Launcher attitude control: some additional design and optimization tools

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Abstract

This paper deals with the launcher attitude control during atmospheric flight. A two step approach combining an $H_{\infty}$ control design and an optimization procedure is proposed. The first step is multi-objective stationary $H_{\infty}$ design based on the Cross Standard Form. It provides easily a first rough solution from a few physical tuning parameters. The second step is a fine tuning using an multi-constraint satisfaction algorithm. This algorithm enables the certification criteria computed on the validation model to be met and is also used to propagate the nominal tuning to the full flight envelope.

Keywords : multi-objective synthesis, performance, robustness, Cross Standard Form, launcher

1 Introduction

In this paper, the low-level control loop of a non-stationary launcher during atmospheric flight is considered. Only the yaw attitude is explored: the problem is formulated in terms of angle of attack regulation in face of a typical wind profile (disturbance rejection problem, see Figure 4) and consumption reduction. Robustness specifications are expressed in the frequency domain for a set of operating instants regularly spaced along the flight path: the open loop transfer ($L(z) = K(z)G(z)$) must satisfy templates on the Nichols chart for various critical configurations sampled in the uncertain parameter space (see Figure 5). Uncertain parameters are the main dynamic parameters on the rigid mode (aerodynamic coefficient, thruster efficiency,...) and on the bending modes (natural frequencies, modal participation factors).

With respect to the pure stationary synthesis problem at one flight instant, there is no methods, to our knowledge, that can handle such a set of specifications (time-domain performance, open-loop frequency-domain specifications and parametric robustness specifications) in a streamlined manner. Then, a two step approach combining a control design, which can provide easily a first rough solution from a few physical tuning parameters, and

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a fine optimization initialized from this solution seems a good alternative between a tedious trials and errors procedure or a blind optimization from an arbitrary initialization.

Although the control design we proposed is an indirect approach, its capability to take advantage of know-how is particularly highlighted in this application: the time-domain performance specification (angle-of-attack peak amplitude in response to typical wind profiles) is handled by a non-conventional LQG (Linear Quadratic Gaussian) synthesis based on physical considerations. Then, this synthesis is incorporated into a standard $H_\infty$ problem in order to meet frequency-domain templates. The final $H_\infty$ synthesis meets all the specifications and produces a low-order compensator in regard to alternative approaches applied on the same problem.

This design is then refined by an optimization procedure. From a practical point of view, the expression of a scalar cost function combining the various objectives, that is heterogeneous terms for instance, the incidence response peak (degree) and the gain margin (dB). Furthermore, when these various objectives correspond to physical specifications, it is more interesting to appreciate the sharpness of the trade-offs between these specifications rather than to perform a pure optimization. For these reasons, it seems more tractable to solve a multi-constraint satisfaction problem than an optimization problem. For the non-stationary launcher application, such a procedure enables to propagate the tuning at one flight instant to the full flight envelope.

In the first part of this paper, the launcher model and the specifications are described. In the second part, the stationary $H_\infty$ design is presented and applied at the flight instant with maximal aerodynamic pressure. The third part is devoted to the multi-constraint satisfaction algorithm. In the last part of the paper, this algorithm is used to propagate the nominal tuning to the full flight envelope and the gain scheduling of the various compensators is presented.

2 Launcher control problem

2.1 Description

This application considers the launcher inner control loop. Referring to Figure 1, the following notation is used:

- $G$: the center of gravity,
- $i$: the launcher angle of attack,
- $\psi$: the deviation angle around axis w.r.t. the guidance attitude reference,
- $V_a$ and $V_r$: respectively, the absolute and the relative velocity,
- $w$: the wind velocity,
- $\beta$: the thruster angle of deflection,
- $\dot{z}$: the lateral drift rate.

The rigid behavior is modeled by a third-order system with state vector: $x^r = [\psi \ \dot{\psi} \ \dot{z}]^T$. This rigid model strongly depends on the 2 uncertain dynamic parameters $A_6$ (aerodynamic efficiency) and $K_1$ (thruster efficiency).
The discrete-time full order validation model $G_f(z)$ considered in this paper (that includes the rigid dynamics, the dynamics of thrusters (order 2), of sensors (order 2) and the first 5 bending modes (order 10). The launcher is aerodynamically unstable. Finally, the characteristics of bending modes are uncertain (4 uncertain parameters per mode).

