Predicting the Surface Tension of Liquids: Comparison of Four Modeling Approaches and Application to Cosmetic Oils

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ABSTRACT: The efficiency of four modeling approaches, namely, group contributions, corresponding-states principle, \( \sigma \)-moment-based neural networks, and graph machines, are compared for the estimation of the surface tension (ST) of 269 pure liquid compounds at 25 °C from their molecular structure. This study focuses on liquids containing only carbon, oxygen, hydrogen, or silicon atoms since our purpose is to predict the surface tension of cosmetic oils. Neural network estimations are performed from \( \sigma \)-moment descriptors as defined in the COSMO-RS model, while methods based on group contributions, corresponding-states principle, and graph machines use 2D molecular information (SMILES codes). The graph machine approach provides the best results, estimating the surface tensions of 23 cosmetic oils, such as hemisqualane, isopropyl myristate, or decamethylcyclopentasiloxane (D5), with accuracy better than 1 mN/m. A demonstration of the graph machine model using the recent Docker technology is available for download in the Supporting Information.

1. INTRODUCTION

Surface tension (ST) is the force existing at a liquid/gas interface, which tends to minimize the surface area; it is caused by asymmetries in the intermolecular forces between molecules located at the interface. It plays an important role in a number of processes where a liquid/gas interface is present; for instance, it drives the shape of small liquid drops and bubbles or the wetting of a solid surface by a liquid. In cosmetics, the spreadability of oils on the human skin reflects both the sensory qualities and the efficacy of the product; Parente et al. reported that the spreadability of oils and the film forming properties could be partly correlated to the surface tension of the cosmetic oil; the lower the surface tension, the more spreadable the oil.

Surface tensions can be measured by numerous methods (e.g., Wilhelmy plate, Du Noiyy ring, maximum bubble pressure, drop-weight, or hanging-drop) accounting for the large amount of experimental data available in the literature. However, for a given compound, significantly different surface tension values can be found, depending on the method used and/or the purity of the liquid. Furthermore, when virtual liquids are generated in silico, the experimental determination of surface tension is not possible. In this case, it is crucial to predict reliably the surface tension values of candidate compounds in order to select the most promising ones, which should be synthesized first. Unfortunately, predicting this property is not straightforward because of the complexity of the phenomenon, i.e., the necessity to break bonds in order to bring a molecule from the bulk phase to the surface. Various prediction methods for surface tension values were described previously by several authors. In this paper, four modeling approaches, namely, group contributions, corresponding-states principle, \( \sigma \)-moment-based neural networks, and graph machines, are applied to the prediction of the surface tension of liquid compounds. These methods are described below.

Molecular dynamic simulations were first used in 1976, when Rao et al. highlighted the possibility of computing the surface tension of a liquid film from statistical thermodynamic relations. In 2016, Ghoul et al. calculated the surface tension by modeling the liquid/gas interface. Density functional theory has also been used successfully for the surface tension prediction of 18 pure nonpolar fluids. Nevertheless, models based on molecular dynamics are computer demanding and not suitable for structures with more than a few tens of atoms such as some we are interested in.

Introduced by Brock and Bird in 1955, the corresponding-states principle allows surface tension estimations from the critical constants \( P_c, V_c, \) and \( T_c \). Curl and Pitzer improved this model, which is restricted to short and weakly polar molecules by introducing an acentric shape parameter. Escobedo et al. used this model for the surface tension prediction of 94 compounds.
and Li et al. presented a new method based on a combination of corresponding-states and group contribution models.18

Group contribution-based (GC) methods are popular approaches to predict a wide range of physicochemical properties.19,20 In such methods, the molecular structure of any organic compound is viewed as an assembly of functional groups with specific property attributes. The properties of the studied molecule are computed as a function of the contributions of the different groups, with respect to their occurrence (first-order contribution), with respect to adjacent groups (second order), and with respect to other molecular attributes (third order). Regarding the prediction of surface tension, a model was introduced by Egemen et al. in 2000 to predict the surface tension of 44 organic compounds,21 and an extended model was proposed by Conte et al.7

Models based on quantum chemistry are also effective to predict a wide range of physicochemical properties of organic compounds. In particular, the COSMO-RS method (Conductor-like Screening MOdel for Real Solvents), introduced by Klamt in 1995, is based on unimolecular quantum chemical calculations combined with exact statistical thermodynamics.22–24 It provides the necessary information for predicting chemical potentials in liquids, from which one can compute many other thermodynamic equilibrium properties such as activity coefficients, solubility, partition coefficients, vapor pressure, and free energy of solvation,25 without requiring any experimental input. For instance, the interfacial tension between two liquids, i.e., the free energy per unit surface area that is required for maintaining an interface between two condensed phases, can be calculated using COSMO-RS.26 However, the prediction of surface tensions remains a challenge, yet unsolved by COSMO-RS (Klamt personal communication). Interestingly, Kondor et al. proposed to predict surface tensions at different temperatures by a regression method, using the so-called Klamt set of COSMO-RS σ-moments. This set of five descriptors is well known to encode as much chemical information as the complete set of 15 COSMO-RS σ-moments. The five σ-moments calculated from the σ-profiles of the molecules under study25,26,27 and the temperature were selected as inputs of a nonlinear Quantitative Structure–Property Relationship (QSPR) model,7 based on neural networks.

