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Parametric and non-parametric models for lifespan modeling of insulation systems in electrical machines

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Abstract – This paper describes an original statistical approach for the lifespan modeling of electric machine insulation materials. The presented models aim to study the effect of three main stress factors (voltage, frequency and temperature) and their interactions on the insulation lifespan. The proposed methodology is applied to two different insulation materials tested in partial discharge regime. Accelerated ageing tests are organized according to experimental optimization methods in order to minimize the experimental cost while ensuring the best model accuracy. In addition to classical parametric models, the life-stress relationship is expressed through original non-parametric and hybrid models that have never been investigated in insulation aging studies before. These two models present the original contribution of this paper. For each material, models are computed from organized sets of experiments and applied on a randomly configured test set for validity checking. The different models are evaluated and compared in order to define their optimal use.

Index Terms—accelerated aging, design optimization, electric machines, lifetime estimation, insulation, model checking, modeling, partial discharges, regression analysis, stress

I. INTRODUCTION

Reliability has become an important issue in the electrical engineering field since the most critical industries, such as urban transports, aeronautics, or space, are moving towards the design of more electrical based systems that will replace heavy mechanical and pneumatic based ones. Such design offers significant benefits in terms of performance, impact on environment, and operating costs [1]. However, more electrical power has to be generated in these systems, requiring higher voltages and operating frequencies [2]. These new operating conditions increase the risk of Partial Discharges (PD) in the electrical machine insulation systems [3], [4]. Given that around 40% of electrical machine failures result from insulation winding failures [4], [5], the lifespan of insulation materials becomes crucial for reliability assessment. In addition to electrical stress, insulation materials are subject to thermal, mechanical and environmental stresses that act simultaneously [6], [7] and can interact. Several models have been derived to describe the effects of these different stresses on the insulation lifespan [6], [8], [9]. However, these models take into account a single stress factor or two factors (mainly the electrothermal stress) at a time and their validity is assessed only for particular materials and in restricted factor ranges. Moreover, they do not include the synergetic effects due to interactions between factors.

In this paper, an original, general and comprehensive statistical approach for the insulation lifespan modeling is introduced. The considered aging phenomenon is mainly due to PD occurring in electrical machine windings. The objective is to provide a reliable lifespan model with a minimum experimental cost for economic purposes. To comply with both the economical and the accuracy constraints, the number of experiments and their configuration were specified in previous work according to two experimental optimization methods: the Design of Experiments (DoE) [9]-[13] and the Response Surface (RS) [9], [11], [12], [14]. The proposed insulation lifespan models included three different stress factors: voltage, frequency and temperature, as they were identified as the predominant factors causing PD [3], [4]. Two different types of insulating materials were tested in two different stress domains, both in PD regime. The experiments have to be organized at some specific points corresponding to some normalized and standardized levels.

In this paper, the effects of the same three stress factors and their different interactions are studied with the classical method that consists in adding all interaction terms to the main factor terms, thus leading to classical full parametric models. Alternatively, these effects are examined with two original methods: piecewise constant (non-parametric) and piecewise linear (hybrid) models. These models result from the classification of the experiments in different ranges of the stress factors according to their individual and combined effects on the lifespan. Non-parametric and hybrid models, that originally relate the insulation lifespan to stress factors, have never been investigated in insulation aging studies before. In this case, experiments could be randomly configured, which differs from the modelling methods previously developed and provides more flexibility.

II. EXPERIMENTAL SETUP

A. Materials

Two test campaigns were carried out on two insulating materials widely used in electric machine wiring insulation: a 200°C (Insulation Material 1, IM1) and a 220°C (Insulation Material 2, IM2) thermal class insulating materials. IM1 is composed of two insulating layers of Poly-Ether-Imide (PEI) and Poly-Amide-Imide.
IM2 is composed of PAI only. Test samples are twisted pairs of 0.5mm diameter copper wires covered with IM1 or IM2. They were manufactured according to a standardized procedure [15], cf Fig. 1a, ensuring the quality of the process and the homogeneity of samples. A typical twisted pair is shown in Fig. 1b. Their upper parts are cut in the middle to prevent from short circuits.

