Open Archive TOULOUSE Archive Ouverte (OATAO)

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: http://oatao.univ-toulouse.fr
Eprints ID: 18181

To link to this article: DOI: 10.1080/10255842.2010.494160
URL: http://dx.doi.org/10.1080/10255842.2010.494160

To cite this version: Swider, Pascal and Ambard, Dominique and Guérin, G. and Søballe, Kjeld and Bechtold, Joan E. Sensitivity analysis of periprosthetic healing to cell migration, growth factor and post-operative gap using a mechanobiological model. (2011) Computer Methods in Biomechanics and Biomedical Engineering, vol. 14 (n° 9). pp. 763-771. ISSN 1025-5842

Any correspondence concerning this service should be sent to the repository administrator: staff-oatao@listes-diff.inp-toulouse.fr
Sensitivity analysis of periprosthetic healing to cell migration, growth factor and post-operative gap using a mechanobiological model

Pascal Swidera, D. Ambarda, G. Guérina, Kjeld Søballeb and Joan E. Bechtoldc

aIMFT UMR CNRS 5502, University of Toulouse, CHU Purpan, Amphithéâtre Laporte, Place Dr Baylac, 31056 Toulouse Cedex, France; bDepartment of Orthopaedic Surgery, University of Aarhus, Aarhus Amtsyggehus, Aarhus, Denmark; cBiomechanics Laboratory, Midwest Orthopaedic Research Foundation and Minneapolis Medical Research Foundation, Minneapolis, MN, USA

A theoretical rationale, which could help in the investigation of mechanobiological factors affecting periprosthetic tissue healing, is still an open problem. We used a parametric sensitivity analysis to extend a theoretical model based on reactive transport and computational cell biology. The numerical experimentation involved the drill hole, the haptotactic and chemotactic migrations, and the initial concentration of an anabolic growth factor. Output measure was the mineral fraction in tissue surrounding a polymethymethacrylate (PMMA) canine implant (stable loaded implant, non-critical gap). Increasing growth factor concentration increased structural matrix synthesis. A cell adhesion gradient resulted in heterogeneous bone distribution and a growth factor gradient resulted in homogeneous bone distribution in the gap. This could explain the radial variation of bone density from the implant surface to the drill hole, indicating less secure fixation. This study helps to understand the relative importance of various host and clinical factors influencing bone distribution and resulting implant fixation.

Keywords: implant fixation; computational cell biology; mechanobiology; osteoblast; anabolic growth factor; reactive transport in porous media

List of symbols

- \( f_s \) (%): structural fraction
- \( C_g \) (ng/mm\(^3\)): growth factor concentration
- \( \alpha_s \) (mm\(^5\)/cell s ng): structural fraction synthesis
- \( C_{g0} \) (ng/mm\(^3\)): initial GF concentration
- \( \rho \) (kg/mm\(^3\)): density
- \( D_g \) (mm\(^2\)/s): growth factor diffusion
- \( \phi_t \) (%): fluid fraction or porosity
- \( r_i \) (mm): implant radius
- \( q_t \) (mm/s): fluid flux
- \( r_d \) (mm): drill-hole radius
- \( C_o \) (cell/mm\(^3\)): cell concentration
- \( r_b \) (mm): host bone radius
- \( \alpha_o \) (m\(^3\)/cell s): cell proliferation
- \( \phi_{di} \) (%): structural fraction at implant
- \( n_o \) (cell/mm\(^3\)): proliferation threshold
- \( \phi_{mn} \) (%): mean structural fraction
- \( D_{o} \) (mm\(^2\)/s): cell random diffusion
- \( \Delta \phi \) (%): heterogeneity index
- \( \chi_o \) (mm\(^2\)/s ng): chemotactic coefficient
- \( u, \bar{u} \): output measures
- \( h_o \) (mm\(^5\)/s kg): haptotactic coefficient
- \( a_i \): coefficients of experimental design

1. Introduction

Joint replacement implants with stable and secure mechanical conditions post-operatively are associated with higher longevity (Hahn et al. 1998). In general, low mineralisation of newly formed tissue or bone that is located heterogeneously (i.e. without complete contact between native bone and implant surface) decreases clinical performance (Søballe et al. 1992; Morshed et al. 2007). The characteristics of the bone that forms in the vicinity of the implant are affected by surgical technique, the amount of implant stability gained at surgery and interactions between biochemical factors such as coatings, growth factors and cells in the periprosthetic space (Albrektsson et al. 1983; Wang et al. 1997; Overgaard 2000; Anderson 2001; Colnot et al. 2007; Schwarz et al. 2007).