2.2 Objectives

The available measurements are the attitude angle $\psi$ and rate $\dot{\psi}$. The control signal is the thruster deflection angle $\beta$. The control objectives for the whole atmospheric flight are as follows:

- performance with respect to disturbances (wind): the angle of attack peak, in response to the typical wind profile $w(t)$, must stay within a narrow band ($\pm i_{\text{max}}$).
  
  This wind profile is plotted in Figure 4 (dashed plot) and corresponds to a worst case wind encountered during launches with a strong gust when aerodynamic pressure is maximal (at time $T_1$),

- closed-loop stability with prescribed margins for both rigid and flexible dynamics.
  
  These specifications can be interpreted as a template on the Nichols locus of the open loop transfer $L = KG$ (see Figure 5 as an example): (i) the locus must cross the axis above the critical point with a Low Frequency Gain Margin $LFGM_{dB}$; (ii) it must cross the same axis under the critical point with a High Frequency Gain Margin $HFGM_{dB}$ (negative value); (iii) resonance associated with flexible modes 2 to 5 must be gain controlled and must stay below a specified level $X_{dB}$ (rolloff specification). Note that the first flexible mode is “naturally” phase controlled (resonance phase around 0 deg) due to the collocation between sensors and actuator and that the flexible modes are not taken into account in the synthesis model. But a roll-off behavior with a cut-off frequency between the first and the second flexible modes must be specified in the synthesis,

- delay margin must be greater than one sampling period ($T_s$).
All these objectives must be achieved for all configurations in the uncertain parameter
domain (22 uncertain parameters including aerodynamics coefficient, propulsion efficiency
and bending modes characteristics). particularly in a number of identified worst cases,
where the combination of parameter extremal values is particularly critical. In this paper,
the robustness analysis is limited to these worst cases as the experience has shown that
they are quite representative of the robustness problem. A complete $\mu$-analysis is presented
in [1].

3 Stationary $H_\infty$ design

The design is based on the CSF (Cross Standard Form) [2] presented as a generalization
of the LQ inverse problem to the $H_2$ and $H_\infty$ inverse problem. The CSF enables to
formulate a standard problem from which an initial compensator can be obtained by
$H_2$ or $H_\infty$ synthesis. The CSF is used to mix various synthesis techniques in order to
satisfy the different specifications of the launcher control problem. The general idea is to
perform a first synthesis achieving some specifications, mainly time-domain performance
specifications. This first solution is then used to initialize a standard problem which is
gradually completed to handle frequency-domain or parametric robustness specifications.

This approach is detailed in [2] and [3]. The standard problem set up for the final $H_\infty$
is depicted in Figure 2. This standard problem depends on:

- the 4 state space matrices ($A^a_d$, $B^a_2d$, $C^a_2$, $D_{22}$) of the discrete-time rigid model aug-
  mented with a first order wind model,
- the state feedback and estimator gains $K^a_d$ and $G^a_d$ of the LQG/LTR\(^1\) design proposed
to fulfill the specifications regarding the rigid dynamics,
- the frequency weighting $F(z)$ introduced to attempt to fulfill the frequency-domain
  specifications on flexible modes 2 to 5 ($X_{dB}$ constraint). $F(z)$ is a high pass second
  order filter with a wide resonance including flexible modes 2 and 3 and their varia-
  tions (see Figure 3). Note that flexible modes 4 and 5 are not significant regarding
  the $X_{dB}$ specification.

The 8 tuning parameters (gathered in a vector $p$) for the whole design are displayed
in Table 1\(^2\).

Of course, at each step of this design (the first step is the LQ control law, the second
step is the introduction of the Kalman filter and the last one is the introduction of the
filter $F(z)$ for the final $H_\infty$ synthesis), the specifications satisfied at the previous step
are perturbed by the new ones taken into account in the following step. This problem is
particularly relevant as this stationary design is built up at the flight instant $T_1$ where the
aerodynamic pressure is maximal and where the performance/robustness trade off is the
more stringent. Figures 4 and 5 show results obtained with a rough tuning (see Table 1)
which was established without any trial and error tuning. We note $K_1(z)$ this LTI (Linear
Time Invariant) controller. One can notice that the constraint $i_{max}$ on the angle-of-attack
response is violated. It also can be shown that the time delay margin ($T_s$) is also violated
while others specifications ($LFGM_{dB}$, $HFGM_{dB}$ and $X_{dB}$) are met.

\(^{1}\)LTR: Loop Transfer Recovery.