Fig. 1a. Manufacturing process of twisted pairs from copper wires  
Fig. 1b. Twisted pair as a test sample

B. Stress Factors

This study focuses on extrinsic insulation aging caused by PD. This phenomenon is ever more prevalent in electrical machines of embedded systems due to the required increase of the supply voltage. The authors of [3], [4] point out that electrical and thermal stresses are the predominant factors causing PD in electrical machines. Therefore, three aging factors are considered in our study: the applied voltage (the amplitude $V$ of a square wave voltage), its frequency $F$ and the ambient temperature $T$.

C. Accelerated Aging Tests

In order to get achievable lifetime measurements, IM1 and IM2 are tested under higher-than-nominal stress factor levels ensuring PD regime. Temperature covers a wide range of operating conditions for an embedded electrical machine but does not exceed the thermal classes of the two materials. Table I lists the factor domains for each tested material. Note that IM2 is tested in more restrictive stress ranges than IM1. Test samples are disposed in a climatic chamber where the temperature is set to the desired value. A power electronic system generates a square voltage controlled in amplitude and frequency. The lifespan of a test sample is the failure time at which a short circuit occurs in the pair. The experimental setup is in Fig. 2.

Fig. 2. Climatic chamber and power electronics as a test bench for the two types of insulation materials

For IM1, 30 experiments were carried out. Among them, 18 were specified according to a classical design method described in section III, while the others have random values for $V$, $F$ and $T$ with 6 samples tested at each experiment. For IM2, 27 experiments were organized and 24 were randomly configured, 8 twisted pairs were tested in each configuration.

D. Lifespan Expressions

In the case of IM1, according to [9], the lifespan logarithm ($\log(L)$) follows an inverse power model depending on $\log(10V)$, $\log(F)$ and $\exp(-bT)$, with $b = 4.825\times10^{-3}$. Indeed, the inverse power law has been widely used and validated for electrical stress effect on insulation lifespan [6]-[8]. In the case of IM2, as proposed in [9], the form of a factor in the lifespan model has been inferred from tests where only this factor varies. Fig. 3 shows the results of three such tests performed on IM2. From these graphs, it can be deduced that the lifespan logarithm of IM2 is linear with respect to $\log(V)$, to $\log(F)$ (as in inverse power law) and to $1/T$ where $T$ is measured in K. Therefore, $T$ influences IM1 and IM2 lifespan in two different ways.

For IM1, the measured lifespans ($L$) range from 28s to 62mn40s, while the measured $L$ for IM2 range from 3mn52s to 64mn18s. The values of $\log(L)$ for some short measured lifespans of IM1 are very close to zero when $L$ values are set in minutes (mn). This can artificially lead to high aberrant values when relative errors are derived for model performance assessment. Therefore, we suggest to compute $\log(L)$ for IM1 from lifespans set in seconds (s).
rather than in (mn). For IM2, the lifespans are longer and thus \( \text{Log}(L) \) can be computed with \( L \) taken in (mn). This scale modification does not affect the model results since it only shifts \( \text{Log}(L) \) by the constant \( \text{Log}(60) \).

E. Effect of the initial conditions

Of course, the type of insulation material will affect the model coefficients but not the lifespan modeling ability of the selected methods. Indeed, it has been proven with different materials and different levels of the stress factors in [9], [11], [12], [13] that the methods based either on DoE or on RS lead to satisfying lifespan modeling results.

This section will investigate the effect of the initial conditions of the insulation material on the results. A first set of experiments have been achieved with healthy twisted pairs covered with IM2 for \( V=1.25 \text{kV}, F=9.1 \text{kHz} \) and \( T=40^\circ \text{C} \). Then, several twisted pairs have been tested with different wounds manually conducted on different parts of the wire and different levels of severity, between light, medium and heavy, which are shown in figures 4a to 4d.

