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Sampling, Corporate Governance and Risk Analysis

Geoffrey J Lyman and Florent S Bourgeois

1. Materials Sampling & Consulting, Principal, geoff.lyman@gmail.com
2. Université de Toulouse, Laboratoire de Génie Chimique, Toulouse, France
florent.bourgeois@inp-toulouse.fr

ABSTRACT

As the prices of metals slide, financial control of mineral processing operations becomes more and more important. Control of a mineral processing operation implies measurement of grades, recoveries and losses, with every effort being made to recover more product from the same feed. Shareholders become more concerned with the transparency of the operations and the corporate governance.

There is only one way to achieve this and this is by sampling and making good use of the results of that sampling. Sampling systems that provide only ‘rough’ figures are not good enough. Similarly, a competitive position in the marketplace demands close control of product quality. Maximisation of product output demands that the product just meet the specification so that product yield is maximised. Decisions based on accurate and precise sampling results carry less risk that those based on imprecise and biased results.

This paper reports some recent advances in sampling theory that permit full quantification of sampling distributions, not just the sampling variance as has been possible in the past. These advances can be used together with simulation methods to tie down sampling precisions and develop calculations that quantify decision making risk when using the sampling results. A number of examples illustrate the use of the new advances and a final example indicates the development of risk analysis curves in the coupling of sampling precision to sale of a commodity.

INTRODUCTION

The control of mineral processing operations is becoming an increasingly complex task as the grades of ores decrease, the scale of operation increases and financial control of the operation becomes more critical. The role of sampling is implicitly gaining in importance as it is the measurement technology that empowers the control of operations. It is no longer adequate to have sampling technology that carries financially significant uncertainty in metal content; the sampling must have an accuracy and precision that matches the demands of shareholders for high quality information concerning production in terms of attainment of objectives and the value of the resource. These are elements of corporate governance and the data feeds directly into the assessment of risk of on-going operations. Further, control of product quality is critical in making the product desirable in the marketplace and ensuring that the product attains maximum value within the contract specification with the product consumers. Indeed, the ability to specify the quality of a consignment of a product within tight tolerances, with documentation supporting the product quality ensures that the seller will realise the value of his product consistently in the face of challenges by the buyer. Close control of a product quality also permits the supply of a product that just meets the agreed specification, maximising the value of the resource.

This paper examines recent developments in sampling theory and shows how these developments can be employed to provide full risk analysis associated with either the purchase or sale of commodities, and in particular, precious metal ores.

SAMPLING THEORY AND COMPUTATIONAL DEVELOPMENTS

Beginning in the 1950s, Pierre Gy (Gy 1953,1967,1982) began the development of a comprehensive theory of sampling for particulate materials. There had been some notable contributions made to the
science and practice of sampling as far back as 1895 (Brunton, 1895,1909) with the development of useful sampling equipment that was capable of taking unbiased samples. However, the adoption of good sampling practice was slow in penetration of the mineral industry, and slower in the sampling of commodities such as grain. Even with the publication in 1945 of Taggart's Handbook of Mineral Dressing (Taggart 1945), there is support for devices and methods of sampling that are 'mechanically incorrect' meaning that there is a substantial risk, indeed an expectation, that the devices will collect biased samples. The theory of sampling espoused in this Handbook is also complex, limited in application and difficult to apply. By 1972, when Gy presented his Doctoral Thesis, he had worked out the basis for a comprehensive theory and practice of sampling. Gy's first book in English appeared in 1979, but is perhaps best known for the second edition of that book that appeared in 1982.

The theory has two parts. The first part deals with the heterogeneity of a particulate material such as a broken ore and provides a basis for the calculation of the variance of sampling that applies to a sample of a given mass with the material in a given state of comminution. The second part deals with the method of calculating the variance of sampling from a process stream in which the grade of the target analyte is fluctuating in time. A number of authors had dealt in various manners with the sampling variance due to the particulate nature of a mineral mixture, but no one had been able to deal with the problem of determining the variance of sampling due to fluctuations in grade when sampling by taking increments of solids periodically from the process flow.

Gy solved this problem by adopting the nascent theory of Geostatistics as it was being put forward by Georges Matheron at École des Mines de Paris at Fontainebleau. In fact, the mathematics of time-series analysis was already available to enable the variance calculation, but this area of mathematics was essentially unknown to mineral processors and miners when Matheron began to construct the discipline of Geostatistics. Adding this capability to sampling theory was Gy's stroke of genius. Gy introduced the concept that the time variation of grade in a process stream could be modelled by a stationary random function in the same way that the geostatisticians model the variation of grade within a domain of a mineral deposit. A stationary random function is one whose statistical properties do not change with time. Figure 1 shows such a random function after removal of long term movement of the mean.