QSPR models are regression or classification models that predict a property of a molecule from selected descriptors thereof; they were used successfully for the prediction of surface tensions of a wide range of organic compounds.1 Kondor et al. used a data set of 1275 surface tension values, collected at different temperatures for 188 molecules, for building and validating the model. When the model was applied to a test set of 225 data points, the squared correlation coefficient R² between the estimated surface tensions and the experimental values was equal to 0.96. Stanton et al. described a 10-descriptor model for the prediction of surface tensions of 146 compounds.8 Kauffmann et al. developed a model based on eight descriptors for a data set of 159 molecules.9 Freitas et al. decreased the number of descriptors of the model to six to successfully predict the surface tensions of 299 compounds.10 Delgado et al. designed a model with six descriptors corresponding to the different intermolecular interactions in the bulk phase and calculated from the molecular structures for the prediction of the surface tensions of 320 compounds.

Goulon et al. developed graph machines as an alternative QSAR/QSPR method.28 Graph machines are regression or classification models that estimate properties of molecules directly from the topological information supplied by their SMILES (Simplified Molecular Input Line Entry Specification) codes. Therefore, they do not require any descriptor. In these models, molecules are described as graphs derived from their 2D-structure, and the parametrized functions that compute the estimation of the property or activity of interest reflect the compound molecular structures. As usual in regression or classification models, the parameters of the graph machines are computed during training from examples present in a database of experimental values of the property or activity of interest. The main benefit of the method is the absence of descriptors, with the SMILES codes being the only required information.

In the present work, surface tension estimations are performed by four different methods, falling into two families: (i) group contribution models and corresponding-states principle models (Section 2.1) and (ii) neural network regression and graph machine regression (Section 2.2). Neural network regression is performed from COSMO-RS σ-moments, whose computation is described in Section 2.3. For the first family, methods are taken from the literature. For the two methods of the second family, predictive models are designed and trained from a database of surface tensions, at 25 °C, of 269 molecules belonging to several chemical families and containing only carbon, hydrogen, oxygen, and silicon atoms, namely, alkanes, ethers, esters, ketones, carbonates, acids, alcohols, silanes, and siloxanes, with straight, branched, or cyclic chains. For all four methods, the predictive ability of the resulting models are assessed by the estimation of the surface tensions of 23 cosmetic oils that are not present in the training data set.

2. EXPERIMENTAL AND THEORETICAL METHODS

2.1. Group Contribution and Corresponding-States Based Models. Surface tension estimations using methods based both on group contribution (GC) or on the corresponding-states principle (CSP) were carried out using the InBioSynSolv (IBSS) property library developed by Heintz et al.27 IBSS software is intended to design new molecules satisfying targeted properties and to predict many physicochemical properties by means of methods encoded in a property model library.28,29 Among the methods available in IBSS for the prediction of surface tensions, the Conte et al. GC method7 was developed by splitting 402 molecules into chemical groups and estimating the parameters of the following relation:

\[
ST = \sum_i N_i C_i + \sum_j M_j D_j + \sum_k O_k E_k
\]

where \(ST\) is the surface tension, \(C_i, D_j,\) and \(E_k\) are the first-, second-, and third-order contributions estimated by successive linear regressions, and \(N_i, M_j,\) and \(O_k\) are the occurrences of the molecular groups. The reported standard deviation of the estimation error (i.e., of the difference between the estimated and measured values of the surface tension) is 1.47 mN·m⁻¹, and the mean absolute estimation error is 1.05 mN·m⁻¹ over the 402 data points of the database used for estimating the parameters \(C_i, D_j,\) and \(E_k\) in a range between 15 and 52 mN·m⁻¹.30 Once the contribution values are estimated, the GC method is intended to predict the property value of any molecular structure. Difficulties can appear when using such methods, e.g., a nonunique decomposition of some structures or compounds with fragments for which no group contributions have been defined yet. This is the case for siloxane derivatives, which are at the core of the present study.
Other estimation methods have been developed, based on the principle of corresponding states. This principle states that some dimensionless properties will follow the same relation over dimensionless variables of state and other dimensionless quantities for all compounds, typically, the reduced temperature \( T_r = T/T_C \) and reduced pressure \( P_r = P/P_C \), where \( T \) and \( P \) are the critical temperature and pressure. The related CSP models are the so-called two-parameter corresponding-states principle relation. For surface tension, an additional parameter, Pitzer’s dimensionless acentric factor \( \omega \), was introduced to better match the CSP relation and the experimental data, leading to the following relation:\(^{16,32}\)

\[
ST = \left[ \frac{1.86 + 1.18\omega}{19.05} \left[ \frac{3.75 + 0.91\omega}{0.291 - 0.08\omega} \right]^{2/3} (1 - T_r)^{11/9} \right]^{1/3}
\]

Zuo and Stenby used an 86-component database containing almost exclusively hydrocarbons (linear and branched alkanes and alkenes, aromatics) to develop the model for the surface tension based on a CSP approach. It includes two reference fluids, methane and \( n \)-octane:\(^{33}\)

\[
ST_r = \ln \left( 1 + \frac{ST}{P_c^{2/3} T_c^{1/3}} \right)
\]

\[
ST_r = ST_r^{(1)} + \frac{\omega - \omega^{(1)}}{\omega^{(2)} - \omega^{(1)}} (ST_r^{(2)} - ST_r^{(1)})
\]

where

\[
ST_r^{(1)} = 40.520(1 - T_r)^{1.287}
\]

and

\[
ST_r^{(2)} = 52.095(1 - T_r)^{1.21548}
\]

1: methane; 2: \( n \)-octane

In relations 2 and 3, with the critical pressure and temperature expressed in bar and Kelvin, respectively, \( ST \) is obtained in mN·m\(^{-1}\). The authors claimed a five percent accuracy for nonpolar fluids and reported larger deviations for more polar substances.