Table II gives the corresponding lifespans, from which several interesting conclusions can be derived. First, a light wound does not affect lifespan. Second, any of the medium wound reduces the insulation lifespan regardless their location and a high level of wound dramatically affects lifespans. Then, a bi-modal distribution of the lifespans should be observed when certain samples are wounded and others not, which will not be the case in our results. Consequently, it could be concluded that samples are all healthy since no-bimodal distribution was obtained in any of our test populations.

### III. Parametric Models

The basic approach to evaluate the effects of the factors and their interactions consists in computing a full parametric model where all these terms are included. Parametric lifespan models can be expressed as multi-linear regression models: \( Y = X\beta \) where \( Y \) is the vector of measured lifespan, \( X \) is experimental matrix composed of predictor variables and \( \beta \) is the vector of model coefficients that can be estimated by the Ordinary Least Square (OLS) method [16].

A. Design Optimization Methods

In the context of an electro-thermal aging study on insulation materials, experimental data sets required for lifespan model estimation are restricted due to various experimental constraints: cost of tested materials, availability of the test bench, limited experimental time, etc. Therefore, the number and the configuration of experiments must be optimized in a way to minimize experimental cost while ensuring the best model accuracy. In this paper, we aim to achieve orthogonal experimental designs. Orthogonality is one of the most interesting optimality criteria as it guarantees the best model accuracy with uncorrelated estimation of model coefficients [10]. A design is orthogonal if its experimental matrix \( X \) is orthogonal which requires that its information matrix \( XX^T \) and its dispersion matrix \( (XX)^{-1} \) are diagonal.

The most efficient method to evaluate the effects of several factors and their interactions on a response variable is the basic DoE [10] that has been already validated for insulation lifespan models in [9], [11], [12], [13]. According to DoE [9], [10], 2 levels (±) are assigned to each factor. With \( k \) factors, \( 2^k \) experiments are necessary such that each experiment involves one of the \( 2^k \) combinations between the levels of each factor. The obtained design is called \( 2^k \) Full Factorial Design (FFD2) having an orthogonal experimental matrix [10]. With 3 factors, the DoE lifespan model can be written as (1):

\[
Y = \text{Log}(L)_{\text{DoE}} = M + E_v X_v + E_f X_f + E_T X_T + I_{T\nu} X_{T\nu} X_T + I_{T\nu} X_{T\nu} + I_{T\nu} X_{T\nu} X_T
\]

where \( L \) is the lifespan, in (s) for IM1, in (mn) for IM2, \( X_v, X_f \) and \( X_T \) are the three factor levels corresponding to the values of \( \text{Log}(V), \text{Log}(F) \) and \( \exp(-bT) \) for IM1 or \( 1/T \) for IM2. \( M \) is the model constant, \( E_v, E_f \) and \( E_T \) are the three factor effects, \( I_{T\nu}, I_{T\nu} \) and \( I_{T\nu} \) are the different interaction effects. However, it may be of interest, for a better approximation of the lifespan model, to include quadratic terms of the main factors. The appropriate optimization method for second order models with interactions is the Response Surface (RS) method [14]. RS lifespan model can be written as (2):

\[
Y = \text{Log}(L)_{\text{RS}} = M + E_v X_v + E_f X_f + E_T X_T + I_{T\nu} X_{T\nu} + I_{T\nu} X_{T\nu}^2 + I_{T\nu} X_{T\nu} + I_{T\nu} X_{T\nu} X_T + I_{T\nu} X_{T\nu} X_T
\]

where \( I_{T\nu}, I_{T\nu} \) and \( I_{T\nu} \) are the factor quadratic effects.
According to [14], it is impossible to achieve the orthogonal experimental matrix property for second order designs. However, orthogonality can be obtained if the experimental matrix \( X \) is excluded from its first line and its first column. The design is then called “almost orthogonal”. There are two popular almost orthogonal RS designs for the second order models [14]:

1) **3\(^k\) Full Factorial Designs (FFD3):**
   In this design, three levels (-1, 0 and +1) are required for each factor [14], 3\(^k\) experiments are required.