\(^{2}\)the LQ weighting on the angle of attack $i$ is normalized to 1, that is $J_{LQ} = \int_0^{+\infty}(i^2 + p(1)\ddot{z} + p(2)\dot{\beta}^2)\,dt$. 4
Figure 2: Set-up for final $H_\infty$ synthesis.

Figure 3: Singular values: $F(z)$ (black) and $G_f(z)$ (grey).

<table>
<thead>
<tr>
<th>parameter</th>
<th>signification</th>
<th>initial rough tuning (at flight instant $T_1$)</th>
<th>final tuning at flight instant $T_1$</th>
<th>final tuning at flight instant $T_2$</th>
</tr>
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<tr>
<td>p(1)</td>
<td>LQ weighting on $\dot{z}$</td>
<td>$10^{-4}$</td>
<td>$1.29 \times 10^{-4}$</td>
<td>$1.16 \times 10^{-4}$</td>
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<tr>
<td>p(2)</td>
<td>LQ weighting on $u = \beta$</td>
<td>$1$</td>
<td>$0.693$</td>
<td>$0.543$</td>
</tr>
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<td>p(3)</td>
<td>wind model dynamics</td>
<td>$-0.1$</td>
<td>$-0.0795$</td>
<td>$-0.215$</td>
</tr>
<tr>
<td>p(4)</td>
<td>LTR weighting</td>
<td>$10^{-5}$</td>
<td>$6.58 \times 10^{-6}$</td>
<td>$8.96 \times 10^{-6}$</td>
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<tr>
<td>p(5)</td>
<td>general state to measurement noise covariance ratio</td>
<td>$10^{-7}$</td>
<td>$1.06 \times 10^{-7}$</td>
<td>$1.86 \times 10^{-7}$</td>
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<tr>
<td>p(6)</td>
<td>rate to position measurement noise covariance ratio</td>
<td>$10$</td>
<td>$12.0$</td>
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<td>p(7)</td>
<td>static gain of $F(z)$</td>
<td>$30$</td>
<td>$27.6$</td>
<td>$20.0$</td>
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<tr>
<td>p(8)</td>
<td>central resonance frequency of $F(Z)$</td>
<td>$30$</td>
<td>$29.4$</td>
<td>$28.7$</td>
</tr>
</tbody>
</table>

Table 1: Vector $p$ of tuning parameters: signification and numerical values (normalized units).
4 Fine tuning

Instead of a long trial and error procedure to fulfill exactly all the requirements, a multi-constraint satisfaction procedure is proposed. The specific features of such a procedure w.r.t. a pure criterion optimization are the followings:

- one can take into account the end-to-end requirements computed on full order validation model, that is the certification specifications,

- of course, these specifications can not be optimized with this procedure. One can only check if a set of constraints can be achieved. But this enables the trade-off between specifications to be evaluated. Note also that the definition of a scalar index function including the various specifications is always a tedious task,

- such a procedure is quite interesting when a good initialization can be provided as it is the case in our problem: the rough solution previously presented is not so far from the final objective,

- this procedure is also applied to propagate the nominal tuning found at flight instant $T_1$ to the others operating flight instants where new specification values are prescribed.

Five performance indexes are considered in this problem and gathered in the vector $c$:

- $c(1)$: absolute value of the angle of attack time-response peak,
- $c(2)$: low frequency gain margin,
- $c(3)$: high frequency gain margin,
- $c(4)$: peak of the frequency response of the open loop transfer $L(z) = K(z)G(z)$ computed around flexible mode 2, 3, 4 and 5,
• $c(5)$: delay margin.

This vector $c$ is a function of the tuning parameters $c = C(p)$ and must satisfy the following constraints:

$$
c(1) < i_{\text{max}}, \quad c(2) > LFGM_{dB}, \quad c(3) < HFGM_{dB} < 0, \quad c(4) < X_{dB} < 0, \quad c(5) > T_s .
$$

(1)

Considering the current vector of tuning parameters $p$, one can derive numerical value of the sensitivity matrix $J(p)$ around $p$ (that is the local derivative matrix of the vectorial function $C(p)$): this matrix gives the relative variations of the performance indexes w.r.t. the relative variations of the tuning parameter $3$:

$$\delta_c = J(p)\delta_p .$$

The procedure aims at satisfying all the constraints (1) by a gradient type exploration. The 2 scalar tuning parameters of this procedure are $N_{\text{max}}$, the maximal iteration number, and $\varepsilon$, the step length in the gradient direction:

**initialization:** $p = p_0$, $N_{\text{max}} = 100$, $\varepsilon = 0.03$, $i = 1$, $c = C(p)$, $J = J(p)$,

**while** (1) is not met and $i < N_{\text{max}}$:

- $\delta_c^d(1) = (i_{\text{max}} - c(1))/i_{\text{max}}, \quad \delta_c^d(2) = (LFGM_{dB} - c(2))/LFGM_{dB}, \quad \delta_c^d(i) = (LFGM_{dB} - c(3))/LFGM_{dB}, \quad \delta_c^d(4) = (X_{dB} - c(4))/X_{dB}, \quad \delta_c^d(5) = (T_s - c(5))/T_s,$
- $\delta_c^d = 1.02 \delta_c^d$ (this factor 1.02 is introduced to ensure that the procedure will overpass the constraint),
- $\delta_p = J^T(JJ^T)^{-1}\delta_c^d$,
- $p \leftarrow p(1 + \varepsilon \delta_p)$ ($\varepsilon$ is the step length along the gradient direction),
- **hard constraint verification** $4$,
- $i \leftarrow i + 1$, $c = C(p)$, $J = J(p)$,

**end while**

This procedure is applied to fulfill the specifications at time $T_1$ from the rough tuning. The relative evolution of the 5 performance index versus the iteration number is depicted in Figure 8: one can notice that the both violated constraints (that is the angle of attack peak and the delay margin) are satisfied in 47 iterations $5$. This diagram enables also the trade-off with the 3 others specifications to be evaluated. This controller is noted $K_2(z)$. Figures 6 and 7 highlight that the specification are met (to be compared with Figures 4 and 5).

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$3$This calculus is done considering 5% relative variation for each of the 8 parameters.

$4$The hard constraints on the vector $p$ aim to ensure that:
- the wind model dynamics $p(3)$ is stable,
- the LQ weighting $p(2)$ is strictly positive,
- the measurement noise covariance $p(5)$ is strictly positive.

$5$In Figure 8 all the relative performance must be positive to solve the multi-constraint problem.
5 Linear Time Variant (LTV) design

The previous procedure was applied forward from $T_1$ to $T_f$ (last instant of atmospheric flight) and backward from $T_1$ to $T_i$ (initial time of atmospheric flight) to fulfill all the specifications defined for a set of operating points regularly spaced between $T_i$ and $T_f$. At each operating point, the tuning parameter vector is initialized on the solution found at the previous computed point. Figure 9 presents, for instance, the evolution of the performance indexes at time $T_2$ from the tuning parameter vector previously found at time $T_1$ (see table 1). All the specifications are fulfilled within 26 iterations.

This procedure enables to find a LTI controller at each point avoiding long trial and error designs. The last problem is the gain-scheduling of these LTI controllers w.r.t. time.
It is important to notice that the final $H_\infty$ synthesis on the problem described by Figure 2 is performed using the LMI solver of Matlab. The advantage of this solver is that it provides the best $H_\infty$ performance index among all the available solvers. The drawback is that the state space representation of the resulting controller has no physical meaning and cannot be mastered due to internal changes of variable in this solver to optimize numerical calculus. Therefore the direct linear interpolation of those state matrices provide a LTV controller $K(z,t)$ with a chaotic behavior on intermediate point. This is highlighted in Figure 10 where the singular value of the LTV controller is plotted versus the frequency (between 0 and the half sampling frequency) and versus time, the gain scheduling variable (between $T_i$ and $T_f$).

To solve this problem, we propose to compute the observer-based realization of each LTI controller using the procedure described in [4]. In this new realization, the controller states become meaningful variables (that is: estimates of plant states) and the linear interpolation of new state space matrices provides a new LTV controller $K_{LQG}(z,t)$ with smooth transition between various controllers as it can be seen in Figure 11 (see also [3] for more details).

![Figure 10: $K(z,t)$: singular value w.r.t. time.](image1)

![Figure 11: $K_{LQG}(z,t)$: singular value w.r.t time.](image2)

6 Conclusions

The conjonction of an $H_\infty$ design based on the Cross Standard Form and a multi-constraint satisfaction problem solver provides efficient tools for the design of non-stationary launcher pilots. From a practical point of view, the main advantage of such an approach is that the trade-off between the various specifications can be handled. This property will be used in the next future to evaluate if new control architectures (involving new sensors) would make possible to push back the limits in the trade-off tuning.

References


