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Modeling quasi-static magnetic hysteresis: a new implementation of the play model based on experimental asymmetrical B(H) loops

A. Giraud, A. Bernot, Y. Lefèvre, J.F. Llibre

Abstract — This paper relates a new model of quasi-static magnetic hysteresis based on the Play model hysterons, which builds the magnetic field density $B$ from the magnetic field $H$. In the original model, $H$ is discretized into temporal values $H(t_n)$, which is itself modeled by a hysteron chain of $m$ sub-values. $B$ is then reconstructed from these sub-values through a function experimentally determined by measuring $B(H)$ centered cycles, using a constraint optimization method. The new proposed method is to measure asymmetrical $B(H)$ loops, which give additional equations leading to a fully determined linear square invertible system. The asymmetrical $B(H)$ loop is included in a bigger symmetrical loop with a magnetic flux density turnaround in order to be regulatable.

Index Terms—Asymmetrical loops, Electrical machines, Hysteresis, Hysterons, Iron losses, minor loops, Play model, Steel sheet

I. INTRODUCTION

THE iron losses have been studied for decades, especially in electrical machines [1]. Following the development of power electronics and new controlling systems such as Pulse Width Modulation (PWM), high frequency harmonics due to switching frequency must be treated inside electrical machines [2]. Moreover, high speed applications require high fundamental frequency of functioning. The increase of frequencies induces a high iron losses growth [3]. Therefore, models must be adapted.

Several models exist and generally two kinds are considered: post-processing models based on physical considerations [4] which need experimental characterization, and pre-processing models based on the $B(H)$ loop building, which need also experimental measurements. Both types have their own issues and advantages, but the pre-processing method appears to be more accurate. Its major disadvantage is the high calculation power needed. Indeed, the most famous pre-processing model is Preisach model [5], which needs the highest calculation power but provides the best accuracy in quasi-static hysteresis building. Play and Stop models [6] derive from Preisach model and have been proved as mathematically equivalent [7]. The Play model proposes to build the magnetic flux density $B$ from the applied magnetic field $H$. The Stop model is equivalent and builds the applied magnetic field from the magnetic flux density. Both are based on a double deconstruction of $H$ and $B$. First, for each time step a value of $B$ and $H$ is obtained. Then, strings of intern variables called hysterons are assigned to each value of $H$ or $B$. The global functioning is described further in the article.

This paper proposes a new modeling of Play model based on asymmetrical hysteresis loops. First, the principle of original Play model is described. Then, the issues induced by this way of modeling are presented and a method is finally developed in order to improve the efficiency of the model.

II. DEVELOPMENT OF THE METHOD

A. Original model description

The principle of the original model [8] is simple: in order to obtain the magnetic flux density $B$ from the applied magnetic field $H$, both are temporal signals, $H$ is first temporally discretized. Hence, $H$ is decomposed in $M$ temporal values $H(t_i)$ separated by a specific step $\varepsilon_0$. Then, each value $H(t_i)$ is also decomposed in intern variables $h_i$. Those intern variables are called hysterons and follow a specific algorithm presented in [9]. Hysterons could be compared to a spring network: each hysteron in the chain has an influence on another one. The algorithm is given as:

- $h_1 = H(t)$
- For $i \in \{1, \ldots M\}$, $h_i$ preferentially changes to $h_{i+1}$ with a $\varepsilon_0$ step
- For $i \in \{1, \ldots M\}$, $|h_i - h_{i+1}| = \varepsilon_0$
- The starting point must be known, the saturation point is preferred

Then, for each $h_i$ corresponds a specific $b_i(h_i)$ defined as:

$$b_i = \sum_{j=1}^{M-i+1} u_{ij} w_j(h_i)$$

(1)

With

$$w_j(k, \varepsilon_0) = \begin{cases} 1 & si k = j \\ 0 & si k \neq j \end{cases}$$

Where $u_{ij}$ are piece-wise linear functions which need to be identified. Finally $B(t)$ is obtained with:
\[ B(t) = \sum_{i=1}^{M} b_i(h_i) \] (2)

The general functioning is presented in Fig. 1.

Fig. 1: Global functioning of Play model described in [8] and [9]

However, the \( u_{ij} \) coefficients have to be identified to use the model. The method proposed in [8] is based on major hysteresis loop measurement. The number \( l \) of major loops is equal to \( M \); the more \( M \) is high, the more the system is accurate. Then, \( u_{ij} \) are identified with the hysteresis loop measurement as presented Fig. 2 with \( l = 2 \).