2) **Central Composite Designs (CCD):**
   A CCD has the advantage of requiring less number of experimental points than a full 3\(^k\) design. However, five levels of each factor are needed instead of three. A CCD is composed of [9], [12], [14]:
   - A complete 2\(^k\) DoE design,
   - Two axial points on the axis of each factor at a distance \( \theta \) from the design center, i.e. two extra levels (±\( \theta \)),
   - \( N_0 \) central points at the design center.

An almost orthogonal CCD is obtained if [14]:

\[
2^{(2^k + 2k + N_0)} = (2^{2k} + 2^k)^2 \tag{3}
\]

Second order RS model have been also applied for the insulation lifespan modeling in [9], [11] and [12]. In this paper, a more detailed analysis of DoE and RS model performance and applicability is presented. Since the factor domain of IM1 allows more levels to be investigated than that of IM2, organized experiments were specified according to a CCD for IM1 and a 3\(^k\) FFD3 for IM2. From these two designs, first and second order models with interaction terms can be computed. For IM1, the CCD is composed of the 8 experiments of the 2\(^3\) FFD2, 4 central points and 6 axial points with \( \theta = \sqrt{2} \) to satisfy (3). Factors levels are given in Table III.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>STRESS FACTORS LEVELS APPLIED ON INSULATION MATERIAL 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>Log(10V - kV)</td>
</tr>
<tr>
<td>-( \sqrt{2} )</td>
<td>Log(10(^{0.1}))</td>
</tr>
<tr>
<td>-1</td>
<td>Log(10(^{0.1}),74)</td>
</tr>
<tr>
<td>0</td>
<td>Log(10(^{0.1}),73)</td>
</tr>
<tr>
<td>+1</td>
<td>Log(10(^{2.554}))</td>
</tr>
<tr>
<td>+( \sqrt{2} )</td>
<td>Log(10(^{3}))</td>
</tr>
</tbody>
</table>

For IM2, the design is composed of the 27 combinations between the levels -1, 0 and +1 of the three factors. These levels are given in Table IV.

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>STRESS FACTORS LEVELS APPLIED ON INSULATION MATERIAL 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>Log(10V - kV)</td>
</tr>
<tr>
<td>-1</td>
<td>Log(10(^{0.7}))</td>
</tr>
<tr>
<td>0</td>
<td>Log(10(^{0.93}))</td>
</tr>
<tr>
<td>+1</td>
<td>Log(10(^{1.25}))</td>
</tr>
</tbody>
</table>

For both test campaigns, randomly configured experiments (test sets) are carried out in order to test the model validity. Organized (blue and red points) and random (green points) experiments are represented in Fig. 5a and Fig. 5b for IM1 and IM2 respectively.

![3D representation of factor levels in the 1st test campaign for IM1](image1)

![3D representation of factor levels in the 2nd test campaign for IM2](image2)

**B. Model Prediction Performance**

The validity of parametric DoE and RS models in the factor domains given in Table I can be checked by applying them to their respective test sets. The prediction performance of the models can be evaluated by comparing the predicted and measured values of \( Y \) and \( L \) through relative errors. For each experiment, we have a set of repeated measurements for lifespan logarithms (\( Y_{meas} \)), and one value (\( Y_{pred} \)) is predicted by the model. The set of \( Y_{meas} \) corresponding to the same experiment can be averaged leading to \( Y_{av} \) and a 95% Confidence Interval (CI) of \( Y_{av} \) can be computed by assuming a normal distribution of the set of repeated \( Y_{meas} \).