![Figure 1. Example of a random function. Variation in grade of a process stream after trend removal (real data).](image)

The nature of the random function is characterised by a covariance function or a variogram, as is used by the geostatisticians. There is nothing new in modelling process variations as random functions (stochastic processes); the methodology has been in use in chemical engineering and general control engineering since the 1960s. But it was certainly new to the theory of sampling when introduced by Gy.

The variograms corresponding to the first 3000 and the last 3000 points in the data set shown are given in Figure 2. The two variograms are essentially identical and show a diurnal component with a
period of 24 hours as well as a mixing component (an exponential covariance). The last component is a measurement variance which is uncorrelated between one reading and the next. This latter component shows up as a non-zero intercept when the variogram is extrapolated to zero time lag. A close-up of the start of the variogram to the right in Figure 2 shows the uncorrelated component (called the nugget value) which is an estimate of the variance due to the intrinsic heterogeneity of the sample increment and the subsequent sample preparation and analysis variances.

![Figure 2. Left: Variograms corresponding to the first (red) and last (green) 3000 points in the data set of Figure 1. An approximate 95% confidence interval on the red variogram is indicated by the feint blue lines. The black line is a fitted variogram (covariance) model made up from an uncorrelated component, an exponential correlation and a damped periodic correlation. Right: a close-up of the region of the variogram near the origin, showing the estimated nugget component of 0.126.](image)

To make a complete estimate of the variance of sampling due both to the variation of grade in the process stream, the particulate nature of the material and sample preparation and analysis, one must recognise that the variances due to grade variation are independent of the other uncertainties and can therefore be summed to provide the total sampling variance. The theory developed by Gy completely describes the total sampling variance when all stages of sampling are mechanically correct.

In a statistical problem, the analysis is not complete until it is possible to determine the entire probability distribution for the quantity under scrutiny. The variance of a statistical quantity measures the spread of the distribution, but gives no information beyond that as to the full nature of the distribution.

The authors have applied a mathematical method to the particulate sampling problem that permits the calculation of the full distribution of sampling (Lyman 2014). This development provides the full distribution of one of the three components of uncertainty of sampling. Lyman et al. (2016) present an example with estimation of the distributions of all three components of sampling uncertainties for a gold ore.

Next, due to computational developments in the mid 1990s (Dietrich and Newsam, 1997), it has become possible to generate very large and accurate realisations of Gaussian random functions in any number of dimensions. Twenty years later, these computational methods, known as circulant embedding, are just beginning to be incorporated into the geostatistical packages offered by software specialists. Materials Sampling & Consulting has its own software implementing these methods as well as powerful covariance function estimation tools.

Having captured the nature of the time variation of grade in the process stream using a variogram and the distribution of the grade about the mean, which may or may not be normal (Gaussian), it is necessary to determine the distribution of the uncertainty which derives from the sampling of the process stream according to some protocol (systematic or stratified random sampling). When a process stream is sampled by periodically extracting increments from the process stream, there is a difference between the true average grade of the process stream over the sampling period and the
grade of the sample formed from the collection of the increments. It is very difficult to determine the
distribution of this uncertainty theoretically, but the time variation of the grade in the process flow can
be simulated on a very fine time scale using circulant embedding methods and the sampling likewise
simulated by extracting values from this process grade simulation. By generating many possible
realisations of the grade variation and simulating the sampling, for each realisation, the distribution of
the uncertainty due to sampling can be captured.

Figure 3 shows a simulation of the grade of the feed to a plant treating a mixture of low (75% at 2 g/t)
and high grade (25% at 10 g/t) gold ore in a circumstance where the two ore types cannot be
effectively blended prior to feeding. For a 200 mm top size of the ore and a feed rate of 1000 t/h, the
primary increment mass is 277 kg. The simulation was carried out on a 277 kg basis (1 second of
feed) for an 8 hour period, with 32 samples taken. The simulation required realisations of a random
function modelling the grade containing 28,800 points. An accurate simulation of this magnitude is
not possible within a reasonable time-frame without the circulant embedding method.