In periprosthetic healing, predominately intramembranous bone forms without a cartilagenous phase. Osteoblasts are the cells promoting bone formation and mineralisation, and various growth factors can help to promote bone matrix formation (Conover 2000). Osteoclasts and osteoblasts interact, but this is not included here. There are many anabolic growth factors, and the TGF-\( \beta \) superfamily is known to promote bone formation. It has mitogenic action on osteoblasts and can stimulate matrix formation via autocrine, paracrine and endocrine modes (Roberts 2000). Growth factors regulate the cell differentiation, proliferation and motility (Linkhart et al. *)

*Corresponding author. Email: swider@cict.fr
were considered negligible compared to those of the structural and fluid phases. As a consequence, the sum of $\phi_S$ and $\phi_F$ was equal to 1.0 and $\phi_F$ also expressed the porosity of the multiphasic medium.

The osteoblast concentration $C_o$ was described by the conservative Equation (A2) involving random diffusion and active migrations (Maheshwari and Lauffenburger 1998; Fall et al. 2002; Meinel et al. 2003). The gradient of growth factors $\partial C_g/\partial x$ directed the chemotactic flux and the gradient of structural fraction $\partial \phi_S/\partial x$ directed the haptotactic flux. Both influenced the convection and source terms of Equation (A2). The cell source represented the proliferation process using a logistic law where $a_o$ and $n_o$ were the rate of cell divisions and the proliferation threshold, respectively.

The growth factor concentration $C_g$ was described by the conservative Equation (A3). It includes random diffusion and convection which was dependent upon the fluid flux $q_f$ and the porosity gradient $\partial \phi_f/\partial x$. To limit the complexity of the problem, no production or consumption term was considered.

\[
L \frac{\partial \Omega}{\partial t} = D \Delta u - C \nabla \phi_S + \Omega. \tag{1}
\]

Equation (A1) expresses the balance of the fluid fraction $\phi_S$. The volumes of cell and growth factor phases

2. Materials and methods

The method was based on our original formulation combining mechanical conditions with computational cell biology (Ambard and Swider 2006). The initial description of the model was followed by the numerical experimentation.

2.1 Governing equations

The tissue volume unit shown in Figure 1 was considered to express the diffusive-convective-reactive Equation (1). The output measure $u$ was the structural fraction or mineralised fraction of newly formed tissue $\phi_S$, the interstitial fluid flux $q_f$, the osteoblast concentration $C_o$, and the anabolic growth factor concentration $C_g$. Coefficients $L$, $C$, $D$ and $\Omega$ were detailed in appendix (Equations (A1)–(A4)).

\[
L \frac{\partial \Omega}{\partial t} = D \Delta u - C \nabla \phi_S + \Omega. \tag{1}
\]

Equation (A1) expresses the balance of the fluid fraction $\phi_S$. The volumes of cell and growth factor phases
The synthesis of the structural fraction $f_s$ was modelled using the source term in Equation (A4). The synthesis was directed by $\alpha_s$ acting together with the fluid, the cells and the growth factors. The source was proportional to the cell and growth factor concentrations.

2.2 Parametric sensitivity analysis

2.2.1 Reference computational model

The implant described in Figure 2(a) is a polymethylmethacrylate (PMMA) stable canine implant that was previously evaluated in vivo (Vestermark et al. 2004). The distribution pattern of newly formed tissue was symmetric around the cylindrical implant. The distribution pattern of the structural fractions $f_s$ was investigated with four transverse histological slices as shown in Figure 2(b). The average result is shown in Figure 3. As shown in Figure 2, the region of interest was defined by three parameters: the implant radius $r_i$, the drill-hole radius $r_d$ and the host bone radius $r_b$.