The evaluation criteria of the model prediction performance on the test set are:

- Relative errors between predicted and measured average
  \( \log(L) \):
  \[
  REL = 100 \times |Y_{av} - Y_{pred}|/Y_{av} \tag{4}
  \]

- Relative errors between measured average \( L \) in the original scale (\( L_{av} \)) and predicted \( L \) (\( L_{pred} \)) obtained by applying the logarithmic back transformation on \( Y_{pred} \):
  \[
  REL = 100 \times |L_{av} - L_{pred}|/L_{av} \tag{5}
  \]

**C. Parametric Lifespan Models for IM1**

The first order lifespan model for IM1 is estimated from the 8 blue points of Fig. 5a and the second order model is estimated using the 18 points of the CCD of Fig. 5a (blue and red points with 4 replications of the center). The estimated coefficients of the two models are given by the diagrams of Fig. 6a and 6b.
that, for IM2, and in the domain given by Table I, the frequency and the temperature have the highest effects, individually and in interaction.

Therefore, DoE model shows good prediction performance on its test set with an average error of 3% for predicted \( Y \) and of 14% for predicted \( L \). However, the RS model presents higher errors on the test set with respect to DoE model. Therefore, the addition of 3 levels, 3 quadratic terms and 10 experimental points to the training set of DoE model over-fits the data and thus does not improve its prediction quality. Indeed, the oversizing of the model surely leads to an extremely accurate modelling of the training data. However, the counterpart is a decrease of the model flexibility and thus of its capacity to adapt to different experiment scenarios as those of the test set [16].

### D. Parametric Lifespan Models for IM2

The first order model for IM2 is estimated from the 8 blue points of the extreme 2\(^3\) FFD2 of Fig. 5b where each factor has the extreme levels \(-1\) and \(+1\). The second order model is estimated using the 27 points of the 3\(^3\) FFD3 of Fig. 5b (all blue points). The estimated coefficients of DoE and RS models are given in Fig. 8. These diagrams show that, for IM2, and in the domain given by Table I, the frequency and the temperature have the highest effects, individually and in interaction.

Therefore, as in IM1 parametric models, the least important interactions are those between the least important factors \((I_{IV})\) and between the three factors \((I_{III})\). RS model shows a high quadratic effect for \( I_{IV} \) rather than \( I_{IV} \) in the case of IM1 RS model. These models are applied on the green points of Fig. 5b to evaluate their prediction performance. Results are summarized in Fig. 9 and Table VI.

### IV. NON-PARAMETRIC MODELS

Previous models assume a multi-linear relationship between \( \text{Log}(L) \) and the three main factors, their quadratic forms and their interactions, allowing to quantify their respective effects. However, the interactions are introduced as independent explanatory variables through the product of the corresponding factors. This choice has no physical justification. Therefore, the interpretation of the resulting coefficients is not evident. Thus, it may be of interest to...
define another lifespan-stress relationship using the RS training set that could be more easily interpreted and could fit better the data than a second order model. Non-parametric Regression Trees (RT) present a first alternative approach to linear regression models which are especially appropriate when interactions exist between factors. To date, RT which have never been applied in insulation aging studies, will be used as a new method for the lifespan modeling of the two insulation materials.

A. Overview
Classification and regression trees were introduced by Breiman in 1984 [17]. They allow to explain the relationship between a single response variable (output) and a set of predictor variables (inputs). The principle of RT is to recursively split the training data set into smaller and more homogeneous groups. At each node, the splitting explanatory variable and its corresponding threshold value are selected so that the homogeneity of the two resulting groups is maximized. At the end, each leaf is characterized by the mean value of the response in the corresponding final group [17]. For a new observation, the response can be easily predicted by following the appropriate path throughout the tree. The order of variable appearance in the tree allows to compare their relative importance.

For both materials, RT will be constructed using the RS training sets (blue and red points in Fig. 5a and 5b) with factor levels $X_I$, $X_F$ and $X_T$ as inputs and $\log(L)$ as an output. For a better readability, factors will be represented in the tree by $V$, $F$ and $T$ instead of $X_I$, $X_F$ and $X_T$. RT performance will be evaluated on test sets (green points of Fig. 5a and Fig. 5b) with criteria defined in section III.B.

