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CA-CFAR Detection Based on an AWG Interference Model in a Low-Complexity WCP-OFDM Receiver

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Abstract—In this paper, we consider a previously described low-complexity WCP-OFDM radar receiver and focus on the self-interference component induced by targets throughout this processing. Particularly, we assume and verify by simulation that this self-interference can be modeled in the range-Doppler map as an additive white Gaussian process independent from the internal noise. To that end, we propose an expression for the signal-to-interference-plus-noise ratio and verify by simulation that the expected performance of the well-known CA-CFAR detector are recovered and thus predictable.

I. INTRODUCTION

In modern applications such as vehicular technology, merging communication and radar systems is tempting to save resources (e.g., spectrum, energy, weight, volume). To that end, several waveforms were proposed to jointly perform data transmission and radar sensing [1]–[3]. Particularly, cyclic prefix orthogonal frequency-division multiplexing (CP-OFDM) multicarrier modulations have shown interesting capabilities to fulfill this dual function [2], [4]. More specifically in [4], the authors described a low-complexity radar receiver suited for such a waveform that consists of: i) linear estimation of the data symbols; ii) symbols removal; iii) bi-dimensional discrete Fourier transform (2D-DFT) to obtain the traditional range-Doppler map.

Recently we extended this 3-stage radar processing to a more general class of multicarrier waveforms referred to as weighted cyclic prefix (WCP)-OFDM [5]. It allows non-rectangular short pulses to be used while preserving perfect symbol reconstruction and low-complexity implementation [6]. Orthogonal time-frequency localized (TFL) pulses [7] have thus revealed their benefit over CP-pulses in presence of large Doppler shifts at close range [5].

Still, since the low-complexity receiver of [5] does not perform matched filtering, interference terms appear throughout the 3-stage processing. Visually, this manifests as a loss on the target peak that converts to a white-looking process in the range-Doppler map. Yet, this interference, or self-interference from a target point of view, has been formally neglected so far. This paper focuses on deriving statistical properties of these self-interference terms for detection purposes.

To that end, the paper is organized as follows. Section II recalls the existing WCP-OFDM radar model and gives an expression of the induced interference. In Section III, we provide an analytical expression of the signal-to-interference-plus-noise-ratio (SINR) in the range-Doppler map of the system that allows the performance of the so-called cell-averaging constant-false-alarm-rate (CA-CFAR) detector [8] to be predicted. Finally, Section IV concludes and outlines potential prospects.

II. WCP-OFDM Radar Model

Herein we express the target self-interference terms appearing in the range-Doppler measurements of the WCP-OFDM receiver presented in [5] (Fig. 1(a)-(b)).

A. Linear WCP-OFDM transmitter

Let \( \{c_{k,m}\}_{(k,m) \in I} \) be a sequence of complex symbols to transmit with \( I = \{0, \ldots, K - 1\} \times \{0, \ldots, M - 1\} \) where \( K \) and \( M \) denote the number of sub-carriers and blocks (i.e., radar sweeps), respectively. The baseband output of the transmitter, sampled at critical rate \( 1/T_s \), is [6]

\[
s[l] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} c_{k,m} g[l - mL] e^{j2\pi k l / K}, \quad l \in \mathbb{Z}
\]

where \( g[l] \) is the transmit pulse shape; \( 1/K \) and \( L \) represent elementary symbol spacing in frequency and time, respectively. A WCP-OFDM transmitter is characterized by: (i) \( L/K > 1 \), (ii) \( g[l] = 0 \) for \( l < 0 \) or \( l > L - 1 \). As a result the transmitter can be implemented by a blockwise processing based on the inverse discrete Fourier transform (IDFT):

\[
s_m[l] = \begin{cases} 
\sum_{k=0}^{K-1} d_{k,m} e^{j2\pi k l / K} g[l] & \text{if } l \in \{0, \ldots, L - 1\} \\
0 & \text{otherwise}
\end{cases}
\]

such that

\[
s[l] = \sum_{m=0}^{M-1} s_m[l - mL], \quad l \in \mathbb{Z}
\]

where we introduced the precoded symbols \( d_{k,m} = c_{k,m} e^{j2\pi k mL} \) applying a rotation to each data symbol \( c_{k,m} \).
B. Single target: linear time-varying channel model

To express the self-interference terms, we consider a single point target perfectly located in a range gate $l_0$, with a constant radial velocity $v$ during the coherent processing interval (CPI). We further assume no range ambiguities and a slow moving target, namely $l_0 < L$ and $v < v_a$ where $v_a = c/(2f_cL)$ represents the ambiguous velocity, with $c$ the speed of light and $f_c$ the carrier frequency. In a narrowband scenario, the received signal can be expressed as

$$r[l] = \alpha s[l - l_0]\exp(j2\pi f_d l/L) + n[l]$$

where $\alpha$ and $f_d = v/v_a$ are the complex amplitude and the normalized Doppler frequency of the target; $n$ is the receiver noise. Range resolution is given by $\delta_r = cT_s/2$.