### 2.2. Neural Network and Graph Machine Modeling

#### 2.2.1. Database Construction

The data set required for the training of neural networks and graph machines was carefully built with the following requirements in mind: (i) to obtain a representative sample of the surface tension variation with the chain length for compounds belonging to homologous families, (ii) to select molecules that contain the constitutional atoms usually found in the cosmetic oils, i.e., carbon, hydrogen, oxygen, and silicon, (iii) to select, for each molecule, at least two consistent measured values for the surface tension provided by different literature references, and (iv) to include only molecules that are in the liquid state at 25 °C under atmospheric pressure. A data set of 269 molecules (hereinafter termed “complete set”) with a measured surface tension ranging from 12 to 48 mN·m\(^{-1}\) was built from data extracted from reliable sources\(^{34-36}\) and retrieved from technical datasheets.\(^{39,40}\) When measured data were not available at 25 °C, but for several other temperatures nearby, a simple linear equation was used to interpolate the surface tension at 25 °C.

#### 2.2.2. Model Selection

Model selection was a crucial step in the design of machine-learning based models. Its purpose is to find, given the available data, the appropriate model complexity that provides the best generalization, i.e., that provides the most accurate estimations on data that are not present in the training set. The available data set of 269 molecules was partitioned into a training/validation set of 244 molecules and a test set of 25 molecules for performance assessment. The molecules of the test set were chosen so that (i) the distributions of molecules among the chemical families considered were very similar in the complete and test sets and that (ii) the distribution of the surface tension values was as uniform as possible on the range of measured values. The distribution of the molecules among chemical families is displayed in Figure 1.

Model selection was performed by computing the virtual leave-one-out score (defined in Section 3.2) of each model, at the end of the training process performed with the training/validation set. Further details are provided in Sections 2.2.3 and 2.2.4.

After model selection, the performance of the selected neural network model and the performance of the selected graph machine model were assessed on the 25-molecule test set. As a final step, the surface tensions of 23 cosmetic oils were measured as described in Section 2.4 and predicted by the neural network and the graph machine models as described in Section 3.3.

#### 2.2.3. Neural Networks

A neural network is a nonlinear parametrized function.\(^{41}\) In the framework of nonlinear regression for estimating a quantity of interest from a training set of \( N_P \) elements, the neural network parameters are estimated from available data during a training phase, which consists of minimizing the sum of squared training errors \( J(\theta) \) with respect to the parameters

\[
J(\theta) = \sum_{i=1}^{N_P} (y_i - g_i(\theta))^2
\]

where \( y_i \) is the measured value of the quantity of interest for the \( i \)th element of the training set, and \( g_i(\theta) \) is the value of the quantity of interest estimated by the neural network. In the present case, the quantity of interest is the surface tension of liquids, hence \( y_i \) is the measured value of the surface tension of molecule \( i \) of the database, \( g_i(\theta) \) is the estimated value of the surface tension of molecule \( i \), and \( N_P = 244 \) as explained in the previous section. In the present study, the minimization of \( J(\theta) \) was performed by the Levenberg–Marquardt algorithm.

For nonlinear regression by neural networks, the usual form of \( g_i(\theta) \) is a linear combination of nonlinear functions called
“hidden neurons”. Each hidden neuron is a continuous, nonlinear function (typically a tanh function) of a linear combination of the variables. Such a neural network is called a multilayer Perceptron (MLP). It is particularly suitable for nonlinear regression because it is a universal approximator: any continuous nonlinear function can be approximated with arbitrary accuracy by an MLP having a finite number of hidden neurons. An MLP with zero hidden neurons is a linear model, hence is trained by traditional least-squares fitting.

As usual in nonlinear regression, the generalization ability of an MLP model depends on its complexity, which depends basically on the number of hidden neurons. A neural network with too many neurons approximates the training data very accurately but is unable to generalize, i.e., to predict accurately data that are not present in the training set; a neural network with too small a number of neurons is unable to learn the training data, hence to generalize (this phenomenon is known as the “bias-variance dilemma”). Therefore, the design of an MLP requires a model selection phase.59 In the present study, MLP selection was performed by virtual leave-one-out, as described in Section 3.2.

2.2.4. Graph Machines. As mentioned previously, a graph machine is intended to perform classification or regression on graphs; in QSAR/QSPR applications, the graphs under consideration are the 2D structures of the molecules. In the present case, a graph machine provides an estimate of the surface tension; the latter quantity being continuous, the task considered here is a regression task.

The design of a graph-machine based model includes the following steps:

- Construction of the 2D-graph of the molecule from its SMILES representation: Each node of the graph is a non-H atom, and each edge is a chemical bond of the graph. Each node has at least two labels: its nature and its degree (the number of chemical bonds that bind it to its adjacent non-H atoms). For molecules that contain cycles, hence are represented by a cyclic graph, edges are deleted in order to form an acyclic graph in which every path of the graph has its end at a specific node called “output node”.

- Construction of the computational structure: For each acyclic graph, a function is generated by implementing, at each node of the graph, a parametrized nonlinear function, called node function (typically an MLP). All node functions are identical and have the same parameters within each graph and for all graphs; therefore, the number of parameters of the resulting model is equal to the number of parameters of the chosen node function. As a result of this construction, the value computed by the output node of each model, which is intended to be an estimate of the property of interest, depends solely on the 2D structure of the molecule and on the values of the parameters of the node function.

- Estimation of the parameters of the node function by training from the database: This is done by minimizing the sum of squared errors \( f(\theta) \) defined in Section 2.2.3.

Details of the above steps are provided in a previous paper42 and references therein. The bias-variance dilemma applies to graph machines as well as to any machine learning method, so an appropriate model selection procedure must be performed. The selection method for graph-machine based models is described in Section 3.2.

2.3. COSMO-RS Computing Procedure. COSMO-RS uses a combination of quantum chemistry and statistical thermodynamics in order to compute the chemical potential \( \mu \) of a solute in a liquid phase, which can then be converted into physicochemical properties.22 As mentioned in the Introduction, no model for the estimation of surface tension is available from COSMOtherm: the so-called COSMO-RS \( \sigma \)-moments.