Fig. 2: Parameter identification

The parameters obtained are:

\[
\begin{align*}
q_{2,2} &= u_{1,2} + u_{2,1} \\
q_{2,1} &= u_{1,1} + u_{2,1} \\
q_{2,0} &= u_{2,1} \\
q_{2,-1} &= -u_{1,1}
\end{align*}
\]

Therefore, a matrix system describes the problem as below:

\[ C \times U = Q \] (3)

Where \( U \) is the matrix representing the coefficients \( u_{ij} \) of (1). The matrix \( Q \) corresponds to the experimental measurements and \( C \) is a matrix fulfilled by 0, 1 or -1.

However, (3) induces a major issue: the problem described here is not solvable. Matrix \( C \) has to be reduced as a square one with a specific optimization algorithm [9]. Unlike the model proposed in [10] which used a 3-D vector for \( H \) and \( B \), the method presented here is in one dimension. In this way, a finite element calculation is not required to get the hysterons from \( B \) and \( H \), but the method does not take into account the space dependency of hysterons. Indeed, the model described here is applied on electrical steel sheet which limit space effect, whereas ring core are used in [10].

\[ B \cdot \text{Principle of the new method} \]

The new method proposed here is typically following the same process. The magnetic field is temporally discretized and then decomposed in intern variables. The only difference in the global functioning is the mean to get \( b_i(h_i) \). Indeed, to avoid the utilization of piece-wise linear functions, it is proposed to directly use the measurements to identify \( b_i(h_i) \).

The interpolation used in the original model is suppressed. Hence, (3) becomes:

\[ C \times B_l = Q \] (4)

Where \( B_l \) is the matrix of \( b_i(h_i) \). For two loops \( (l = 2) \), matrices of (4) can be detailed from the Table 1:

<table>
<thead>
<tr>
<th>Loop n°</th>
<th>( H ) step</th>
<th>( h_i )</th>
<th>( q_{l,k} )</th>
<th>( b_l(h_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( [2\varepsilon_0, \varepsilon_0] )</td>
<td>( q_{2,2} )</td>
<td>( b_{1,2} + b_{2,1} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \varepsilon_0 )</td>
<td>( q_{2,1} )</td>
<td>( b_{1,1} + b_{2,1} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( [0, \varepsilon_0] )</td>
<td>( q_{2,0} )</td>
<td>( b_{1,0} + b_{2,1} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( [-\varepsilon_0, 0] )</td>
<td>( q_{2,-1} )</td>
<td>( b_{1,-1} + b_{2,0} )</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( [\varepsilon_0, 0] )</td>
<td>( q_{1,1} )</td>
<td>( b_{1,1} + b_{2,0} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( [0, 0] )</td>
<td>( q_{1,0} )</td>
<td>( b_{1,0} + b_{2,0} )</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, for this example, the system changes as:

\[
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
b_{1,1} \\
b_{1,0} \\
b_{1,2} \\
b_{2,0} \\
b_{1,1} \\
b_{2,1}
\end{bmatrix}
\times
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
q_{2,2} \\
q_{2,1} \\
q_{2,0} \\
q_{2,-1} \\
q_{1,1} \\
q_{1,0}
\end{bmatrix}
\]

Although, the simple matrix system obtained is not exploitable. Indeed, \( \text{rank}(C) = 5 \), so \( C \) is not invertible.
more equations are needed. Using only major loops with the presented method involves a non-invertible system. So the method has to be adapted. That is why we have decided to change the common method of identification using asymmetrical loops.

C. Asymmetrical loops introduction

As presented before, more equations are needed. With an eye to add equations, the concept of using only major loops must be necessarily changed. Regarding to the global functioning of hysterons building, adding more major loops would be useless. Indeed, it would involve new hysterons and the ones without enough information would be still undetermined.

The solution proposed here is to add asymmetrical inner loops, as presented in Fig. 3. Starting from the saturation point, when a hysteron is undetermined, a minor loop allows to go back and get it. These asymmetrical loops allow to reach new magnetization states unreachable with only symmetrical loops, thus leading to new equations containing the unknown intern variables $b_i$.

Fig. 3: Hysteresis loop with asymmetrical loop in NO20 - 1000A/m at 10Hz

However, an asymmetrical regulation is not possible. The minor loop is then inserted inside a bigger symmetric loop, as in Fig. 4. The symmetric opposite of the minor loop is also added, in order to have a controlled set of $H$ points. The bigger loop and the inferior minor loops have no other use than allowing the superior minor loop to be regulated.