The set of continuous governing Equations (1) was solved using a spatio-temporal finite difference scheme with a meshing of 0.02 mm in the radial direction $r$. The healing process was computed up to 8 weeks post-operatively according to in vivo experiments. All fluxes were zero at the implant surface (at radius $r_i$), and at the boundary of the zone of interest (radius $r_b$). Input data for this reference model are shown in Table 1. Geometric dimensions were based on the canine implant model and biochemical factors were estimated from literature values (Lauffenburger et al. 1983; Puleo et al. 1991; Soeber et al. 1992; Linkhart et al. 1996; Maheshwari and Lauffenburger 1998; Dee et al. 1999; Conover 2000; Roberts 2000; Bailón-Plaza and Van der Meulen 2001; Cowin 2001).

2.2.2 Numerical experimentation

The robustness of the predictive model was dependent upon several mechanobiological factors. We focused on the structural fraction distribution and we implemented a statistical experimental design (Box et al. 2005) involving four variables represented at two levels, high and low, denoted by (+) and (−), respectively. The clinical aspects were represented by varying the initial growth factor concentration $C_{g0}$ and the drill-hole radius $r_d$. Among the biological factors, we selected the chemotactic coefficient $x_0$ and the haptotactic coefficient $h_0$ which were critical because they related to cell active migrations. To calculate sensitivities, the initial conditions of distribution were always the same constant. Other initial conditions (diffusion and concentration) related to the cell population, and the phase of growth factors were kept unchanged for all computations.

First, the output measures $f_{sm}$, $f_{si}$ and $\Delta f_s$ were computed using the initial value of four variables and the response of the numerical model was noted $\tilde{u}$. High levels (+) and low levels (−) were used to successively compute the new responses $u$ noted. Second, the discrepancies between responses $u$ and $\tilde{u}$ were linearised using a

![Figure 2](image-url)

Figure 2. Canine experimental device. (a) In vivo implantation: the stable PMMA implant was fixed into the femoral epiphysis. (b) Histological slice showing the distribution pattern of structural fraction (Vestermark et al. 2004). In digitised histological slices, the summation of structural fraction pixels in successive concentric zones was divided by the pixel overall summation of the zone of interest to derive the radial distribution of structural fraction. Mean value was established on a base of four slices. $r_i$, implant radius; $r_d$, drill-hole radius; $r_b$, host bone radius.
Cells (osteoblasts) were measured between the implant and the host trabecular bone in the radial direction and y-axis represents the magnitude of \( \phi_i \) of implant radius; \( r_d \), drill-hole radius; \( r_b \), host bone radius.

polynomial interpolation in the form of Equation (2). Coefficients \( a_0 - a_3 \) described the order 1 direct effects of parameters variations on the output measures, whereas coefficients \( a_4 - a_8 \) expressed the order 2 combined effects.

\[
\begin{align*}
\tilde{u} - \hat{u} = a_0 r_d + a_1 C_g + a_2 \chi_0 + a_3 h_0 + a_4 r_d C_g \\
+ a_5 r_d \chi_0 + a_6 r_d h_0 + a_7 C_g \chi_0 + a_8 C_g h_0 \\
+ a_9 \chi_0 h_0.
\end{align*}
\]

The statistical significance of \( a_i \) was established using a 95% confidence interval from a bilateral Student test as expressed by Equation (3). In this equation, \( p \) is the number of coefficients \( a_i + 1 \) and \( s \) is the variance of \( a_i \). Parameter \( n \) represents the number of calculations equal to \( 2^4 \) because the procedure involved four parameters with two levels each. The coefficient 4.03 was obtained from the Student table (Box et al. 2005).

\[
\text{CI}_{0.95} = \left[ a_i - 4.03 \sqrt{s^2/(n-p)}; \ a_i + 4.03 \sqrt{s^2/(n-p)} \right]
\]

with \( i = 0, 9 \).

Our experience with the experimental canine implant model allowed the establishment of a reliable range of variation of drill-hole radius \( r_d \) to \( \pm 7\% \). In return, the discrepancies of the biochemical parameters were unknown and poorly documented. As a basis, we used data from the literature (Lauffenburger et al. 1983; Puleo et al. 1991; Søballe et al. 1992; Linkhart et al. 1996; Maheshwari and Lauffenburger 1998; Dee et al. 1999; Conover 2000; Roberts 2000; Bailón-Plaza and Van der Meulen 2001; Cowin 2001; Geris et al. 2008) to define plausible ranges of \( \pm 55\% \) for \( h_0 \), \( \pm 75\% \) for \( \chi_0 \) and \( \pm 25\% \) for \( C_g \).

