Multilevel assessment method for reliable design of composite structures

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ABSTRACT
This work aims at demonstrating the interest of a new methodology for the design and optimization of composite materials and structures. Coupling reliability methods and homogenization techniques allow the consideration of probabilistic design variables at different scales. The main advantage of such an original micromechanics-based approach is to extend the scope of solutions for engineering composite materials to reach or to respect a given reliability level. This approach is illustrated on a civil engineering case including reinforced fiber composites. Modifications of microstructural components properties, manufacturing process, and geometry are investigated to provide new alternatives for design and guidelines for quality control.

1. Introduction
Design and optimization of composite structures generally suffer from the variability of several parameters (constituent properties, microstructural morphology, structural geometry, manufacturing process, loading conditions). In the context of deterministic approach, such a weak point is taken into account by the introduction of important safety factors. Associated reduction of mechanical properties- above all strength-considered for calculations causes outsized geometries and excessive costs. For a few years now, reliability-based analyses that model the random character of design properties have led to a new consideration of the risk. Reliability indicators derived by these methods (such as the failure probability) provide a clear representation of the influence of uncertainties [1].

For composites, this significantly contributes to the development of materials and structures, regarding both design solutions and maintenance programs (see reviews of Chiachio et al. [2] and Sobey et al. [3]). Most existing approaches developed in this way aim at determining the reliability level of structures according to the Joad and macroscopic parameters, such as geometry and material strength. This helps, for instance, to determine most probable failure modes, identify driving parameters of failure mechanisms, estimate safety margins, compare materials performances, or propose new design solutions (for instance (3-7)).

Associating micromechanics to the reliability framework clearly enhances such representation. Material constitutive laws derived from homogenization techniques account for the physical mechanisms involved at the microstructural scale (8-10). Introducing micromechanical arguments within probabilistic approach leads then to an enriched modeling of variabilities inherent to composites. On the one hand, the ability to represent uncertainties on microscopic, macroscopic, and structural features could explain the consequences of variability of composites from small to larger scales. Regarding that application field, micromechanical schemes are often examined and compared to find the most suitable prediction of homogenized equivalent properties of these heterogeneous materials/structures [8, 11-13] or used to incorporate microscale variables and capture their effects on the reliability of structures [14-16]. On the other hand, the description of the physical origin of the material failure deeply justifies the reliability analysis and provides a robust assessment of structures reliability. Indeed, the lack of knowledge about the physics of the damaging and failure process may become a critical issue and induce some major inconsistencies in the reliability estimation, especially when considering large scale structures [2, 17].

The present work intends to demonstrate the interest of the association of micromechanics and reliability methods regarding engineering design and optimization. Based on a case study in civil engineering, new tools for reliability-based design are presented. Special attention is given to available alternatives to achieve a reliability constraint and to guidelines derived from the analysis to improve engineering choices or quality control.

Initiated in the case of material engineering to develop new solutions for composite materials [13, 15, 8], such methodology is extended here at a structural scale. After a general recall of the methodology and case study, we present subsequently the design interest for a multiscale optimization. Based on the integration of uncertainties at micro, macro, and structure scales, the results and discussion focus on new design solutions provided by the coupled approach.

2. General framework
Theoretical developments and implementation steps of the coupled approach suggested by the authors are detailed in [13, 15]. For clearness, we briefly recall here the major milestones of this type of study.

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The first key point of the association of micromechanics and reliability methods stands in the selection of random variables \( X = \{ X \} \). They should include all variable data of the problem, and possibly as much as possible of design parameters. For composites, microstructural features play an important role and should then be considered. The limitations at this stage concern the availability of the statistical distribution of random variables and the size of the problem (number \( N \)) to avoid prohibitive calculation time. Significant random parameters of the considered problem may be identified by means of sensitivity analyses [1]. In this work, random variables \( X \) taken into account concern several scales of the problem (from microstructural to macrostructural data) and different aspects (mechanical properties, manufacturing parameters, load).

The second step deals with the definition of the mathematical function \( G \) representing the failure scenario regarding either strength achievement or serviceability (failure domain is defined by \( DT = \{ X, G(X) : S(O) \} \)). In order to provide a sound basis for results, this requires a relevant representation of the mechanical behavior of the structure and a physical definition of the limit state. For both of these issues, the association with micromechanics is a clear advantage: on the one hand, homogenization techniques derive the effective behavior of a representative volume element (RVE) of a heterogeneous material from the knowledge of its microstructural characteristics; on the other hand, they provide a local load sustained by constituents for a given macroscopic load.