The sampling simulation was carried out 5000 times to obtain a good estimate of the distribution of
the sampled (punctual) grade and the true grade. Surprisingly, while the distribution is symmetric,
indicating unbiased sampling as it should, the distribution of not Gaussian. Instead, it is a Laplace
distribution (double-sided exponential), which has heavier tails than the normal distribution.

The last component of uncertainty is the analytical uncertainty which is expected to be normal. This
uncertainty arises from the random errors that come into play when diluting solutions, titrating or
weighing. With instrumental methods of analysis, the errors are usually due to solution aspiration,
instrument electronics, calibration, and matrix effects.

The three components of uncertainty in a sampling result are statistically independent and it is
therefore possible to correctly combine their distributions using the method of characteristic functions.
This fact has not been appreciated until recently proposed by one of the authors (Lyman 2015). This
methodology makes it possible to accurately determine the total sampling distribution due to all three
components of uncertainty.
Good corporate governance demands that reports to shareholders and stock market regulators are accurate. Just how accurate the reporting of resources and their reconciliation with actual production must be is a matter for regulatory bodies or company directors. The determination of production is ultimately tied down when the product is sold and relies on precise, unbiased sampling of the product. Prior to product sale, the mass of metal produced is tied to sampling of the products of the beneficiation processes employed and material balances around the plant. Determination of losses of metal values is also based on sampling and material balances around processes. Reconciliation of mine output with plant feed may be based solely on sampling of plant feed or may involve the material balances on the plant as a whole. This reconciliation is critical to the assessment of the resource estimation and grade control procedures. The accuracy with which these balances can be carried out is based solely on the accuracy of sampling and the distribution of sampling results, both within the resource itself and around the beneficiation processes.

Risk analysis in metal mining operations depends not only on the results of sampling but also on the distribution of uncertainty attached to the results. To state that the estimated recovery of metal is going to be 87% over the next month is not meaningful until a confidence interval has been placed on the estimated recovery. If the figure is known to ±10%, such uncertainty may be completely unacceptable, while an interval of ±1% may enable close planning of product shipments to consumers. The specification of a confidence interval is based on knowledge of the distribution of the figure involved. To simply assume that the distribution is normal introduces additional levels of risk in using the result. First, the distribution may be symmetric, but it may not be normal. The distribution of Figure 4 is a perfect example of being misled by an unjustified assumption of normality. The significance of the departure from normality is case variable. As an example, the standard deviation of the difference in grade, which is 0.507 g/t, would lead to a normal 95% confidence interval of 1.96×0.507 g/t = ±0.994 g/t. However, the actual 95% confidence interval shown in Figure 4 is ±1.175 g/t, which is nearly 20% wider. Second, the distribution may be skewed, making the confidence intervals asymmetric; simply given a variance value, there is no way of knowing whether there is asymmetry present.

The sampling of ores containing coarse gold can lead to unexpected problems if the size distribution of the gold is not well understood. Consider a case in which the ore is being treated by a third party in their mill and the mill feed must be sampled to establish the value of the ore. The mill is equipped with a sampling system through which all toll ore passes. The equipment in the system reduces the ore to 95% passing 6 mm and delivers a 20 kg sample to the laboratory for each 8 hour period of operation. The sample is crushed to pass 2 mm and divided down to 2 kg prior to further
pulverisation prior to fire assay. The sample grade is nominally 2 g/t. Figure 5 shows the assumed size distribution of the gold particles, assuming also that they are reasonably compact in shape.

While the sample protocol may seem to be reasonable, it is possible to determine the sampling distribution for 2 kg samples of this ore, using the full sampling distribution method (Lyman, 2014). The distribution is shown in Figure 6. It is wide, skewed and has a relative standard deviation of 19.7%. In fact, even the 20 kg sample from the sampling system will have a standard deviation of 6.23%. The skewness of the distribution will tend to produce sampling results slightly below the true average value (the most probable value, or mode is lower than the mean). However, in the long term, the sampling is unbiased. For commercial purposes, one would hope to achieve a more precise result. For a commodity other than gold ore, an acceptable standard deviation is of order 2% (4% at 95% confidence).

An appropriate procedure in this case is to retain a larger sample mass to represent the period of sampling, grind it fine while employing an accelerated leaching process to recover the gold and then recover the leached and ground residue for subsampling and fire assay. Sampling for gold is a case in which comminution of the ore does not reduce the sampling uncertainty until breakage of the gold particles actually takes place.