Table 1. Model data of the stable PMMA canine implant (Lauffenburger et al. 1983; Puleo et al. 1991; Søballe et al. 1992; Linkhart et al. 1996; Maheshwari and Lauffenburger 1998; Dee et al. 1999; Conover 2000; Roberts 2000; Bailón-Plaza and Van der Meulen 2001; Cowin 2001).

<table>
<thead>
<tr>
<th>Constant data</th>
<th>Stat. exp. design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td></td>
</tr>
<tr>
<td>( n_i = 3.25 \text{ mm} )</td>
<td>( r_b = 7 \text{ mm} )</td>
</tr>
<tr>
<td>Cells (osteoblasts)</td>
<td>( a_0 = 1.9 \times 10^{-16} \text{ m}^3/\text{cell s} )</td>
</tr>
<tr>
<td>( n_g = 10^5 \text{ cell/mm}^3 )</td>
<td>( C_g = 10^3 \text{ cell/mm}^3 )</td>
</tr>
<tr>
<td>Growth factors</td>
<td>( D_g = 4.8 \times 10^{-6} \text{ mm}^2/\text{s} )</td>
</tr>
</tbody>
</table>

Note: The statistical experimental design involved four variables; \( r_g, h_o, \chi_0 \) and \( C_g \) represented at two levels, high and low, denoted by (+) and (−), respectively.
3. Results

The experimental and theoretical results of the reference model (Ambard and Swider 2006) are shown in Figure 3. The structural fraction increased from 50% in the host bone to 70% at the drill hole. The averaged fraction $\phi_{sm}$ was 60% and the heterogeneity index $\Delta \phi_s$ was 30%. At the implant surface, the structural fraction $\phi_{su}$ was 70%.

Bar diagrams in Figure 4 presented coefficients $a_i$ and associated confidence intervals for $\phi_{su}$ (Figure 4(a)), $\phi_{sm}$ (Figure 4(b)) and $\Delta \phi_s$ (Figure 4(c)). The polynomial Equation (2) was reduced to Equation (4) using the significant coefficients $a_i$. The algebraic sign of $a_i$ demonstrated whether the effect was favourable on the output measure $u$ (+); (−) signified a negative effect.

$$\phi_{su} - \tilde{\phi}_{su} = 10.9 \times r_d + 11.5 \times C_g + 11.9 \times \chi_o. \quad (a)$$
$$\phi_{sm} - \tilde{\phi}_{sm} = 7.8 \times r_d + 8 \times C_g + 6.5 \times \chi_o - 3.1$$
$$\times h_i + 1.2 \times r_d C_g + 1.8 \times C_g \chi_o. \quad (b)$$
$$\Delta \phi_s - \Delta \tilde{\phi}_s = 10.4 \times h_i. \quad (c)$$

Equation (4a) and Figure 4(a) show that three parameters had a predominant effect on the structural fraction $\phi_{su}$ at the implant surface: the drill hole, the growth factor concentration and the chemotactic migration. As expressed by Equation (4b) and plotted in Figure 4(b), the mean structural fraction $\phi_{sm}$ in the post-operative gap was influenced by the same parameters but it was noticed that haptotactic migration showed a negative contribution. At this point, two combined effects involving growth factor, drill hole and chemotactic migration were observed. Equation (4c) and Figure 4(c) show the predominant role of haptotactic migration on the heterogeneity index. We also observed that larger confidence intervals were obtained for the structural fraction at the implant surface $\phi_{su}$.

4. Discussion

The theoretical model based on reactive transport in porous media showed a good ability to represent the tissue pattern with a set of conditions of a stable PMMA implant (Vestermark et al. 2004). The fluid phase strongly interacted with the cell and growth factor phases. The fundamental mechanisms were modelled using diffusive properties combined to convection and source terms in which the role of active migrations was significant.

The initial conditions of the healing process were critical and it was challenging to mimic in vitro conditions because of variability of the experimental model and surgical technique. We assumed that the osteoblast concentration was initially the same for the post-operative gap and into the underlying host bone. To model the initial bleeding due to surgery, we assumed that growth factors were concentrated into the post-operative gap and their concentration into the host bone was comparatively negligible.