Because of polar covalent bonds, molecules bear a charge density \( \sigma \) on the so-called \( \sigma \)-surface, which is a slightly inflated van der Waals surface of the compound. The full 3D information about the charge density repartition on the \( \sigma \)-surface can be reduced to a curve \( p(\sigma) \), the \( \sigma \)-profile of the molecule \( X \), which is a smoothed histogram expressing how much of the \( \sigma \)-surface lies in the polarity interval \([\sigma - d\sigma/2, \sigma + d\sigma/2]\).23 Figure 2 gives an example of the \( \sigma \)-profiles of three cosmetic oils: propylheptyl caprylate, dioctyl ether, and tetradeacne. None of these compounds has Lewis acidity (hydrogen-bond (HB) donor region), but Lewis basicity (HB acceptor region) is present. The \( \sigma \)-profiles show the difference in the Lewis basicity, propylheptyl caprylate having the strongest one since the curve exhibits the most significant peak above 0.01 eÅ\(^2\).

Klamt suggested an expression of the partition coefficient \( K \) of a solute \( X \) between two phases as a development on the \( \sigma \)-moments (eq 8).23 Usually, a development to \( m = 6 \) is sufficient to estimate satisfactorily the partition coefficient \( K \).

\[
RT \ln K = c_{\text{dev}} M^X_{\text{dev}} + c_{\text{don}} M^X_{\text{don}} + \sum_{i=0}^{m} c_i M^X_i
\]  

(8)

The \( \sigma \)-moments \( M^X_i \) are calculated from the \( \sigma \)-profiles \( p^X(\sigma) \) of the studied compound \( X \), according to eqs 9–11:

\[
M^X_{\text{dev}} = \int_{+\sigma_{1/2}}^{+\infty} p^X(\sigma)(\sigma - \sigma_{1/2}) d\sigma
\]  

(9)

\[
M^X_{\text{don}} = \int_{-\infty}^{-\sigma_{1/2}} p^X(\sigma)(-\sigma - \sigma_{1/2}) d\sigma
\]  

(10)

\[
M^X_i = \int_{-\infty}^{-\infty} p^X(\sigma)\sigma^i d\sigma
\]  

(11)

Among the 15 \( \sigma \)-moments, the first ones have a simple physical meaning. The zero-order \( \sigma \)-moment \( M^X_0 \) is the whole surface area of the molecule, expressed in Å\(^2\). The first-order one \( M^X_1 \) is the polarization charge of this surface, expressed in e. For uncharged molecules, this moment is equal to zero. The second-order \( \sigma \)-
moment $M_{\sigma}^2$, expressed in $e^2 \cdot \text{Å}^{-2}$, is the polarity of the studied molecule. The third-order $M_{\sigma}^3$ represents the asymmetry of the $\sigma$-profile $p_{\sigma}(\sigma)$. The other $\sigma$-moments $M_{\sigma}^4$, $M_{\sigma}^5$, and $M_{\sigma}^6$ have no known physical meaning. Finally, $M_{\sigma}^2$, and $M_{\sigma}^2 \text{out}$ expressed in e, are the “hydrogen-bonding” $\sigma$-moments. They represent the ability of the molecule to interact with hydrogen-bond acceptors and donors, respectively. They have a nonzero value when the $\sigma$-profile of a molecule outstrips the range $[-\sigma_{\text{HB}} + \sigma_{\text{HB}}]$, where $\sigma_{\text{HB}}$ is the hydrogen-bond cutoff equal to 0.01 e $\text{Å}^{-2}$.

The 14—not 15 since $M_{\sigma}^3$ is always equal to zero for the uncharged molecules of our study—calculated $\sigma$-moments values for the complete and test sets of molecules are given in Tables S3 and S4 of the Supporting Information.

2.4. Surface Tension Measurements for Cosmetic Oils.

To assess the accuracy of the estimations of the developed model, a test data set of 23 cosmetic oils that are not present in the complete set was built. The surface tensions of these cosmetic oils were determined experimentally, using two different methods: measurements were both performed (i) with the Kruess K100 tensiometer which measures the force occurring when a probe (ring, plate or rod) is completely wetted with a liquid and (ii) by the pendant drop method associated with the Kruess DSA10 tensiometer. It has thus been verified on a few molecules that values obtained for surface tensions are consistent from one method to the other. The values measured with the K100 tensiometer at 23.5 °C are presented in Table 3. The rod is immersed in each liquid studied, with an immersion rate of 10 mm min$^{-1}$ and an immersion depth of 2 mm. The force acting on the rod is recorded and allows the calculation of the surface tension of the liquid. Before each series of measurements, calibration with pure water is carried out. Additionally, surface tensions of four liquids belonging to the training/validation set, whose values (reported in the literature) can be found in Table S1 of the Supporting Information, were measured for the following results (all values with a standard deviation of 0.2 mN m$^{-1}$): n-undecane ST = 23.1 mN m$^{-1}$, n-pentadecane ST = 26.9 mN m$^{-1}$, triacetine ST = 35.1 mN m$^{-1}$, and decamethycyclopentasiloxane ST = 18.0 mN m$^{-1}$. These values are consistent with those of the literature. During a liquid measurement, the surface tension values were sampled every 3 s until the standard deviation was below 0.1 mN m$^{-1}$. Three measurements were taken in a row for each sample. The means and standard deviations of the measurements performed on the 23 compounds are reported in Table S2 of the Supporting Information. No temperature correction was made to the previous measured values since the observed deviations—between 0.1 and 0.7 mN m$^{-1}$—were larger than the temperature corrections, expected to be below 0.2 mN m$^{-1}$.