The final stage lies in the calculation of probabilistic indicators, such as the probability of failure:

\[
P_f = \text{Prob}(G(X) : S(O)) = \int_{x} f_{X}(x) D_{x},
\]

with \( f_{X} \) the joint probability density function. When \( f_{X} \) cannot be directly expressed and/or for problems dealing with a large number of design variables, it is much more efficient to use numerical methods, especially if the coupling with finite-element simulations is required. Among them, approximation methods FORM/SORM (first-order and second-order reliability methods) constitute very interesting solutions. Working in the standard normal space, these methods compute by optimization algorithm the design point \( P^* \) of the failure domain that exhibits the highest failure probability. The distance between the design point and the standard space origin is called the reliability index \( \phi \).

In what follows, usual intrinsic notations are employed. The tensor products of two second-order tensors \( a \) and \( b \) are defined by:

\[
[a \otimes b] : x = (b : x)a, \quad [a \otimes b] : x = a \cdot x \cdot b,
\]

\[
(a \otimes b) : x = a \otimes x \cdot b, \quad a \otimes b = (a \otimes b) + (a \otimes b)
\]

for any second-order tensor \( x \), the term \( a \otimes i = a \otimes a. \otimes a \) represents the \( i \)th tensor product of a tensor \( a. \) \(

3. Application case: The Laroin footbridge

The case study considered in this article is a civil engineering structure that includes carbon fiber reinforced polymers. The innovative pedestrian footbridge of Laroin (France, 2002) is composed of a 110-m-length steel deck held by composite stay cables. At each sicle of the footbridge, eight stay cables and one steel stending cable are joined to a steel reversed V-shaped pylon of 20.60-m height. Each stay cable includes two or three strands of seven unidirectional cylindrical composite rods (Figure 1) [19].

So as to demonstrate the interest of the approach for the design of composite materials and structures, two types of reliability investigations are presented in what follows:

- The first one called ‘Material’, initiated in [18], that focuses on the elementary composite rod;
- The second one called ‘Structure,’ which integrates the entire footbridge.

3.1. Material study

Based on high-strength carbon fibers (Torayca T700SC-12K) and epoxy resin (Bostik Findley Eponal 401), composite rods have been manufactured by pultrusion by Toray Carbon Fibers Europe company (mean fiber volume fraction \( \bar{f} \) of 67%). Inside the cables, each cylindrical rod (with diameter \( \varphi \)) is subjected to uniaxial tensile load (denoted \( F \)).

3.1.1. Mechanical mode

The cylindrical rod corresponds to the RVE of the composite material. Its behavior is described by means of a micromechanical approach, that provides both:

- the macroscopic properties (effective behavior) from the components properties and morphology; and
- the local solicitations from the corresponding macroscopic quantities on the RVE.

In the present context, the formulation of Mori and Tanaka [20] constitutes a well-adapted constitutive framework to derive these expressions. Matrix medium corresponds here to the epoxy resin, fibers are modeled as infinite cylindrical inclusions, and constituents are assumed to be isotropic (demonstration details are given in [13]). Using that scheme, the effective stiffness tensor \( C_{eff} \) that links the macroscopic stress \( a \) and strain \( \varepsilon \)
on the RVE ($\mathbf{a} = \mathbf{Cell} \cdot x$) is thus given by:
\[
\mathbf{C}_{eff} = \left[ \mathbf{S} + f_j / (S_\text{j} - S_\text{r}) : (\mathbf{A})_f \right]^{-1},
\]
with $\mathbf{S}$ the elastic compliances of the epoxy resin ($i = r$) and fiber ($i = f$):
\[
\mathbf{S} = \frac{1 + \nu;}{E;} \mathbf{1} \otimes \mathbf{1} - \frac{\nu;}{E;} \mathbf{1} \otimes \mathbf{1}.
\]

$E; \text{ and } \nu; \text{ represent the Young modulus and Poisson ratio of constituents. On the other hand, the stress concentration tensor } (\mathbf{A})_f \text{ of Eq. (3) relates the average local stress over the fiber phase } (\mathbf{a})_f \text{ and macroscopic stress:}
\[
(\mathbf{a})_f = (\mathbf{B})_f : \left[ (1 - f_j) \mathbf{I} \otimes \mathbf{I} + f_j (\mathbf{B})_f \right]^{-1},
\]
with
\[
(\mathbf{B})_f = \mathbf{S}^{-1}_f : \left[ \mathbf{1} \otimes \mathbf{I} + f_j \mathbf{S} : (\mathbf{S}^{-1}_j - \mathbf{S}^{-1}_r) \right]^{-1}.
\]