C. Linear WCP-OFDM receiver and radar processing

Let us now follow the 3-stage low-complexity processing of [4]:

i) A linear estimation of the symbols is performed at the radar receiver, as in the first stage of a conventional linear multicarrier receiver, namely, for $(k', m') \in I$,

$$\hat{c}_{k',m'} = \sum_{l=-\infty}^{+\infty} r[l]g[l-m' L] e^{-j2\pi \frac{f_k l}{K^{1/2}}}, \quad (k', m') \in I$$

where $g[l]$ denotes the receive pulse shape. In the WCP-OFDM framework, $g[l]$ also verifies $g[l] = 0$ for $l < 0$ or $l > L - 1$. Thus, by truncating the received signal (2) to its first $ML$ samples, a DFT-blockwise operation can again be implemented:

$$\tilde{d}_{k',m'} = \sum_{l=-\infty}^{+\infty} r_{m'}[l]g[l] e^{-j2\pi \frac{f_k l}{K^{1/2}}}, \quad (k', m') \in I$$

with

$$r_{m'}[l] = \begin{cases} r[l + m' L] & \text{if } l \in \{0, \ldots, L - 1\} \\ 0 & \text{otherwise.} \end{cases}$$

and $\tilde{c}_{k',m'} = \tilde{d}_{k',m'} e^{-j2\pi \frac{f_k m' L}{K^{1/2}}}$. Note that since precoding is a linear and invertible operation, the remaining of the processing can be expressed as a function of the $\tilde{d}_{k',m'}$.

ii) Symbols removal is then achieved as symbols are known at the radar receiver

$$\tilde{d}_{k',m'} = \tilde{d}_{k',m'} / \tilde{d}_{k',m'}, \quad (k', m') \in I.$$

Using (2) and the finite support of $g, \tilde{g}$ in WCP-OFDM, this gives

$$\tilde{d}_{k',m'} = \hat{d}_{k',m'} + \hat{d}_{k',m'}^\prime + \hat{d}_{k',m'} + \bar{n}_{k',m'}$$

where each term of the sum, detailed thereafter, represent the useful signal (4a), the self-interference induced from the current (4b) and previous (4c) multicarrier blocks, and the noise (4d). Interestingly enough appears in (4) the cross-ambiguity function of $\tilde{g}, g$

$$A_{\tilde{g},g}(l, f) = K^{-1} \sum_{p=-\infty}^{+\infty} \tilde{g}[p]g[p - l] e^{j2\pi fp}$$

sampled at different points as exemplified in Fig. 2. The channel’s selectivity is in fact responsible for breaking the biorthogonality of the WCP-OFDM transmission system, denoted as $A_{\tilde{g},g}(pL, q/K) = \delta_{p,0}\delta_{q,0}$ with $\delta$ the Kronecker symbol. As a result, the system is no longer able to ensure a perfect reconstruction of the symbols following (3) and the target peak incurs a loss $A_{\tilde{g},g}(l_0, f_d / L)$ for the benefit of both interference terms, driven by $(L,1/K)$-spaced versions of $A_{\tilde{g},g}(l_0, f_d / L)$.

iii) If we neglect at first the interference contributions as in [4] and [5], the range-Doppler map of the scene is deduced from (4a) by computing an IDFT and a DFT along the slow-frequency and slow-time domains, respectively. This is abusively summarized as

$$\{x_{k',m'}\} = 2D\text{-DFT}\{\tilde{d}_{k',m'}\}.$$  

However, given the actual expression of (4) and since DFT is linear, we get, similarly, for $(k', m') \in I$,

$$x_{k',m'} = x_{k',m'} + x_{k',m'}^\prime + x_{k',m'} + x_{k',m'}.$$  

III. CA-CFAR DETECTION PERFORMANCE

From (4) we are able to derive an analytical expression of the SINR in the target cell of the WCP-OFDM radar’s range-Doppler map. This expression allows us to predict the performance of a CA-CFAR detector when applied to the output of the system.