3. RESULTS AND DISCUSSION

3.1. Dependence of Surface Tension on Chemical Function and Chain Length. The variation of the surface tensions of several chemical families (alkanes, ethers, esters, ketones, carbonates, acids, alcohols, silanes, and siloxanes) with the number of carbon atoms is plotted in Figure 3, gathering experimental values from at least two independent sources. For silicon-containing compounds, the variation of the surface tension is plotted against the sum of the number of carbon and silicon atoms. For instance, decamethycyclopentasiloxane $(\text{C}_{10}\text{H}_{30}\text{O}_{5}\text{Si}_{5})$ is plotted at abscissa 15 on the X-axis of Figure 3. For all chemical families, the surface tension increases with the number of carbon or (carbon+silicon) atoms. Surface tension is closely related to the intermolecular forces: the stronger these forces, the more tightly bound the molecules, hence the higher the surface tension. Because long chains are more likely to form more intermolecular bonds than small ones do, the surface tension increases with chain length. Silicones exhibit drastically low surface tension. This is due to their inorganic polar backbone on which apolar organic groups are grafted, able to rotate freely around the Si–O bonds. These apolar groups shield the Si–O backbone, preventing the dipolar interactions thereby decreasing the surface tension. Apart from silicones, alkanes and ethers have the lowest surface tensions because only London interactions take place between CH$_{12}$/CH$_3$ groups. Compounds containing polar ester groups have higher surface tensions because of the Keesom and Debye interactions. Finally, alcohols have higher surface tension values because these molecules carry a free hydroxyl group that is involved in strong intermolecular hydrogen bonds. As explained in the Introduction, a low surface tension results in high spreadability. That is why silicones are used in cosmetic formulations. Figure 3 gives an indication on how chemical functions affect the surface tension of linear molecules, showing that alkanes and ethers are the carbon compounds whose surface tensions are closest to those of silicones. It also focuses on the effect of chain length on the surface tension for series of linear molecules (except cyclic silicones), highlighting the increase in intermolecular forces with the chain length, responsible for an increase in surface tension. Obviously, this property does not depend only on those two factors, but also on topological modifications such as cyclization or branching of compounds. Thus, results presented in the following sections of this study also deal with branched and cyclic compounds.

3.2. Modeling Surface Tensions of the Liquids of the Complete Set. 3.2.1. Surface Tension Estimation from $\sigma$-Moments by Neural Networks (Multilayer Perceptrons). In the present section, we discuss the design and test of a multilayer Perceptron for the estimation of the surface tension of the molecules of the complete data set described in Section 2.2.1, comprising 269 compounds that carry diverse functional groups (ethers, ketones, acids, carbonates, esters, silanes, siloxanes, alkanes, and alcohols) and that possess straight or branched chains, cycles, etc. The compounds contain only carbon, hydrogen, oxygen, and silicon atoms because those are the
Training and of the Predicting Ability for Increasing Neural Network Complexity

Table 1. Surface Tension Estimation from \( \sigma \)-Moments by Neural Networks (Multilayer Perceptrons). Estimation of the Quality of Training and of the Predicting Ability for Increasing Neural Network Complexity

<table>
<thead>
<tr>
<th>number of parameters</th>
<th>29</th>
<th>36</th>
<th>43</th>
<th>50</th>
<th>57</th>
<th>64</th>
<th>71</th>
<th>78</th>
<th>85</th>
<th>92</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of hidden neurons</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>RMSTE ( ^{1} )</td>
<td>2.55</td>
<td>2.41</td>
<td>2.22</td>
<td>1.94</td>
<td>1.75</td>
<td>1.66</td>
<td>1.46</td>
<td>1.28</td>
<td>1.25</td>
<td>1.13</td>
</tr>
<tr>
<td>VLOO score ( ^{b} )</td>
<td>3.54 (0.02)</td>
<td>3.29 (0.05)</td>
<td>3.17 (0.09)</td>
<td>2.98 (0.15)</td>
<td>2.96 (0.11)</td>
<td>2.83 (0.21)</td>
<td>2.72 (0.08)</td>
<td>2.71 (0.13)</td>
<td>2.71 (0.12)</td>
<td>2.74 (0.10)</td>
</tr>
</tbody>
</table>

\( ^{1} \)RMSE value (mN m\(^{-1} \)) of the model (out of 100) having the smallest RMSTE for the 244 molecules of the training/validation set. \(^{b}\)Mean and standard deviation (in parentheses) of the VLOO scores (mN m\(^{-1} \)) averaged over the 10 models (out of 100) having the smallest VLOO scores for the 244 molecules of the training/validation set.

only atoms present in the backbone of cosmetic oils. The database was partitioned into a training/validation set of 244 compounds and a test set of 25 compounds as described in Section 2.2.3. In the spirit of Kondor et al., the estimation was performed by a multilayer Perceptron from five COSMO-RS \( \sigma \)-moments: \( M_{0}, M_{1}, M_{2}, M_{ac}, \) and \( M_{a}, \) [Neural network computations were performed with a homemade software package based on the NeuroOne kernel, a former product of Netral S.A.] Feature selection by the random probe method confirmed the fact that for our data \( M_{0},M_{2}, \) and \( M_{a}, \) are the most relevant for ST estimation.