The Eshelby tensor $\mathbf{S}$ of infinite cylindrical fiber inclusions within the resin is provided by (see [21] for instance):
\[
\mathbf{S} = \frac{4}\nu; \left( 1 - \nu; \right) \mathbf{O} \left( 1 - \nu; \right)
\]
\[
+ \frac{3 - 4\nu;}{4(1 - \nu;)} \left( 1 - \nu; \right) \mathbf{O} \left( 1 - \nu; \right)
\]
\[
- \nu; \left( 1 - \nu; \right) \mathbf{O} \left( 1 - \nu; \right)
\]
\[
+ \frac{\nu;}{2(1 - \nu;)} \left( 1 - \nu; \right) \mathbf{O} \left( 1 - \nu; \right).
\]

Due to several manufacturing defects (fiber misalignment, impregnation defects, etc.), it appears that only a part $\text{Pact} : S$ of the fiber fraction is involved in the mechanical response of the composite. Accordingly, the fiber volume fraction $f_j$ appearing in the Mori-Tanaka scheme (Eqs. 3 and 6) has been adjusted to account for the manufacturing process efficiency:
\[
f_j \rightarrow f_j = \text{Pact} \times f_t.
\]

Identification of the quality parameter $\text{Pact}$ has been done on the pultrusion production line of Toray Carbon Fibers Europe [13].

### 3.1.2. Random variables

Among all design parameters entering the mechanical model, only those whose variability clearly affects the composite material and footbridge reliability have been defined as random variables. It has been demonstrated that some properties or aspects can be considered as deterministic (for instance, components elastic properties) or neglected (for instance, matrix porosity, fiber elliptical shape) without modifying significantly the reliability estimation [13]. For the Material study, one has thus considered the following random variables $X = \{ X \} : \text{M, S}$ (Table 1):

<table>
<thead>
<tr>
<th>Study</th>
<th>Random variables</th>
<th>$X_i$</th>
<th>$\sigma_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>Fiber yield strength (MPa)</td>
<td>$\alpha_f (\mu)$</td>
<td>6870</td>
</tr>
<tr>
<td>MS</td>
<td>Fibre volume fraction</td>
<td>$\gamma$</td>
<td>67</td>
</tr>
<tr>
<td>MS</td>
<td>Active fiber fraction (%)</td>
<td>$\eta$</td>
<td>95</td>
</tr>
<tr>
<td>MS</td>
<td>Rod diameter (mm)</td>
<td>$\phi$</td>
<td>6</td>
</tr>
<tr>
<td>M</td>
<td>Axial load (kN)</td>
<td>$\beta$</td>
<td>76</td>
</tr>
<tr>
<td>S</td>
<td>Weight (kN/m)</td>
<td>$\omega$</td>
<td>8650</td>
</tr>
<tr>
<td>S</td>
<td>Exploitation (kN/m)</td>
<td>$\lambda$</td>
<td>8076</td>
</tr>
</tbody>
</table>

- Components mechanical properties: fiber axial yield strength $\alpha_f (\mu)$;
- Manufacturing process parameters: fiber volume fraction $f_j$, upper part of Figure 2).
- Load conditions: uniaxial tensile load $\mathbf{F}$.

### 3.1.3. Failure criterion

The reliability assessment is defined as the mechanical failure of composite rods under tensile load. The considered limit state function accounts for the microstructural origin of the failure, namely, the fiber failure. The failure domain is thus assumed to be reached when the average principal value of the average local stress over the fiber phase $\langle \mathbf{a} \rangle_j$ exceeds the fiber axial failure strength $\langle \mathbf{a} \rangle (\mu)$:
\[
G = \sigma;_j (\mathbf{a})_j - (\mathbf{a})_j.
\]

In Eq. (10), $\langle \mathbf{a} \rangle_j$ is deduced from the micromechanical model! (Eq. 5) and $\langle \mathbf{a} \rangle (\mu)$ is a material data provided by manufacturers.