The successful marketing and sale of a commodity is heavily dependent on good sampling of the consignments delivered to the client. When a product is to be sold at a specified content of valuable (or diluent in the case of coal), the maximum value of the product is realised by ensuring that the consignment just succeeds in meeting the specification both in terms of grade of the primary element
and the allowable levels of deleterious elements, unless there is a premium paid for higher grade material that operates from the specified grade upwards. To this end, the seller should have in place sampling and blending systems that permit very close control of product quality.

Consider a producer of an ore with a specification of >44% contained metal which commands a price of 3.0 USD/dmtu FOB, but no premium for supplying higher grade. A tonne of ore at 44% is then worth 132 USD. However, to ensure that they do not incur a penalty for providing a product with less than 44% metal, they ship at 45% metal. Had they shipped at 44%, by blending lower grade material into the 45% product, they would have shipped 2.27% more ore. On a 50,000 tonne shipment, this is worth about an extra 0.15 million out of 6.6 million for the 50,000 tonnes at 45% metal. For an annual production of 3.5 million tonnes, the benefit is about 10.5 million USD. Such a dollar advantage can justify the investment in a quite excellent sampling and analysis system.

The connection between sampling precision and ability to ‘sail closer to the wind’ is conveniently expressed by what is usually called an operating characteristic (OC) curve. However, it can simply be interpreted as a ‘risk’ curve. A true OC curve for a shipment of product would be based solely on the accuracy of sampling as a ship is loaded. Once the ore is on the belt on the way to the ship, it is usually too late to adjust the grade of the shipment unless the port is equipped with a very fast turn-around robotic lab which can return an assay for a sampling unit within minutes rather than hours.

Let’s say that a contract negotiation begins with a discussion of how the value of the product is to be decided. The seller explains that he has a very good sampling system and that its performance has been documented in detail to show that the sampling system is ‘mechanically correct’ and therefore unbiased. He has also documented the tests and calculations that have been carried out to prove the system and his analyses are precise; the seller has a ‘defensible’ sampling system. The buyer does not have such a good system and no solid documentation other than a statement that his system complies with ISO specifications. He cannot prove the accuracy of his system. It is logical (and hopefully realistic) to conclude that settlement will be made on samples taken by the seller’s sampling system, although final samples may be exchanged for assay by the buyer’s third party laboratory.

The seller knows from the analysis of his sampling system that it will operate with a standard deviation of say 0.1% metal for a 50,000 tonne consignment and, for the payable element in the ore, he knows that the chemical analysis has a standard deviation of 0.11% for a single analysis and he will analyse five 10,000 tonne sampling units in duplicate. The analytical uncertainty is then \(\sqrt{(0.11)^2 / 10} = 0.0348\%\) so the total sampling and analysis standard deviation is \(\sqrt{0.1^2 + 0.0348^2} = 0.106\%\). On the assumption that his sampling uncertainty follows a normal distribution (which in truth may not be a valid assumption), he can now draw a curve which will indicate his probability of delivering a consignment with a grade below 44%, given that he loads ore at 44 + x%. If the sampling system standard deviation can be reduced to 0.05% for the total consignment, by sampling more frequently or carrying out more analyses per shipment, the total standard deviation will be 0.0609%.

It is evident that if the sampling precision is improved and the risk of an off-spec consignment is held at 5%, the grade offset can be reduced from 0.175 to 0.1, permitting more product to be produced from the same resource.

Of course, it is necessary to analyse, in conjunction, the precision with which a stockpile can be built to a specified grade. That task requires similar considerations to those described here. Taken together with the sampling system for loading the product, these risk curves can be used to estimate how much expenditure on sampling can be justified to provide a payback time of a few years. It is only with this kind of analysis that the costs of excellent sampling systems can be put into perspective.
CONCLUSIONS

Recent advances in sampling theory together with powerful simulation methods can be used to provide a full picture of the distributions of sampling uncertainty. Previously, the Gy theory of sampling provided only an estimate of sampling variance. The theory could not be used to determine whether sampling distributions were Gaussian (normal), or skewed. The advances also permit the correct combining of distributions of uncertainty due to the sampling from the process stream, the particulate nature of the material and the chemical or physical analysis. In this way, a total picture of sampling uncertainties can be built up.

The picture can be used to optimise the sampling at any stage of a process and particularly at the sampling of the final product. Adherence to an ISO Standard does not guarantee the required precision.

The ability to carry out risk analysis at any stage of sampling is an important new tool enabled by the recent advances and the application of good quality control principles and statistics.

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