Model selection was performed by training models of increasing complexity (number of hidden neurons). For each model complexity, 100 models were trained with different random initial parameter values. The Root Mean Square Training Error (RMSTE) and the Virtual Leave-One-Out (VLOO) score were computed:

\[
RMSTE = \sqrt{\frac{1}{N_{T}} \sum_{i=1}^{N_{T}} (f(\theta_{m}) - y_{i})^{2}},
\]

where \( f(\theta) \) is the sum of square errors (eq 7), \( \theta_{m} \) is the vector of parameters of the model after completion of training, and \( N_{T} = 244 \) is the number of molecules in the training/validation set:

\[
VLOO \text{ score} = \sqrt{\frac{1}{N_{T}} \sum_{i=1}^{N_{T}} (y_{i} - g(\theta_{m}^{\text{VLOO}}))^{2}},
\]

where \( y_{i} \) is the surface tension value measured for molecule \( i, \) and \( g(\theta_{m}^{\text{VLOO}}) \) is the surface tension value for molecule \( i \) that would have been estimated by the model if molecule \( i \) had been left out the training set (i.e., if the model had been trained on all molecules of the training/validation set except molecule \( i \)). The virtual leave-one-out score provides an unbiased estimation of the generalization ability of the model. The 10 models (out of 100) having the smallest VLOO scores were stored.

The third row of Table 1 shows the values of the RMSTE on the 244 molecules of the training/validation set for the best trained models (models with minimum RMSTE) of increasing complexity. As expected, RMSTE decreases monotonously with increasing complexity; by contrast, the mean value of the VLOO scores of the 10 stored models (fourth row) first decreases, reaches a minimum for the model with 78 parameters (multilayer Perceptron with 11 hidden neurons), and subsequently increases. This illustrates the bias-variance dilemma explained in Section 2.2.3. The VLOO score difference between the 71- and 78-parameter models is very small, and the recommended practice in such a situation is to select the most parsimonious (least complex) model. Therefore, the multilayer Perceptron with 10 hidden neurons was selected for testing. The results of the models with less than four hidden neurons are not displayed, as they have much higher values.

For the selected model—with 10 hidden neurons—the VLOO ST estimates for molecules of the training/validation set and the ST estimates for the molecules of the test set are plotted against their measured values in Figure 4. The RMSE computed for the training/validation and test sets are equal to 1.91 and 3.60 mN m\(^{-1} \), respectively, and the determination coefficients \( R^{2} \), displayed for both sets, are equal to 0.89 and 0.76, respectively. The surface tension is correctly estimated around the center of the range but is clearly inadequate for the outer examples, which impairs the \( R^{2} \) value for the test set where outliers, being at the ends of the range, have large leverages.

As mentioned in the Introduction, Kondor et al. were the first to use the COSMO-RS \( \sigma \)-moments for nonlinear regression by neural networks. To build their model, Kondor et al. used a database of 1500 surface tensions recorded for 188 molecules at different temperatures. The initial set was partitioned into a training/validation set of 1275 examples and a test set of 225 examples. The results, obtained with a neural network whose variables were the COSMO-RS \( \sigma \)-moments mentioned above, were fair for both sets. However, the method used for partitioning the set remains unclear since there are more examples in the test set than the number of initial molecules. Therefore, some compounds are inevitably in both sets at least once, though at different temperatures. This is clearly identified
for CO₂ with three data points belonging to the training set and one to the test set. A solid assessment of the prediction ability of Kondor’s model would request a test with fresh examples not pertaining to any of the previous sets and possessing known surface tension values at various temperatures. To the best of our knowledge, this is yet to be done.

### 3.2.2. Surface Tension Estimation from SMILES by Graph Machines

In the present section, we discuss the design and test of graph machines (described in Section 2.2.4) for the estimation of the surface tension of the molecules of the complete data set described in Section 2.2.1, comprising 269 compounds, and partitioned into a training/validation set of 244 compounds and a test set of 25 compounds.

The model design methodology for graph machines is the same as described in Section 3.2.1 for multilayer Perceptrons: graph machines of increasing complexity (number of hidden neurons of the node function) were trained. For each model complexity, 100 models were trained with different random initial parameter values. RMSTE (eq 12) and the VLOO scores (eq 13) were computed, and the 10 models having the smallest VLOO scores were stored.

The third row of Table 2 shows the RMSTE values of the best-trained models (models with smallest RMSTE) with different complexities for the 244 molecules of the training/validation set. As expected, the RMSTE decreases with increasing complexity, while the mean value of the VLOO scores of the 10 stored models (fourth row) goes through a minimum (model with 95 parameters, i.e., node function with seven hidden neurons) and subsequently increases. Therefore, graph machines with seven hidden neurons were selected for further testing.

As an illustration, the VLOO ST estimates for molecules of the training/validation set and the ST estimates for the molecules of the test set are plotted against their measured values in Figure 5.

![Figure 5](image)

**Figure 5.** Surface tension estimation from SMILES by graph machines. VLOO surface tension (ST) estimates computed by graph machines (node function with seven hidden neurons) for the 244 molecules of the training/validation set (gray hollow circle) and surface tension estimates for the 25 molecules of the test set (black filled circle) vs measured values of the surface tension. The VLOO estimates for the molecules of the training/validation set and the estimates for the molecules of the test set are computed with the 10 models (out of 100) that have the smallest VLOO scores. The dashed gray line is the regression line for the training/validation set, while the dashed black line is the regression line for the test set.