Actually, the asymmetrical loop is calculated to start at each $H$ step of the major loop. Then, following the asymmetrical loop, $H$ increases up to the saturation point and goes back to its starting point. However, the “history” of hysterons sequence keeps evolving step by step like for the original model. But after the point of U-turn, the $H$ changes its direction of variation, so as the sequence of hysterons. In this way, we add new sequences of hysterons for each $H$ step.

A combination of two sine waves, 1st and 3rd harmonics as in (6), is chosen to regulate the measurement sequence of Fig. 4. This is chosen because it is the lowest possible harmonic content curve to generate the measurement sequence.

$$H(t) = H_1 (\sin(\omega t) + \alpha \sin(3\omega t))$$  \hspace{1cm} (6)

With

$$\alpha = \frac{H_3}{H_1}$$  \hspace{1cm} (7)

Where $H_3$ and $H_1$ are respectively the magnitudes of the third harmonic and the fundamental.

In order to control the minor loops, the ratio $r$ between harmonic and fundamental has to be calculated. The general form of the magnetic field waveform is given in Fig. 5.

Considering the relation between maximum and minimum, the ratio can be expressed as:

$$r = \frac{1 - \alpha}{\sin(\arcsin(A)) + \alpha \sin(\arcsin(A))}$$  \hspace{1cm} (8)

With

$$A = \left( \frac{9\alpha - 1}{12\alpha} \right)$$  \hspace{1cm} (9)

$H_1$ is in the end adjusted to fit the required sequence of points.

Actually, the ratio $r$ allows a complete control of the lowest $H$ point of the asymmetrical loop. Basically, when the ratio $r = 1$, the waveform is a pure sine wave, related to the major loop.

Finally, the method proposed here uses (8) and (9) in a specific algorithm to equilibrate the system with enough variables and equations.

Fig. 4: Real hysteresis loop used to regulate applied magnetic field in NO20- 1000A/m at 10Hz
D. A new algorithm

The basic principle is to insert several asymmetrical loops to obtain new experimental values. Finally, more equations are obtained without adding more unknown variables to due to the addition of experimental values \( q_{l,k} \) such as presented on Fig. 7 and Fig. 8. The global functioning of this new algorithm is described on the chart of Fig. 6.

Here, a 60 × 60nm NO20 single sheet (Fe-Si) was used to obtain 2 loops (related to the example of Fig. 2). We can see that with the asymmetrical loop, more experimental data at the same \( H \) step is obtained, twice more for this case.

Considering \( M \) symmetrical loops (and so \( M \varepsilon_0 \) H steps) and the applied field \( H \) as in (6), Table 2 describes the functioning of our algorithm in order to choose the parameters.

Then, both magnitude and ratio change following a specific evolution based on the \( H \) step. Therefore, for the same quantity of \( b_l \) we obtain more experimental values \( q_{l,k} \), still following (4) but with different sizes for matrices:

\[
C \times \begin{bmatrix}
    b_{1,n} \\
    b_{1,n-1} \\
    \vdots \\
    b_{1,-n} \\
    b_{2,n-1} \\
    \vdots \\
    b_{2,-(n-1)} \\
    \vdots \\
    b_{n,1} \\
    b_{n,0} \\
    b_{n,-1}
\end{bmatrix} = Q
\]  

(10)

The matrix of \( b_{ij} \) is still a vector of \( M(M+2) \) components, such as the number of column in \( C \).

<table>
<thead>
<tr>
<th>( \text{Highest } H ) of the loop</th>
<th>( \text{Smallest } H ) of the loop</th>
<th>( \text{Ratio } r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M \varepsilon_0 )</td>
<td>( -M \varepsilon_0 + \varepsilon_0 )</td>
<td>( \frac{1}{M \varepsilon_0} )</td>
</tr>
<tr>
<td>( -M \varepsilon_0 + 2\varepsilon_0 )</td>
<td>( -M \varepsilon_0 + 2\varepsilon_0 )</td>
<td>( \frac{1}{M \varepsilon_0} )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( -M \varepsilon_0 + (2M - 1)\varepsilon_0 )</td>
<td>( -M \varepsilon_0 + (2M - 1)\varepsilon_0 )</td>
<td>( \frac{1}{M \varepsilon_0} )</td>
</tr>
<tr>
<td>( (M - 1)\varepsilon_0 )</td>
<td>( - (M - 1)\varepsilon_0 + \varepsilon_0 )</td>
<td>( \frac{1}{(M - 1)\varepsilon_0} )</td>
</tr>
<tr>
<td>( (M - 1)\varepsilon_0 + 2\varepsilon_0 )</td>
<td>( (M - 1)\varepsilon_0 + 2\varepsilon_0 )</td>
<td>( \frac{1}{(M - 1)\varepsilon_0} )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( (M - 1)\varepsilon_0 + (2M - 1)\varepsilon_0 )</td>
<td>( (M - 1)\varepsilon_0 + (2M - 1)\varepsilon_0 )</td>
<td>( \frac{1}{(M - 1)\varepsilon_0} )</td>
</tr>
</tbody>
</table>