### 3.1.4. Simulation procedure

An explicit coupling between the probabilistic code FERUM (Finite element reliability using Matlab) and the micromechanical model has allowed the calculation of reliability indicators (upper part of Figure 2). Precisely, we have chosen to use the FORM approximation method, mainly for its computational efficiency. In that case, the failure probability $P_f$ can be directly computed from the reliability index $\beta_3$. Such a procedure requires the estimation of failure function $G$ for several sets of random variables. At first, FERUM thus generates realizations of random variables $X = \{ X \}_{\text{M, S}}$ according to their individual distribution law and distribution parameters. From the axial load $\mathbf{F}$ and rod diameter $\phi$, the macroscopic stress is given by $\sigma_\text{maj} : \mathbf{S}$, while fiber volume fraction $f_j$ and active fiber fraction $\mathbf{P}_{\text{act}}$ provide the stress concentration tensor $(\mathbf{A})_f$ through Eq. (6) and then the average local stress over the fiber phase $\langle \mathbf{a} \rangle_j$ through Eq. (5). Finally, comparison of the latter with $\langle \mathbf{a} \rangle (\mu)$ allows estimating criterion (10).

### 3.2. Structure study

Calculation of the footbridge is based on European Civil Engineering Standards [19, 22]. Structural loads applied are the own weight of the structure (denoted $W$), the variable exploitation load related to the pedestrian's passage (denoted $E$), and the cables adjustment forces. Regarding the mechanical modeling,
3.2. Random variables

It has been demonstrated that the scatter in cable adjustment forces has a negligible effect on the probability of failure [15]. Regarding the Structure problem, six random variables $X = \{X; J\}$ have thus been considered (Table 1):

- Components mechanical properties: fiber axial yield strength $a(n)$;
- Manufacturing process parameters: fiber volume fraction $f_j$, active fiber fraction $P_{act}$, rod diameter $d$;
- Load conditions: weight of the structure $W$, exploitation load $E$.

3.2.2. Failure criterion

It is assumed that all steel components of the footbridge (deck, reversed V-pylons, standing cables) exhibit an elastic behavior with high strength. While keeping the same failure function $G$ used for the Material study, failure mechanism of the footbridge is defined as the failure of the most loaded stay cables. The latter are cables of greater length (based on three strands, see Figure 1) when they are submitted to the most critical loading configuration. Several finite-element calculations with ABAQUS code have shown that such a situation is induced when the variable exploitation load $E$ is applied in a centered manner on 60% of the bridge span [19].

3.2.3. Simulation procedure

Such multiscale analysis has been performed by means of an additional coupling between finite element code ABAQUS and previous micromechanics-probabilistic coupling (Figure 2). Indeed, reliability assessment of the footbridge requires several interactions between the micro, macro, and structural scales:

- Homogenization steps to derive the effective mechanical behavior of the composite rod $C_{eff}$ and the stress concentration tensor $(A)$ from microstructural parameters $\mathbf{U}_J$ and $P_{act}$ through Eqs. (3) and (6);
- Finite-element calculations to determine the macroscopic axial tension stress $a$ applied to the rod according to the geometry of the rod $(/<J>$, the composite rod effective behavior $C_{eff}$ and the structural load applied to the footbridge $(W$ and $E)$;
- Localization steps to derive the average local stress over the fiber phase $(a)$ through Eq. (5) from the macroscopic stress $a$ applied to the rod.

As for the Material study, comparison of $(a)$ with $a(n)$ allows finally to estimate criterion (10).

4. Micromechanical-based reliability analysis

4.1. Reference point

Deterministic and statistical data of the problem were provided by manufacturers of composites constituents (Toray, Bostik), composite materials (Toray Carbon Fibers Europe), and structural footbridge (Freyssinet).

Elastic properties of matrix and fiber were following: $E_r = 2800$ MPa, $\nu_r = 0.4$, $E_f = 230,000$ MPa, $\nu_f = 0.3$. Random variables follow a normal law for which mean value and standard deviation are respectively denoted $X$; and $S_X$. Ali distribution parameters are detailed in Table 1 and will be considered as the reference point in what follows. Without specific data, we assume also a normal distribution for the loads. Regarding the Material investigation, the study is centered around the operating point $F = 76$ kN that corresponds to a security level approved in civil engineering structures ($f > 3$ or, equivalently, failure probability $P_f$ greater than $10^{-3}$). Precisely, one obtains in this case $f_r = 3.0514$. For the Structure study, data related to the own weight $W$ and exploitation load $E$ are fixed, respectively, by
the geometry and materials of the footbridge and by European standards (19, 22). This leads to a value $\mu_{\Delta} = 18.90$ far above the previous case, since it includes a safety margin to account for anchorage effects and durability aspects.