B. RT for the Lifespan Modeling of IM1

1) Classification of the Training Set Data:
The RT constructed from the 18 experimental points of the CCD of Fig. 5a is shown in Fig. 10a. This RT is first split by the voltage at its root and includes three voltage zones:
- Low Voltage (LV) where $X_V < -0.5$ ($V < 1.43$ kV),
- High Voltage (HV) where $X_V > 0.5$ ($V > 2.10$ kV),
- Medium Voltage (MV) where $-0.5 < X_V < 0.5$.

As in parametric IM1 RS model, with RT, the voltage is the most important variable that first split the data. The temperature is the next splitting variable in both LV and MV zones and finally comes the frequency. This order is coherent with factor effects estimated by RS model in Fig. 6b. However, frequency and temperature order of influence is inverted in HV zone. This fact reveals that the lifespan model is different in this voltage zone and that two different models exist depending on the voltage range: one corresponding to HV and the other to LV and MV zones that can be combined in one zone called MV&LV.

2) Prediction Performance:
When used to predict the test set lifespan logarithms ($Y$) of this campaign (green points of Fig. 5a), the RT displayed on Fig. 10a is less accurate than RS parametric model, with relative errors (ER) up to 33%. Intrinsically, RT are piecewise constant models and thus have lower prediction accuracy than parametric continuous models. To illustrate this, Fig. 10b compares the measured $Y_{av}$ of the test set points, their values predicted by the RT of Fig. 10a and by a linear model computed from the same training set and including only the three main factor terms ($X_I$, $X_F$ and $X_T$). It is clear from Fig. 10b that the linear model better fits the test set points than the RT.

C. RT for the Lifespan Modeling of IM2

1) Classification of the Training Set Data:
The RT constructed from the 27 points of the $3^3$ FFD3 of Fig. 5b is shown in Fig. 11. It is first split by the frequency at $X_F = 0.5$ ($F = 11.695$ kHz) that divides the experiments into 2 zones: Low Frequency (LF) / High Frequency (HF). As in parametric IM2 RS model, with RT, the frequency is the most important variable that first splits the data. After the frequency split, temperature is the next splitting variable and voltage appears last, after these two factors, in both LF and HF zones. This is coherent with the three factor effects estimated by parametric models in Fig. 8b. However, there are two different classifications in LF and HF zones. In LF zone, there are three temperature zones with the same order of classification in each (frequency then voltage). However, in HF zone, the voltage appears just after the first temperature splitting and there are no nodes with the frequency as a splitting variable. Therefore, two different models exist depending on the frequency range.

Fig. 10a. Regression tree constructed from IM1 RS training set

Fig. 10b. Comparison of test set prediction accuracy between the RT and a linear model including only the main factors (IM1)
2) Prediction performance

RT of Fig. 11 is used to predict the lifespan logarithms (\(Y\)) of the test set points (green points of Fig. 5b). As for IM1, predictions are less accurate than those computed by RS model with relative errors (ER\(_Y\)) up to 24% with RT. Fig. 12 shows that a linear model computed from the same training set and including only the three terms (\(X_V\), \(X_F\) and \(X_T\)) better fits the test set points than the RT of Fig. 11.

Fig. 12. Comparison of test set prediction accuracy between the RT and a linear model including only the main factors (IM2)

It can be confirmed that RT have lower prediction accuracy than parametric models since they are piecewise constant. However, RT identifies different ranges of the main factors corresponding to different models.

V. HYBRID MODELS

In light of the above, we can see that each presented model has its advantages and drawbacks. Parametric DoE and RS models allow quantifying the effects of each factor, of their quadratic terms and of their interactions on the lifespan with good prediction performance on the test set points belonging to the same experimental domain of the training set points. However, the second order models appear to over-fit the data although estimated from an optimized training set. On the other hand, interpretation of the interactions through the product terms is not obvious. With RT, a simple and graphical life-stress relationship is obtained. Relative importance of the main factors can be deduced from the hierarchical structure of the RT. This structure also allows to identify ranges of the main factor where more specific and thus more accurate models can be derived. However, RT are piecewise constant and have lower prediction performance on the test set than parametric models. Therefore, we suggest in this section to combine these two approaches in a piecewise linear model in order to benefit from the advantages of the two methods and to overcome their drawbacks. The proposed model is called Hybrid Model (HM) and is presented as an original method based on RT for lifespan modeling.