Fig. 6: Chart of global functioning of the method
The number \( n_q \) of lines in \( C \) and \( Q \) depends on the total number of loops \( M \):

\[
n_q = M(M + 1) + \sum_{i=1}^{M} \sum_{k=1}^{2(i-1)} k
\]  

(11)

Hence, the system is still non-invertible and has to be turned in a square matrix one. First we can assume that:

\[
\forall_{i,j \in \{2, \ldots, M\}} b_{i,-(n-j+1)} = -b_{i,n-j+1}
\]

(12)

The components from (12) have to be considered useless and the corresponding columns are suppressed. Then, the useless lines of \( C \) and \( Q \) are easily suppressible with a basic algorithm which removes the concerned lines and keeps the ranks of the matrices unchanged. Finally, the system presented in (10) is square and invertible and all the \( b_{i,j} \) are determined, then allowing the building of hysteresis loop and iron losses calculation.

III. VALIDATION OF THE MODEL

A. Experimental validation

As for Fig. 3, Fig. 4 and Fig. 5, a NO20 steel sheet is used in order to identify \( b_{i,j} \). First we tried the algorithm for \( M = 2 \). We obtained the following values for \( b_{i,j} \):

\[
\begin{align*}
    b_{1,2} &= 0.2455 \\
    b_{1,1} &= 0.0854 \\
    b_{1,0} &= -0.6588 \\
    b_{1,-1} &= -1.3780 \\
    b_{1,-2} &= -b_{1,2} \\
    b_{2,1} &= 1.2905 \\
    b_{2,0} &= 0.0879 \\
    b_{2,-1} &= -b_{2,1}
\end{align*}
\]

(13)

Then, it is easily possible to construct \( B \) step by step from \( H \) as described in Table 3. In this way, only the superior part of the hysteresis loop is built, but the whole loop is obtained with a central symmetry. Then, a comparison between the simulated loop using the algorithm and the measured one is foreseeable, as presented in Fig. 9.

With an eye to develop the validation, we also used the algorithm for \( M = 4 \) and the comparison between the simulated loop and the measured one is presented in Fig. 10.

**TABLE 3: CONSTRUCTION OF HYSTERESIS LOOP FOR M=2**

<table>
<thead>
<tr>
<th>( H ) step</th>
<th>( h_i ) sequence</th>
<th>( b_i ) sequences</th>
<th>Values of ( B ) (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2 \varepsilon_0 )</td>
<td>( [2 \varepsilon_0, \varepsilon_0] )</td>
<td>( B = b_{1,2} + b_{1,1} )</td>
<td>1.536</td>
</tr>
<tr>
<td>( \varepsilon_0 )</td>
<td>( [\varepsilon_0, \varepsilon_0] )</td>
<td>( B = b_{1,1} + b_{1,1} )</td>
<td>1.376</td>
</tr>
<tr>
<td>0</td>
<td>( [0, \varepsilon_0] )</td>
<td>( B = b_{1,0} + b_{1,1} )</td>
<td>0.6317</td>
</tr>
<tr>
<td>( -\varepsilon_0 )</td>
<td>( [-\varepsilon_0, 0] )</td>
<td>( B = b_{1,-1} + b_{1,0} )</td>
<td>-1.29</td>
</tr>
<tr>
<td>( -2 \varepsilon_0 )</td>
<td>( [-2 \varepsilon_0, -\varepsilon_0] )</td>
<td>( B = -b_{1,2} - b_{1,1} )</td>
<td>-1.536</td>
</tr>
</tbody>
</table>

**Fig. 7: Example with two symmetrical loops and asymmetrical loops**

**Fig. 8: Zoom of Fig. 7 and addition of experimental values explanation**

**Fig. 9: Comparison between simulated and measured hysteresis loop for \( M = 2 \) in NO20 - 1000A/m at 10Hz**
The new method is working and allows to obtain the magnetic flux density $B$ from the applied magnetic field $H$. Obviously, a low value of $M$ involves a low accuracy of the hysteresis loop construction. However, both comparisons enable the verification of the good functioning of this method.