We found that the initial growth factors concentration was always favourable to the structural fraction synthesis into the post-operative gap and at the implant surface. The influence on the heterogeneity was not significant. Among anabolic growth factors, TGF-β1 is the major regulator of bone formation. It involves a mitogenic action on osteoblasts and it shows the ability to stimulate matrix formation via autocrine, paracrine and endocrine modes (Conover 2000; Roberts 2000). Substrate properties also modify the amount of growth factors produced by osteoblasts (Puleo et al. 1991; Ramanurti et al. 1997; Overgaard 2000; Pessková et al. 2007; Popat et al. 2007; Puthumanapully et al. 2008). Owing to a lack of reliable data in the literature, we generally overestimated the range of variation of the initial growth factor concentration. The resulting theoretical model generally showed the same tendencies for positive bone growth as those observed clinically.

We fixed the drill-hole size and its variation from our experience with the experimental canine implant. The model showed that the presence of an initial gap between the implant surface and the host bone allowed apposition of mineralised tissue with no significant influence on the heterogeneity. This result was obtained with initial growth factor concentrations and initial osteoblast concentrations, which were constant independent from the gap size. Therefore the amount of growth factors and cells increased with the drill-hole size. This constituted good conditions for the osteoblastic phase to find an available volume for proliferation and bone apposition.

Stable conditions with a non-critical gap were the initial conditions of the investigated implant. These conditions checked in vivo were taken into account in the theoretical model. Clinically, it is known that the larger the gap is, the more likely the implant is not well secured. Micromotion is unfavourable to the long-term survival of the implant fixation. Another issue is that critical gaps might delay the healing process to the detriment of primary stability. The process does not occur via an intramembranous ossification anymore but via an endochondral process, and our model finds a limitation. The surgical technique might modify the initial response of the host bone. Alteration of microvessels could modify nutrient sources and influence subsequent angiogenesis. Combination with mechanical stimuli needs to be investigated in further studies.

Material properties and surface modification of implants affect cell adhesion and proliferation, and growth factor production (Wakefield et al. 1990; Kieswetter et al. 1996; Anderson 2001; Kilpadi et al. 2001; Colnot et al. 2007; Popat et al. 2007; Rausch-Fan et al. 2008). It is
observed that sensitivity to surface topology is significant (Neidlinger-Wilke et al. 1994; Fermor et al. 1998; Pesskóva et al. 2007) and generally, osteoblastic proliferation decreases when surface roughness increases (Martin et al. 1995; Mustafa et al. 2001; Rosa and Beloti 2003).

Figure 4. Sensitivity analysis of structural fraction in the periprosthetic tissue to four variables: the drill-hole radius $r_d$, the growth factor concentration $C_g$, the chemotactic coefficient $x_0$, and the haptotactic coefficient $h_0$. Bars represent the magnitude of polynomial coefficients $a_i$ of Equation (2) and their confidence intervals. (a) Structural fraction $f_{si}$ at the implant surface (Equation (4a)), (b) mean structural fraction $f_{sm}$ (Equation (4b)), (c) heterogeneity index $\Delta f_h$ (Equation (4c)).
We previously proposed a preliminary model of interactions with bioactive surfaces (Guérin et al. 2009). Osteoblast adhesion properties and growth factor source were locally updated to model biochemical interactions of implant with its environment. We found that the decrease in cell diffusion significantly improved the amount of mineralised tissue on the implant surface. However, the model also confirmed that implant bioactive properties should play a limited role to reduce heterogeneity of newly formed tissue.

The parameters associated with active migrations are poorly documented in the literature, especially in an in vivo setting, yet their roles are of prime importance in the healing process. Our model considered cells as a continuous phase and it allowed active migrations to be involved in convective terms and source terms. To determine the tendencies, we created ranges for biochemical factors based on literature review. We varied the haptotactic coefficient by $\pm 75\%$ and varied the chemotactic coefficient by $\pm 25\%$.

We observed that the haptotactic coefficient reduced apposition of structural fraction in the newly formed tissue, and its influence on the heterogeneity index was predominant. To potentially explain this result, we observed that the greater porosity gradient was initially located at the drill hole. At the drill-hole edge, the cells migrating from the host bone towards the implant could find favourable adhesion sites to proliferate and synthesise mineral matrix. The cells initially present in the post-operative gap could be attracted towards the drill hole also because of adhesion sites. Finally, the decrease in local porosity because of mineralised tissue apposition limited the access to new cells and favoured heterogeneity of the healing.