A. Construction of HM

The principle of HM is first to identify the most important factor and its splitting values through the RT. Then, by the means of dummy variables, one coefficient for each of the two other factors is defined in each range of the main factor. This model structure allows to:

- Refine the parametric model by examining the life-stress relationship in each identified range,
- Explicit interactions with the main factor by examining the effect of the main factor range on the coefficients of the other two factors, (interaction between the least important two factors have a very low effect according to parametric models),
- Improve the prediction quality of regression trees.

As RT, HM will be computed from the RS training sets of the two campaigns (blue and red points in Fig. 5a and 5b) using the factor levels \(X_V\), \(X_F\) and \(X_T\) as predictors and \(\log(L)\) as a response where \(L\) is in (s) for IM1 and in (\(\mathrm{mn}\)) for IM2. Then the model prediction performance will be evaluated on the corresponding test set (green points of Fig. 5a and 5b) with criteria defined in section III.B.

B. Hybrid Lifespan Models for IM1

For IM1, voltage was identified as the most important factor dividing RS training set into two ranges: HV and MV&LV at \(X_V = 0.5\). The HM can thus be written as (6):

\[
Y = \log(L)_{\text{HM}} = M + E_V X_V + E_{VH} X_H + E_{MV&LV} X_{MV&LV} + E_{VH} X_H + E_{MV&LV} X_{MV&LV} + E_{VT} X_T
\]
where \( \delta_{HF} \) (respectively \( \delta_{MV&LV} \)) is a dummy variable equal to 1 when \( X_T \) belongs to HV (respectively MV&LV) zone and 0 elsewhere.

Equation (6) has the general form of a multi-linear regression model between the response \( \log(L) \) and the predictor variables \( X_V \), \( \delta_{MV&LV}X_V \), \( \delta_{HV}X_F \), \( \delta_{MV&LV}X_F \), \( \delta_{HV}X_T \) and \( \delta_{MV&LV}X_T \). Model coefficients can thus be estimated by OLS method. The coefficients of HM (6) estimated from the 18 points of IM1 RS training set (blue and red points of Fig. 5a) are represented in the diagram of Fig. 13.

This model confirms once again that voltage is the most important factor for IM1. It also confirms the existence of interactions between the voltage and the frequency (respectively the temperature) since two different coefficients exist for the frequency (respectively the temperature) depending on the voltage zone. In addition, the relative effects of frequency and temperature in each zone are coherent with their order in the RT of Fig. 10a: the frequency effect is lower than the temperature effect in HV, but higher in MV&LV, but higher in HV. HM (6) prediction performance on IM1 test set (green points of Fig. 5b) is summarized in Fig. 14 and Table VII. Obviously, HM model improves the prediction quality of both RT and RS models regarding the test set.

For IM2, frequency was identified as the most important stress factor dividing RS training set into two ranges: HF and LF at \( X_F = 0.5 \). The HM can thus be written as (7):

\[
Y = \log(L)_{HM} = M + E_VX_V + E_{HV}\delta_{HV}X_F + E_{MV&LV}\delta_{MV&LV}X_F + E_{HF}\delta_{HF}X_T + E_{LV&LF}\delta_{LV&LF}X_T
\]

where \( \delta_{HF} \) (respectively \( \delta_{LF} \)) is a dummy variable equal to 1 when \( X_T \) belongs to HF (respectively LF) zone and 0 elsewhere. HM (7) coefficients estimated from the 27 points of IM2 RS training set (blue points of Fig. 5b) are given in the diagram of Fig. 15.

This model confirms that the voltage has the lowest effect for IM2 in each frequency zone. It also confirms the existence of interactions between the frequency and the voltage (respectively the temperature) since two different coefficients exist for voltage (respectively the temperature) depending on the frequency zone. The prediction performance of HM (7) on IM2 test set (green points of Fig. 5b) is summarized in Fig. 16 and Table VIII. As in the case of IM1, HM improves prediction quality of both RT and RS models regarding the test set.