From the sensitivity analysis provided by the FORM method, one could get some information on the origin of the scatter in the material response and about the respective influence of each design parameter. Indeed, FORM procedure allows obtaining elasticities, that is derivatives of $\theta$ according to distribution parameters, such as mean value $(\mu)$ or standard deviation $(\sigma_f, \sigma)$ at the design point $P*$. As said before, it helps above

![Image](image-url)

**Figure 3.** Evolution of reliability index $\beta$ around the reference point with respect to the mean value $\mu_X$ of each random variable $X$.  

**Table 2.** Scope of the distribution parameters of random variables around the reference point.

<table>
<thead>
<tr>
<th>Random variables</th>
<th>$X_i$</th>
<th>$\sigma_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber yield strength (MPa)</td>
<td>$f_y$</td>
<td>(4384, 5356)±0.2435</td>
</tr>
<tr>
<td>Fiber volume fraction(%)</td>
<td>$V_f$</td>
<td>[64.70]±0.05</td>
</tr>
<tr>
<td>Active fiber fraction( %)</td>
<td>$P_{act}$</td>
<td>[90, 100]±0.1</td>
</tr>
<tr>
<td>Rod diameter (mm)</td>
<td>$d_r$</td>
<td>[5.8, 6.2]±0.005</td>
</tr>
<tr>
<td>Axial load (kN)</td>
<td>$F$</td>
<td>[64.6, 87.4]±0.3</td>
</tr>
<tr>
<td>Weight (kN/m)</td>
<td>$w$</td>
<td>(9.052, 12.247)±0.532</td>
</tr>
<tr>
<td>Exploitation (kN/m)</td>
<td>$E$</td>
<td>(8.865, 9.288)±0.404</td>
</tr>
</tbody>
</table>

±10%; ±5%; ±5.2%; ±3%; ±15% of $\mu_X$; ±15% of $\mu_f$; ±15% of $\mu_W$; ±15% of $\mu_E$.  

![Image](image-url)
all to identify variables of the problem whose deviation has a weak influence on the reliability and that can be considered as deterministic (see details in [13, 15, 18]). Yet, such analysis is not a practical tool for engineers to bring out the consequences of design choices. We intend in what follows to illustrate an alternative representation for reliability assessment, which highlights specific advantages of the micromechanics-based approach.

4.2. Investigation of new solutions

New design solutions are investigated by means of evolutions around the basic design of the reference point. Acceptable solutions should respect the materials and manufacturing machines capabilities and the model assumptions. Table 2 illustrates the scope of the distribution parameters of variables considered in the present study (range ratio relative to the reference point is indicated for clearness). Uniform range has been assumed for the standard deviation (note that $S_x = 0$ corresponds to the deterministic case).

Reliability analyses of several configurations have been performed. Evolution according to each distribution parameter of each random variable is considered independently, other parameters being fixed to the reference point. On the numerical point of view, explicit formulation of the Mori-Tanaka scheme and the small number of variables lead to a reasonable number of calls to function $G$ for the estimation of the reliability index $R$: around 50 calls to $G$ for the Material study and between 200 and 1000 for the Structure analysis. In this latter case, the more

Figure 4: Evolution of reliability index $R$ around the reference point with respect to the standard deviation $S_x/S_r$ of each random variable $X_i$. 
important number of calls is related to the very weak failure probability.

5. Results and discussion

The evolution of index $\beta$ according to the mean value of each random variable is depicted on Figure 3. An increase (respectively decrease) of $\beta$ with $X_i$ is obtained for strength variables (resp. loading variables). For both the Material and Structure studies, we obtain a quasi-linear evolution of $\beta$ with respect to mean values $\bar{X}_i$ of random variables. A similar evolution can be observed for $U_J$ and $P_{act}$ variables, which stands in agreement with the model adjustment defined in Eq. (9). The most influential design parameters (corresponding to the steepest slopes of $B$-curves) differ according to the context: the most significant influence is obtained for $\sigma_f$ at the composite rod scale (Figure 3a) and for $\bar{a}_J(n)$ at the structure scale (Figure 3b).

For designers, such result provides guidelines to reach or to respect a reliability level. Just focusing on the composite material (Figure 3a), the reliability index $\beta' = 3.0514$ at the reference point can be increased of 15% (that is to, $\beta = 3.5$) either by an increase of 2% of the mean fiber yield strength (that is with $\bar{a}_J(n) = 4970$ MPa) or with an increase of 1% of the mean rod diameter (that is with $<i> = 6.07$ mm). Regarding now the footbridge reliability (Figure 3b), the same increase of 2% in $\bar{a}_J(n)$ only leads to an increase of 3% of index $\beta$ (that is up to $\beta = 19.50$) compared to $\beta' = 18.90$ at the reference point.