We found that the chemotactic migration increased the structural fraction between the drill hole and the implant surface, whereas no significant influence on the heterogeneity was detected. We assumed as an initial condition that growth factors were only concentrated into the initial gap. Growth factor concentration was significantly greater in the gap compared to surrounding host bone because of initial bleeding due to surgery. It was assumed that there were initially no growth factors in the host bone and this induced a gradient between these two zones. Even if they diffused and were transported by fluid flux, the algebraic sign of their concentration gradient was unchanged during the healing process and it was always oriented from the host bone towards the implant. This gradient attracted cells located in the host bone whereas those initially present in the post-operative gap were not recruited.

Finally, the combination of haptotactic flux and chemotactic flux strongly influenced the distribution pattern of structural fraction apposition varying from the implant surface towards the host bone.

In conclusion, we could note that the theoretical results observed clinically (Søballe et al. 1992; Vestermark et al. 2004; Franchi et al. 2005). A sensitivity analysis helped to objectively identify the relative dependence of implant healing on several mechanobiological parameters.

To implement a statistical experimental design convenient in interpretation, we initially limited the number of variable parameters to four. However, we plan to add three variables to raise the clinical relevance. The osteoblast initial concentration and the initial porosity of the host bone could be involved to evaluate the role of bone pathologies such as osteoporosis on implant fixation. We also plan to study surgical technique by modifying the drill-hole profile irregularities and size.

Our study also showed that the investigation of growth factor mode of production could be of particular interest. A preliminary model of local interactions with the implant bioactive surface of implant has been proposed (Guérin et al. 2009) and this will be completed by relevant source terms involving the post-operative neovascularisation.

Acknowledgements

The French Minister of Education and Research is acknowledged for its assistance. Related experimental studies were conducted with the support of NIH AR4205, Institute of Pathology at Aarhus University Hospital, Orthopaedic Research and Education Foundation, Danish Research Foundation for Health, Master of a guild, butcher Peter Ryholts Foundation and the Danish Rheumatism Association, and Anna and Jakob Jakobsens Foundation. Biomet donated the PMMA implants.

References


Lauffenburger D, Rothman C, Zigmund SH. 1983. Measurement of leukocyte and chemotaxis parameters with a linear under


Appendix 1

The local variation $L$, the convective term $C$, the diffusive term $D$ and the source term $\Omega$ of governing Equation (1) were expressed by Equations (A1)–(A4). Output measure $u$ was the extracellular fluid flux $q_f$, the osteoblastic concentration $C_o$, the growth factor concentration $C_g$ and the structural fraction $\phi_s$.

<table>
<thead>
<tr>
<th>$u$</th>
<th>$L$</th>
<th>$C$</th>
<th>$D$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_f$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{\partial \phi_s}{\partial t}$ (A1)</td>
</tr>
<tr>
<td>$C_o$</td>
<td>$\phi_t$</td>
<td>$\phi_t \chi_o \frac{\partial C_g}{\partial x} - \frac{\partial \phi_t}{\partial x}(D_o + \rho \phi_t \phi_t)$</td>
<td>$D_o \phi_t$</td>
<td>$C_o \left[ -\frac{\partial \phi_t}{\partial t} + h_o \rho \left( \frac{\partial^2 \phi_t}{\partial x^2} + \left( \frac{\partial \phi_t}{\partial x} \right)^2 \right) - \chi_o \left[ \phi_t \frac{\partial^2 C_g}{\partial x^2} + \frac{\partial C_g}{\partial x} \frac{\partial \phi_t}{\partial x} \right] + \alpha_o \phi_t (n_o - \phi_t C_o) \right]$ (A2)</td>
</tr>
<tr>
<td>$C_g$</td>
<td>$\phi_t$</td>
<td>$-\left( q_f + D_g \frac{\partial \phi_t}{\partial x} \right)$</td>
<td>$D_g \phi_t$</td>
<td>0 (A3)</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$\alpha_s C_g C_o \phi_t^2$ (A4)</td>
</tr>
</tbody>
</table>