Table 2. Surface Tension Estimation from SMILES by Graph Machines. Estimation of the Quality of Training and of the Predicting Ability for Increasing Graph Machine Complexity

<table>
<thead>
<tr>
<th>number of parameters</th>
<th>31</th>
<th>44</th>
<th>59</th>
<th>76</th>
<th>95</th>
<th>116</th>
<th>139</th>
<th>164</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of hidden neurons</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>RMSTE&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.99</td>
<td>1.33</td>
<td>0.94</td>
<td>0.75</td>
<td>0.49</td>
<td>0.40</td>
<td>0.31</td>
<td>0.18</td>
</tr>
<tr>
<td>VLOO score&lt;sup&gt;b&lt;/sup&gt;</td>
<td>2.77 (0.08)</td>
<td>2.40 (0.15)</td>
<td>2.31 (0.27)</td>
<td>2.05 (0.21)</td>
<td>1.71 (0.29)</td>
<td>1.85 (0.26)</td>
<td>1.90 (0.26)</td>
<td>2.46 (0.77)</td>
</tr>
</tbody>
</table>

“RMSTE value (mN·m⁻¹) of the model (out of 100) having the smallest RMSTE for the 244 molecules of the training/validation set. Mean and standard deviation (in parentheses) of the VLOO scores (mN·m⁻¹) averaged over the 10 models (out of 100) having the smallest VLOO scores for the 244 molecules of the training/validation set.

maximum information on all the available data, the complete set of 269 molecules was used as a training/validation set. For graph machines, 250 models with node functions having seven hidden neurons were trained from that set. The 25 models that had the smallest VLOO scores were applied to the cosmetic oils set, and the mean of their estimates of the surface tension was taken as the final estimate. This resulted in a RMS estimation error of 0.9 mN·m⁻¹. For the MLP estimation from σ-moments, 250 models having 10 hidden neurons were trained from the same set. The 25 models that had the smallest VLOO scores were also applied to the cosmetic oils set, and the mean of their estimates of the surface tension was taken as the final estimate. The results are compared with those obtained with the other two methods, group contribution, and corresponding-states method. Deviations from experimentally measured values of surface tension are gathered in Table 3. Because of the absence of contribution for siloxane groups in Conte’s group contribution method, the surface tension for the dodecamethylcyclohexasiloxane (D6) cannot be estimated.

A large deviation is observed between the experimental data and the values predicted by the GC method. We notice that the molecules considered in Table 3 have a larger molecular weight than compounds included in the database used for the predictions. Conte’s original paper did not give a list of data set molecules but provided several examples. Not a single one concerned such large molecules. This hints at the fact that the
The database set used to develop the GC model does not sample the space of molecules to which the test set we are interested in belongs.

Thus, in the following, surface tensions estimated with the corresponding-states methods described by Zuo and Stenby are compared to results obtained with the neural networks and graph machines. Figure 6 presents the comparison between the experimentally measured surface tensions of the 23 cosmetic oils of the test set and the predicted surface tensions with corresponding-states based method, neural networks using the COSMO-RS $\sigma$-moments as descriptors, and graph machines.

Table 3. Difference between Measured and Predicted Surface Tension (ST) for 23 Cosmetic Oils

<table>
<thead>
<tr>
<th>Entry</th>
<th>Molecule</th>
<th>Chemical structure</th>
<th>Meas. ST</th>
<th>Deviation of predicted ST</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dodecamethylcyclododecane (DM)</td>
<td>![Image]</td>
<td>18.8</td>
<td>-3.6</td>
</tr>
<tr>
<td>2</td>
<td>2,2,4,6,6-Pentamethylheptane</td>
<td>![Image]</td>
<td>21.6</td>
<td>+4.0</td>
</tr>
<tr>
<td>3</td>
<td>2,2,4,6,6,8-Octamethylnonane</td>
<td>![Image]</td>
<td>24.2</td>
<td>+6.3</td>
</tr>
<tr>
<td>4</td>
<td>Dodecane</td>
<td>![Image]</td>
<td>24.8</td>
<td>+2.5</td>
</tr>
<tr>
<td>5</td>
<td>Hemisqualane</td>
<td>![Image]</td>
<td>24.9</td>
<td>+3.9</td>
</tr>
<tr>
<td>6</td>
<td>Isodecyl neopentanoate</td>
<td>![Image]</td>
<td>25.4</td>
<td>-11.5</td>
</tr>
<tr>
<td>7</td>
<td>Tetradecane</td>
<td>![Image]</td>
<td>26.1</td>
<td>-2.6</td>
</tr>
<tr>
<td>8</td>
<td>Isocaprylic acid</td>
<td>![Image]</td>
<td>26.2</td>
<td>-3.5</td>
</tr>
<tr>
<td>9</td>
<td>Diocyl ether</td>
<td>![Image]</td>
<td>27.1</td>
<td>+0.6</td>
</tr>
<tr>
<td>10</td>
<td>Propylhexyloxy caprylate</td>
<td>![Image]</td>
<td>27.4</td>
<td>-3.5</td>
</tr>
<tr>
<td>11</td>
<td>Isopropyl isostearate</td>
<td>![Image]</td>
<td>27.6</td>
<td>-3.9</td>
</tr>
<tr>
<td>12</td>
<td>Propanoic acid</td>
<td>![Image]</td>
<td>27.8</td>
<td>-10.6</td>
</tr>
<tr>
<td>13</td>
<td>Neopentyl glycidyl diethylhexanoate</td>
<td>![Image]</td>
<td>28.0</td>
<td>-7.8</td>
</tr>
<tr>
<td>14</td>
<td>Coco caprylate</td>
<td>![Image]</td>
<td>28.3</td>
<td>-4.8</td>
</tr>
<tr>
<td>15</td>
<td>Isomyl laurate</td>
<td>![Image]</td>
<td>28.3</td>
<td>-1.8</td>
</tr>
<tr>
<td>16</td>
<td>Isopropyl myristate</td>
<td>![Image]</td>
<td>28.3</td>
<td>-1.8</td>
</tr>
<tr>
<td>17</td>
<td>Disopropyl adipate</td>
<td>![Image]</td>
<td>28.7</td>
<td>-3.1</td>
</tr>
<tr>
<td>18</td>
<td>2-ethylhexyl cocooate</td>
<td>![Image]</td>
<td>28.7</td>
<td>-3.4</td>
</tr>
<tr>
<td>19</td>
<td>Dicaprylyl carbonate</td>
<td>![Image]</td>
<td>28.8</td>
<td>-5.6</td>
</tr>
<tr>
<td>20</td>
<td>Neopentyl glycidyl didecanoate</td>
<td>![Image]</td>
<td>28.8</td>
<td>-7.4</td>
</tr>
<tr>
<td>21</td>
<td>Triethyhexanoate</td>
<td>![Image]</td>
<td>28.9</td>
<td>-16.8</td>
</tr>
<tr>
<td>22</td>
<td>Diisooctyl adipate</td>
<td>![Image]</td>
<td>29.4</td>
<td>-7.4</td>
</tr>
<tr>
<td>23</td>
<td>Dioctyl adipate</td>
<td>![Image]</td>
<td>29.5</td>
<td>-8.8</td>
</tr>
</tbody>
</table>