### Table VII

<table>
<thead>
<tr>
<th>Test set prediction performance of HM (IM1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max (RE(_{11}))</td>
</tr>
<tr>
<td>-------------------</td>
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<tr>
<td>8.7%</td>
</tr>
</tbody>
</table>

### Table VIII

<table>
<thead>
<tr>
<th>Test set prediction performance of HM (IM2)</th>
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<tbody>
<tr>
<td>Max (RE(_{11}))</td>
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<td>8.3%</td>
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</tbody>
</table>

### D. Discussion

From the two test campaigns, it was confirmed that a HM shows better prediction quality than RS parametric model, both being estimated from the same organized training set.

\[
X'X = \begin{bmatrix}
2^k + 2k + N_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2^{k+1} + 2^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 2^{k+1} & 0 & 0 & 0 \\
0 & 0 & 0 & 2^{k+1} + 2^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 2^{k+1} & 0 \\
0 & 0 & 0 & 0 & 0 & 2^{k+1} + 2^2
\end{bmatrix}
\]

(8)
On the other hand, it can be shown that, in general, for \( k \) factors, the experimental matrices of HM having the same expressions as (6) and (7) and estimated from a CCD or a 3-level FFD3 always satisfy the orthogonality criteria. The information matrix of HM (6) estimated from a CCD can be written in function of \( k, N_0 \) and \( \theta \) as in (8). Therefore, matrix (8) remains diagonal for all CCD, regardless to \( N_0 \) and \( \theta \). The information matrix of HM (7) estimated from a 3-level FFD3 is also diagonal. It can be written in function of \( k \) as (9):

\[
X'X = \begin{bmatrix}
3^k & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 \times 3^{k-1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4 \times 3^{k-2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 \times 3^{k-2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 \times 3^{k-2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 \times 3^{k-2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3
\end{bmatrix}
\] (9)

Orthogonal design property is therefore an additional advantage for HM over RS models. It offers more flexibility for the choice of the organized design of HM training set. On the other hand, in both test campaigns, HM prediction performance is very close to that of DoE models computed from 2-level FFDs. However, the advantage of HM over DoE model is that HM involves a smaller number of variables than DoE model (2k instead of 2^k for \( k \) factors). In addition, all variables in HM are important and they are more easily interpretable than those of DoE model. Therefore, with \( k \) factors and only two levels per factor that can be tested, the best lifespan model configuration in terms of accuracy and experimental cost is a first order model with interaction terms computed from the 2^k experiments of the 2-level FFD2. If more levels per factor can be tested so that a CCD or a 3-level FFD3 can be established, the best lifespan model is a HM configured after identifying the different regions of the main stress factors with a RT constructed on the same training set.

VI. CONCLUSION AND FUTURE WORK

In conclusion, an original statistical method was developed for the lifespan modeling of insulation materials under PD regime. This method was validated on two different insulation materials in two experimental domains corresponding to accelerated aging conditions. The presented models relate the lifespan logarithm to three main aging factors through three different forms: parametric, non-parametric and hybrid models.

These models allow to evaluate the effects of the three factors and of their interactions. While parametric forms are commonly used in modeling tasks, non-parametric regression trees and hybrid models provide original life-stress relationships that have never been investigated in insulation aging studies before. These different models were compared and the optimal use of each was defined accordingly through an original, more flexible and methodological approach.

In future work, the presented methodology will be applied to the lifespan modeling of other thermal class insulating materials and other critical parts of electrical machines. On the other hand, more stress factors will be considered in the insulation lifespan models such as pressure, humidity or mechanical vibrations. Finally, as prognostic is the final goal of this lifespan modelling, model prediction of long life aging during almost normal conditions will be investigated. For this objective, materials will be tested in domains below the PD regime in order to test the validity of the presented models at lower stress levels that are closer to normal conditions.

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REFERENCES