Figure 5. Material study-Cross-analyses of the evolution of reliability index $U_J$ with respect to the mean values of couple of parameters.
Impact of the mean rod diameter is even lower with an increase of 1.3% of $f_3$ for the same increase of 1% of $f_l$.

Such analysis provides also the safety margin related to loading variables. For instance, we note that an increase of 6% of the mean macroscopic load on the rod (that is with $F = 80.6$ kN) leads to a decrease of 40% of index $f_3$ (leading to a remaining value of $f_3 \approx 0.2$). On the other hand, the sensitivity to the mean value of $f_l$ is again lower than that to the mean value of $f_3$ and $E$. If the micromechanics-based reliability approach allows to account for the influence of multiscale parameters, this shows that the modeling of the composite material inside its structural application modifies the amplitude of variation of $f_3$ and, accordingly, the design recommendations.

Considering now the effect of the standard deviation, the influence on the reliability of the uncertainties on variables is given on Figure 4. Obviously, more (resp. less) the variables tend to be reproducible, more reliability increases (resp. decreases). In agreement with the sensitivity analysis presented in [18], the influence of the scatter on manufacturing process parameters $C_l$, $P_{act}$ and $H_l$ can be neglected. Optimization of the structure is here mainly based on the reproducibility of the fiber yield strength in which the $f$-curves exhibit a polynomial dependence. For the same mean value as the reference point, a highly reproducible $\sigma f$ (n) (with $S_f$: 0) can increase the reliability index of 44% for the Material study (Figure 4a) and of 220% for the Structural analysis (Figure 4b).

For such an application, this result confirms then the main importance of the fiber strength in the composite performance, both regarding its mean value and deviation. Also, the variability of the macroscopic load $F$ plays, as before, an important role at the material scale (Figure 4a) but influence of the scatter of structural loads $W$ and $E$ can be neglected for the footbridge (Figure 4b). This last result is in line with safety conditions since the control over loads is quite difficult in such civil engineering cases.

Cross-analyses between random variables provide also extended design solutions, including combined change on parameters at different scales. Figure 5 illustrates the combined investigation of mean manufacturing process parameters (namely the fiber content and the active fiber fraction) and mean material parameters (namely the fiber yield strength) for the Material study. As shown on Figure 5a, the reliability index $f_3$ can be improved of 18% either:
- by increasing the fiber content (+2% with $j_f = 68.8\%$) while keeping the current manufacturing process,
- or by keeping the fiber content and improving the manufacturing process (+2.3% of the mechanically active fiber such that $P_{act} = 97.2\%$).

Including now material features (Figure 5b), same reliability improvement can be obtained either:
- by using fibers with higher strength (+6.4% with $\sigma f(n) = 8180$ MPa) and a reduced fiber content (-4.8% with $f_l = 64\%$),
- or with an higher fiber content (+4.5% with $f_f = 70\%$) and lower strength fiber (-2% with $\sigma f(n) = 4770$ MPa).

Obviously, such a kind of cross-analysis can be done also on standard deviations of variables or even between mean value and deviation of different parameters according to the needs and possible actions of designers. An example on the Structure case is presented on Figure 6 with the cross-influences of the mean value of a manufacturing process (the rod diameter $d$) and of the standard deviation of a material parameter (the fiber yield strength). For such an application, one can demonstrate that the main quality effort should be put on the reproducibility of the fiber yield strength whatever the geometry of the rod.

6. Conclusion

Modeling the variability of design parameters is now absolutely required to develop the use of composite materials. For these materials mainly governed by their microstructure, the association of reliability methods with micromechanics allows to address such issues in a relevant way. Based on two well-established frameworks, the coupled approach extends the scope of design variables from the micro to the structural scale and gives a physical justification to the reliability criterion. This article has illustrated some innovative tools for design and optimization provided by this method. The ability to investigate the influence on the reliability level of several features at different scales, eventually in a combined way, provides an enriched and efficient assessment method for structures. Modifications of components and morphology of materials, manufacturing process, and geometry can be considered and investigated, either for material optimization or structure optimization. Moreover, results have shown the importance of the scale effect. Indeed, reliability analysis should rely on the integration of the composite part within the whole structure to get a clear view of the consequences of design choices. Association of such structural reliability analysis with economic issues (including the prices of materials and manufacturing, see [23] for instance) will provide at the end a sound basis to fully guide optimization.

References


