuration without and with the passive isolator. In the same figures the Campbell diagram, which plots the system modes as function of $\Omega$, highlights which are the $\Omega$-dependent modes. Note how in Fig. 17 the two couple of whirl modes (positive and negative), which split at 100 Hz and 221 Hz, tend to progressively diverge with growing $\Omega$.

### POINTING PERFORMANCES

In this section a simple AOCS based on a proportional-derivative (PD) controlled is derived as suggested in [10]. Let
Figure 14. SINGULAR VALUES OF THE TRANSFER FROM THE DIS-
TURBANCE $d$ TO THE ANGULAR ACCELERATIONS $\dot{q}_{SC}$ FOR THE
CONFIGURATION WITHOUT PASSIVE ISOLATORS.

Figure 15. SINGULAR VALUES OF THE TRANSFER FROM THE DIS-
TURBANCE $d$ TO THE ANGULAR ACCELERATIONS $\dot{q}_{SC}$ FOR THE
CONFIGURATION WITH PASSIVE ISOLATORS.

us consider only the control of the rotations of the spacecraft and
assume that only a wheel with passive isolators is used to control
the spacecraft $z$-axis. Thus the AOCS directly provides the con-
trol torques for $x$ and $y$-axes to the spacecraft as illustrated in Fig.
19. Since for the considered satellite the three axes are almost
decoupled, three decoupled PD controllers can be tuned on the
total static inertia of the spacecraft. The control law results:

$$T_m = -K_D \theta_{SC} - K_P \theta_{SC}$$

(20)

with $K_P = \omega_{des}^2 \cdot [Jx, tot \ Jy, tot \ Jz, tot]$ and $K_D = 2\zeta_{des} \omega_{des} \cdot
[Jx, tot \ Jy, tot \ Jz, tot]$, where $\omega_{des}$ and $\zeta_{des}$ are respectively the requi-
red closed-loop bandwidth and the damping ratio. The values
chosen for this study are: $\omega_{des} = 0.06 \text{rad/s}$ and $\zeta = 0.7$.

The pointing performance is defined as the relative pointing
error ($\theta_{SC}$) using the frequency domain filter [11, 12]:

$$W_{RPE} = \frac{t_\Delta s (t_\Delta s + \sqrt{12})}{(t_\Delta s)^2 + 6(t_\Delta s) + 12}$$

(21)

and by imposing a maximum amplitude $\epsilon_{max} = 10 \text{nrad}/\sqrt{\text{Hz}}$
of the RPE spectral density. If each component $i$ of the input disturbance $d$ is normalized with the worst harmonic perturbation $a_i \Omega^2$, with $a_i$ harmonic coefficient, the three transfers from $d$ to the three pointing errors $\theta_{SC}$ are illustrated in Figs. 20, 21, and 22. The values used for $a_i$, with $i = 1 \ldots 5$ are [7]: $a = [0.785, 0.785, 0.08, 0.324, 0.324]^T$. The cut-off at 0.01Hz is due to the AOCS control bandwidth. The pointing requirement (red line in the figures) is the filter $\varepsilon_{max} \cdot W_{RPE}$. The three figures show that the pointing requirement is satisfied for the three axes also at high frequencies.

**CONCLUSION**

This paper presents a novel model of a Reaction Wheel Assembly based on the Two-Input Two-Output Port approach. Its modularity allows the user to study any configuration of the sy-
system and to test different designs. The direct interconnection with the dynamics of a flexible spacecraft leads to the analysis of the structural interactions among the different flexible elements of the system. A preliminary pointing budget is finally presented for a study case in the frequency domain in order to highlight the potentialities of the presented model for structural/control co-design problems.

ACKNOWLEDGMENT

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Table 1. NOMINAL VALUES OF DATA (REACTION WHEEL AND BALL BEARINGS PARAMETERS ARE ADAPTED FROM [7]).

<table>
<thead>
<tr>
<th>System</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel</td>
<td>( m_w )</td>
<td>Mass</td>
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<td></td>
<td>([ J_{x_j} J_{y_j} J_{z_j} ] )</td>
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<tr>
<td></td>
<td>( d_{f} )</td>
<td>See Fig. 3</td>
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<td></td>
<td>( c_{b_{j_i}}, c_{b_{j_i}} )</td>
<td>Damping (rot.)</td>
<td>0.1884Ns/rad</td>
</tr>
<tr>
<td>Interface</td>
<td>( m_i )</td>
<td>Mass</td>
<td>2.02kg</td>
</tr>
<tr>
<td>Plate</td>
<td>([ J_{x_i} J_{y_i} J_{z_i} ] )</td>
<td>Inertia</td>
<td>([31.6 31.6 128.9] \text{g m}^2)</td>
</tr>
<tr>
<td>Passive</td>
<td>( k_{i_{j_i}}, k_{i_{j_i}} )</td>
<td>Stiffness (trans.)</td>
<td>1.013 \cdot 10^3 \text{N/m}</td>
</tr>
<tr>
<td>Isolator</td>
<td>( k_{i_{j_i}}, k_{i_{j_i}}, k_{i_{j_i}} )</td>
<td>Stiffness (rot.)</td>
<td>2.026 \cdot 10^3 \text{Nm/rad}</td>
</tr>
<tr>
<td></td>
<td>( c_{i_{j_i}}, c_{i_{j_i}}, c_{i_{j_i}} )</td>
<td>Damping (trans.)</td>
<td>512Ns/m</td>
</tr>
<tr>
<td></td>
<td>( c_{i_{j_i}}, c_{i_{j_i}}, c_{i_{j_i}} )</td>
<td>Damping (rot.)</td>
<td>10.24Ns/rad</td>
</tr>
<tr>
<td>SC Main</td>
<td>( m_{SC} )</td>
<td>Mass</td>
<td>5160kg</td>
</tr>
<tr>
<td>Body</td>
<td>([ J_{x_{SC}} J_{y_{SC}} J_{z_{SC}} ] )</td>
<td>Inertia</td>
<td>([25541 26514 11997] \text{kg m}^2)</td>
</tr>
<tr>
<td>Solar</td>
<td>( m_{s_{SA}} )</td>
<td>Mass</td>
<td>31kg</td>
</tr>
<tr>
<td>Arrays</td>
<td>([ J_{x_{SA}} J_{y_{SA}} J_{z_{SA}} ] )</td>
<td>Inertia</td>
<td>([484.5 13.5 498] \text{kg m}^2)</td>
</tr>
<tr>
<td>Antenna</td>
<td>( m_{a_{SA}} )</td>
<td>Mass</td>
<td>18.3kg</td>
</tr>
</tbody>
</table>

\( \begin{align*}
\text{Modes Frequencies} & \quad [16.41 72.71 73.04] \text{kg m}^2 \\
\text{Modes Damping} & \quad 0.011
\end{align*} \)