The root-mean-square error is computed with eq 13 for the 23 molecules of the test set; the estimations for the surface tension are averaged over the 25 models (out of 250) that have the smallest VLOO (validation leave-one-out) scores for the multilayer Perceptron and the graph machines (71 and 95 parameters, respectively).

Thus, in the following, surface tensions estimated with the corresponding-states methods described by Zuo and Stenby are compared to results obtained with the neural networks and graph machines. Figure 6 presents the comparison between the experimentally measured surface tensions of the 23 cosmetic oils of the test set and the predicted surface tensions with corresponding-states based method, neural networks using the COSMO-RS $\sigma$-moments as descriptors, and graph machines.

The method of corresponding states advocated by Zuo and Stenby performs poorly as was expected. For all but for two compounds, the surface tensions are grossly underestimated. The estimations by neural networks from $\sigma$-moments are not very satisfactory, which suggests that the $\sigma$-moments do not contain sufficiently relevant information for our task. In particular, graph machines provide the most accurate estimation as they take into account the structure of the molecule, thereby

"Mean measured surface tension values (ST) are reported in mN m$^{-1}$ at 23.5 °C. Differences between measured and predicted ST using group contribution method implemented in the IBSS software. Differences between measured and predicted ST using corresponding state-based method included in IBSS software. Differences between measured and predicted ST using neural network model based on the five $\sigma$-moments. Differences between measured and predicted ST using graph machines. The root-mean-square error is computed with eq 13 for the 23 molecules of the test set; the estimations for the surface tension are averaged over the 25 models (out of 250) that have the smallest VLOO scores for the multilayer Perceptron and the graph machines (71 and 95 parameters, respectively). gn/a: not applicable."
solely from the molecular structure, provided the most accurate (red circles), neural networks based on

Predicting surface tensions of liquids from selection of features. exempting the model designer from the computation and selection of features.

Figure 6. Predicted surface tension values computed by graph machine (red circles), neural networks based on five \(\sigma\)-moments (blue triangles), and corresponding-states (green crosses) vs experimentally measured surface tension values for the test set of 23 cosmetic oils.

4. CONCLUSIONS

Predicting surface tensions of liquids from first-principles is not straightforward due to the complexity of the interactions taking place at gas/liquid interfaces. It remains still inaccessible even by advanced methods such as COSMO-RS able to calculate accurately interfacial tensions between two liquid phases.\(^{26}\) In the present study, the estimation of surface tensions of cosmetic oils was performed by the group contribution method, by the corresponding-states method, by nonlinear regression (multi-layer Perceptron) from COSMO-RS \(\sigma\)-moments, and by regression on graphs (graph machines). To the best of our knowledge, this is the first study that compares the accuracy of various methods for estimating the surface tensions of the same molecules and where the models are tested on two sets of molecules; it is also the first report of the use of graph machines for surface tension estimation. The database of surface tensions was created with special emphasis on the diversity of the molecules and on the reliability of the experimental results. It was shown that the group contribution method failed to provide accurate estimations (root-mean-square estimation error = 6.7 mN·m\(^{-1}\)); the corresponding-states method and nonlinear regression from \(\sigma\)-moments yielded estimations with similar accuracies (root-mean-square estimation errors = 2.4 and 2.7 mN·m\(^{-1}\) respectively); graph machines, which make estimations solely from the molecular structure, provided the most accurate estimations (0.9 mN·m\(^{-1}\)). Pending the development of methods, based on first-principles, for surface tension estimation only from molecular structures, graph machines are shown to be able to provide, from molecular structures, estimations whose errors are on the order of the experimental uncertainty on the available data. An interactive demonstration of the graph machine computations, based on the Docker free software technology (existing for many computer platforms) is available for download (see the Supporting Information). In the present study, all measurements were taken at the same temperature. As surface tension varies with temperature, we intend to use the ability of graph machines to take into account exogenous variables (e.g., temperature) for prediction.

ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.jcim.7b00512.

Names, SMILES notations, molecular formulas, CAS RN, \(\sigma\)-moment descriptors, and measured surface tensions values in mN·m\(^{-1}\) at 25 °C of the 244 molecules of the training/validation set, the 25 molecules of the test set, and the 23 molecules of the cosmetic oil set; group-contribution Conte method estimated, corresponding-states Pitzer’s method estimated, corresponding-states Zuo-Stenby method estimated, neural network estimated, neural network VLOO estimated, graph machine estimated and graph machine VLOO estimated values of the surface tension in mN·m\(^{-1}\) for the 269 molecules of the complete set; group-contribution estimated, neural network estimated, and graph machine estimated values of the surface tension in mN·m\(^{-1}\) for the 23 molecules of the cosmetic oil set; Docker Client installation, graph machine demonstration with Docker containers and graph machine results with Docker. (PDF)

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REFERENCES