A. SINR expression

We consider the following conventional framework: the receiver noise $n$ is white and circular-symmetric Gaussian with zero-mean and power $\sigma^2$; the data symbols $\{c_{k,m}\}_{(k,m) \in I}$ are independent and uniformly distributed according to a
\[
\begin{align*}
\bar{d}^{(1)}_{k',m'} &= \alpha e^{-j2\pi \frac{K}{L} k'} e^{j2\pi f_{d}m'} A_{g,g}(l_0, f_{d}/L) \\
\bar{d}^{(i,m')}_{k',m'} &= \alpha \sum_{k=0}^{K-1} d_{k,m'} e^{-j2\pi \frac{K}{L} k} e^{j2\pi f_{d}m'} A_{g,g}(l_0, f_{d}/L + (k - k')/K) \\
\bar{d}^{(i,m'-1)}_{k',m'} &= \alpha \sum_{k=0}^{K-1} d_{k,m'-1} e^{-j2\pi \frac{K}{L} k} e^{j2\pi f_{d}m'} A_{g,g}(l_0 - L, f_{d}/L + (k - k')/K) \\
\bar{n}_{k',m'} &= \frac{1}{d_{k',m'}} \sum_{l=-\infty}^{+\infty} n[l]\|\hat{g} - m'L\| e^{-j2\pi \frac{K}{L} (i-m'L)} K^{1/2}
\end{align*}
\]

chosen constellation, with zero-mean and power \(\sigma_c^2\). The constellation is further assumed to verify \(\mathbb{E}\{1/c_{k,m}\} = 0\) and \(\mathbb{E}\{c_{k,m}/c_{k,m}^*\} = 0\), which is actually fulfilled by usual modulations (e.g., quadrature phase-shift keying, or QPSK, and 16-quadrature amplitude modulation, or 16-QAM).

In this context we show that the self-interference terms are zero-mean, uncorrelated to each other and to the thermal noise \(n\). As a consequence, in the target cell of the range-Doppler map, denoted \(x \in \{x_{k',m'}(k',m')\in I\}\), the SINR defined as

\[
\text{SINR} \triangleq \frac{\mathbb{E}\{|x(t)|^2\}}{\mathbb{E}\{|x(n)|^2\} + \mathbb{E}\{|x(i,m')|^2\} + \mathbb{E}\{|x(i,m'-1)|^2\}}
\]

simply boils down to

\[
\text{SINR} = \frac{\mathbb{E}\{|x(t)|^2\}}{\mathbb{E}\{|x(n)|^2\} + \mathbb{E}\{|x(i,m')|^2\} + \mathbb{E}\{|x(i,m'-1)|^2\}}
\]

or in a more compact way,

\[
\text{SINR} = \frac{\sigma_t^2}{\sigma_n^2 + \sigma_i^2}
\]

with \(\sigma_t^2 = \mathbb{E}\{|x(t)|^2\}\), \(\sigma_n^2 = \mathbb{E}\{|x(n)|^2\}\) and \(\sigma_i^2 = \mathbb{E}\{|x(i,m')|^2\} + \mathbb{E}\{|x(i,m'-1)|^2\}\) the power of the useful term, noise and self-interference, in the target cell, respectively. These are found to be

\[
\begin{align*}
\sigma_t^2 &= \mathbb{E}\{|x(t)|^2\} K M |A_{g,g}(l_0, f_{d}/L)|^2 \\
\sigma_n^2 &= \sigma^2 \sigma_{c-1}^2 K^{-1} \|\hat{g}\|_2^2 \\
\sigma_i^2 &\approx \frac{1}{M \geq 1} \mathbb{E}\{\|\hat{g}\|_2^2\} \sigma_c^2 \sigma_{c-1}^2 \\
&\times \left[ \sum_{k=0}^{K-1} |A_{g,g}(l_0, f_{d}/L + k/K)|^2 - |A_{g,g}(l_0, f_{d}/L)|^2 \right] \\
&+ \left[ \sum_{k=0}^{K-1} |A_{g,g}(l_0 - L, f_{d}/L + k/K)|^2 \right]
\end{align*}
\]

where \(\sigma_{c-1}^2 = \mathbb{E}\{1/|c_{k,m}|^2\}\). The Monte-Carlo simulations run in Fig. 3 for different \(L/K\) and varying \((l_0, f_{d}/L)\) confirm these analytical results. In fact for \((l_0, f_{d}/L) = (0,0)\) the SINR (6) does reduce to the SINR as expected, whose expression is known to be [5]

\[
\text{SNR} = \frac{\mathbb{E}\{|x(t)|^2\} K^2 M}{\sigma_t^2 \sigma_{c-1}^2 \|\hat{g}\|_2^2}.
\]