B. Magnetic flux density prediction using the method

To go further, we can build a hysteresis loop with an applied magnetic field, given in Fig. 11, which is different than the one used for parameter identification. In order to stay as close as possible in a quasi-static state, the 10 Hz frequency is conserved. The number of major loops remains equal to $M = 4$.

Then, the magnetic flux density is simulated using the model and compared to the measured one, as presented in Fig. 12.
The comparison of hysteresis loops from measured and simulated magnetic flux density $B$ and the applied magnetic field $H$ is shown on Fig. 13.

The target $B$ zones are reached by the model but with a medium accuracy. The $B$ points of the minor loops are also built whereas they differ from the hysteresis loops used for parameter identification. It is possible to observe those target zones on Fig. 14.

Obviously, only $B$ points corresponding to $H$ steps $s_B$ can be simulated. So with $M=4$ the model cannot be accurate enough and this simulation is not sufficient. Hence, the experimental validation proposed here has to be developed. Nevertheless, it is an additional step in a global validation of the model.

IV. CONCLUSION

The new implementation of the Play model proposed here permits to have magnetic flux density $B$ from the applied magnetic field $H$ with a correct accuracy considering the low value of the number of major loops $M$ tested here. The simulation is based on matrix calculation and is very fast and does not need a high power of calculation. The characterization allowing the identification of the internal variables is not complex but needs a proper experimental approach.

However, to be complete, the experimental validation has to be tested with a higher value of $M$. In this way, several values of $M$ must be experimented, increasing the accuracy and allowing different types of $H$ waveforms in the steel sheet. Other magnetic materials should also be tested.

Moreover, to go beyond, it will be very interesting to have a dynamic model. Indeed, the proposed method is a quasi-static one. As a first step, the proposition of [11] to take dynamic effect into account should be tested. A dynamic model associated with an accurate eddy current model would enable a complete modelling of magnetic material behavior. The iron losses prediction would be more precise and their calculation could be integrated to a global optimization chain for electrical machines design.

V. REFERENCES


VI. BIOGRAPHIES

Alexandre Giraud graduated from the "Ecole Nationale Supérieure d'Electrotechnique, d'Electronique, d'Hydraulique et d'Informatique de Toulouse" (ENSEEIHT – France) in Electrical Engineering in 2014. Since, he is a Ph.D student of the "Institut National Polytechnique de Toulouse" and works in the GREM3 electro dynamics research group of the LAPLACE (Laboratoire des Plasmas et de Conversion d’Energie) and he is working at the IRF Saint-Exupéry. His field of research for his Ph.D is the modeling of iron losses in electrical machines supplied with high frequency complex spectrum.

Alix Bernot graduated from the École Polytechnique, Palaiseau, France in 2009 and obtained his PhD on SMC transverse flux PM machines from CentraleSupélec, Gif-sur-Yvette, France in 2015. He has worked 4 years as an electrical motor designer for Francecol technology, France, and is currently an electrical machines researcher at Zodiac Aerospace, Ausserre, France, on temporary assignment at IRT Saint-Exupéry, Toulouse, France. His research interests are permanent magnet electrical machines design and modelling, and iron losses in soft magnetic materials.

Yvan Lefèvre graduated from the "Ecole Nationale Supérieure d'Electrotechnique, d'Electronique, d'Hydraulique et d'Informatique de Toulouse" (ENSEEIH – France) in Electrical Engineering in 1983 and received his Doctorate degree from the "Institut National Polytechnique de Toulouse" in 1988. Since 1989, he is working as CNRS researcher in the LAPLACE (Laboratoire des Plasmas et de Conversion d’Energie). His field of interest is the modeling of coupled phenomena in electrical machines in view of their design.

Jean-François Llibre received the Ph. D. degree in Electrical Engineering from National Polytechnic Institute of Toulouse in 1997. He is a lecturer since 1998 and teaches electrical engineering in the University Institute of Technology of Blagnac near Toulouse since 2003. He joined the GREM3 electrodynamics research group of LAPLACE laboratory in 2010. His actual main research interests concern the modeling, the analytical field calculation, the optimal design in electrical machines field.