Then as either \(l_0\) or \(f_{d}/L\) increases, the SINR incurs a loss with respect to the SNR, as hinted by Fig. 2. This gap remains constant at low SNR values, and corresponds to the loss \(|A_{g,g}(l_0, f_{d}/L)|^2\) incurred by \(\sigma_t^2\). But as the SNR is enhanced, self-interference contributions become more significant and SINR curves eventually have an horizontal asymptotic behavior (plateau effect). Finally, it is worth noticing that the SINR curves can be improved by increasing \(L/K\), at the cost of a decreased spectral efficiency though.
B. CA-CFAR detection performance

In this section, we characterize the performance of the CA-CFAR detector at the output of the studied WCP-OFDM radar. Considering the framework described in Section III-A, we make the following assumptions within the range-Doppler map:

(i) the self-interference is white, as suggested by Fig. 4;
(ii) the self-interference-plus-noise, or total interference, is circular-Gaussian distributed, zero-mean with variance $\sigma_i^2 + \sigma_n^2$ in each cell;
(iii) the cells are independent to each other.

Under these assumptions, the total interference is independent and identically distributed in the range-Doppler map, thus following an AWGI model. As a result, in our single target scenario, the environment is said homogeneous and the CA-CFAR detector is supposed to provide optimum detection performance at a fixed probability of false alarm (PFA) $P_{fa}$ [8].

The output of the square law rectifier applied to any cell $x \in \{x_{k',m'}\}_{(k',m') \in I}$ is denoted by $z = |x|^2$, as depicted in Fig. 1(c). For a white Gaussian interference, it is well known that the CA-CFAR detection test becomes [8]

$$z \leq \frac{H_0}{H_s} \beta \hat{\sigma}^2$$  \hspace{1cm} (8)

with

$$\beta = N_s \left[ P_{fa}^{-1/N_s} - 1 \right]$$

$$\hat{\sigma}^2 = N_s^{-1} \sum_{n_x=0}^{N_s-1} \tilde{z}_{n_x}$$

where $\beta$ is the so-called CA-CFAR constant for a given $P_{fa}$ and $\hat{\sigma}^2$ is an estimate of the local noise power from $N_s$ secondary range-Doppler cells. Now, restricting the study to Swerling I targets (i.e., $\alpha \sim \mathcal{CN}(0, \mathbb{E}\{|\alpha|^2\})$), the theoretical probability of detection (PD) is furthermore shown to be [8]

$$P_d = \left( 1 + \frac{\beta/N_s}{1 + \text{SINR}} \right)^{-N_s}.$$

In Fig. 5 the PD obtained by Monte-Carlo simulations with the CA-CFAR test (8) is compared to that obtained assuming an AWGI with the derived SINR (6), using TFL-pulses and a QPSK modulation. Both match perfectly thereby supporting the appropriateness of our AWGI model at least for the chosen numerical values. Besides, trends observed in Fig. 3 are recovered: (i) the higher $(l_0, f_d/L)$, the higher the SNR must be to achieve a desired probability of detection; (ii) PD losses are reduced with high $L/K$, i.e., at the cost of a spectral efficiency loss. Yet, note that the plateau effect...
Fig. 5. In fact with the chosen numerical values, $P_{\text{obs}}$ observed in Fig. 3 does not affect the performance curves of Fig. 5. In fact with the chosen numerical values, $P_d = 1$ is achieved before SINR curves reach their floor values. Note that similar performance curves have been observed for CP-pulses and with 16-QAM modulations (not shown here due to space limitation). Finally, the desired $H_a$ is also well retrieved by simulation.

IV. CONCLUSION

In this paper, we derive an expression of the interference generated by a target through a 3-stage WCP-OFDM radar processing previously described. This self-interference is assumed to be uncorrelated from the ambient noise, which allows us to deduce a closed-form expression of the SINR in the target cell of the computed range-Doppler map. Then, by assuming an AWGI model, we show that the expected performance in detection of the so-called CA-CFAR are recovered in simulations. This performance worsens at high range and/or velocity but this can be mitigated by adjusting spectral efficiency parameters. Natural prospects are to study more in depth the reliability of our AWGI model and its limits.

REFERENCES