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Towards a New Framework for Recursive Interactions in Abstract Bipolar Argumentation

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Abstract. This paper proposes a new framework able to take into account recursive interactions in bipolar abstract argumentation systems. We address issues such as “How an interaction can impact another one?”, or in other words “How can the validity of an interaction be affected if this interaction is attacked or supported by another one?”. Thus, building on numerous examples, a new method for flattening such recursive bipolar abstract argumentation systems (ASAF) using meta-arguments is proposed and compared with the original framework defined in \cite{8}.

Keywords. Abstract argumentation, bipolar argumentation, recursive interactions.

1. Introduction

Argumentation has become an essential paradigm especially for reasoning from contradictory information \cite{9,1}, and for formalizing the exchange of arguments between agents in, e.g., negotiation \cite{2}. Formal abstract frameworks have greatly eased the modelling and study of argumentation. For instance, a Dung’s argumentation system (AS) \cite{9} consists of a collection of arguments interacting with each other through an \textit{attack} relation, enabling to determine “acceptable” sets of arguments called \textit{extensions}.

In the last decade, extensions of Dung’s AS were proposed for including a positive interaction between arguments, called \textit{support}. The support relation has been first introduced in \cite{10,17}. In \cite{4}, the support relation is left general so that the obtained bipolar AS (BAS) keeps a high level of abstraction. However there is no single interpretation of support, and a number of researchers proposed specialized variants of the support relation (deductive support \cite{18}, necessary support \cite{13}, evidential support \cite{14}). Each specialization was developed quite independently, based on different intuitions and provided with an appropriate formalization. In order to restate those proposals in a common setting, \cite{6} proposed a comparative study using the BAS. Following the same line, recent works have been proposed that enforce the important role of necessary support (see \textit{e.g.}, \cite{15,16,7,12}). Another line of work extending Dung’s AS regards high-order attacks: attacks to the attack relation \cite{11,3} and attacks to attacks and supports \cite{18}. More generally, \cite{8} proposes an \textit{Attack-Support Argumentation Framework} (ASAF) which allows for attack and support to the attack and support relations, at any level.

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The authors in [8] encode an ASAF by turning it into a BAS with necessary support, and then into an AS by adding extended attacks. As [7] presents different frameworks for encoding necessary support, it is interesting to enrich them with recursive interactions. So, in this paper we propose to translate the ASAF into a special AS using meta-arguments\(^2\) (called MAS), and compare this MAS with the AS obtained in [8]. Note that our aim is not to replace the ASAF with the MAS; although recursive interactions can be modelled using the MAS, the ASAF is a more intuitive and visual representation tool.

This paper is organized as follows: BAS (with necessary support) and its axiomatization are presented in Sect. 2. Background on the ASAF is given in Sect. 3. In Sect. 4 we extend the MAS proposed in [7] to model recursive interactions. Then, Sect. 5 compares this MAS with the ASAF. Finally, Sect. 6 concludes and suggests lines of future work.

2. Bipolar abstract argumentation system


Def. 1 (BAS) A bipolar argumentation system (BAS) is a tuple \( \langle A, R_{\text{att}}, R_{\text{sup}} \rangle \), where \( A \) is a finite and non-empty set of arguments, \( R_{\text{att}} \subseteq A \times A \) is an attack relation and \( R_{\text{sup}} \subseteq A \times A \) is a support relation.

A BAS can be represented by a directed graph with two kinds of edges: \( \forall a, b \in A, aR_{\text{att}}b \) (resp. \( aR_{\text{sup}}b \)) is represented by \( a \rightarrow b \) (resp. by \( a \Rightarrow b \)). Semantics introduced by Dung for AS can only be used if \( R_{\text{sup}} = \emptyset \). They characterize sets of arguments that satisfy some properties and some form of optimality. For instance:\(^3\)

Def. 2 (Preferred extensions of AS) Let \( AS = \langle A, R_{\text{att}} \rangle \) and \( S \subseteq A \). \( S \) is conflict-free iff \( \not\exists a, b \in S, \text{s.t.} aR_{\text{att}}b \). \( a \in A \) is acceptable wrt \( S \) iff \( \forall b \in A \text{ s.t. } bR_{\text{att}}a, \exists c \in S \text{ s.t. } cR_{\text{att}}b \). \( S \) is admissible iff it is conflict-free and \( \forall b \in S, b \) is acceptable wrt \( S \). \( S \) is a preferred extension of \( AS \) iff it is a maximal (wrt \( \subseteq \)) admissible set.

Handling support and attack at an abstract level has the advantage to keep genericity and to give an analytic tool for studying complex attacks and new semantics considering both attack and support relations, among others. However, the drawback is the lack of guidelines for choosing the appropriate definitions and semantics depending on the application. For solving this problem, some variants of the support relation have been proposed recently, including the necessary support. This kind of support was initially proposed in [13] with the following interpretation: If \( cR_{\text{sup}}b \) then the acceptance of \( c \) is necessary to get the acceptance of \( b \), or equivalently the acceptance of \( b \) implies the acceptance of \( c \). Suppose now that \( aR_{\text{att}}c \). The acceptance of \( a \) implies the non-acceptance of \( c \) and so the non-acceptance of \( b \). Also, if \( cR_{\text{sup}}a \) and \( cR_{\text{att}}b \), the acceptance of \( a \) implies the acceptance of \( c \) and the acceptance of \( c \) implies the non-acceptance of \( b \). So, the acceptance of \( a \) implies the non-acceptance of \( b \). These constraints relating \( a \) and \( b \) are enforced by adding new complex attacks from \( a \) to \( b \):

Def. 3 ([13] Extended attack) Let \( BAS = \langle A, R_{\text{att}}, R_{\text{sup}} \rangle \). There is an extended attack from \( a \) to \( b \) iff either (1) \( aR_{\text{att}}b \); or (2) \( aR_{\text{att}}a_1R_{\text{att}}a_2R_{\text{sup}} \ldots R_{\text{sup}}a_n, n \geq 3, \text{ with } a_1 = a, a_n = b; \) or (3) \( a_1R_{\text{sup}} \ldots R_{\text{sup}}a_n, \text{ and } aR_{\text{att}}a_p, n \geq 2, \text{ with } a_n = a, a_p = b \). Graphically:

\(^2\)A similar idea is presented in [18] for representing defeasible attacks and supports.

\(^3\)“iff” means “if and only if”, “s.t.” means “such that” and “wrt” means “with respect to”.


Among the frameworks proposed in [7] for handling necessary supports, we focus on the one encoding the following interpretation: If \( c \text{R}_{\text{sup}} b \), “the acceptance of \( c \) is necessary to get the acceptance of \( b \)” because “\( c \) is the only attacker of a particular attacker of \( b \)

**Def. 4 ([7] MAS associated with a BAS)** Let \( \text{BAS} = \langle A, \text{R}_{\text{att}}, \text{R}_{\text{sup}} \rangle \) with \( \text{R}_{\text{sup}} \) being a set of necessary supports. Let \( A_n = \{ N_{cb} \mid (c, b) \in \text{R}_{\text{sup}} \} \) and \( R_n = \{ (c, N_{cb}) \mid (c, b) \in \text{R}_{\text{sup}} \} \cup \{ (N_{c b}, b) \mid (c, b) \in \text{R}_{\text{sup}} \} \). The tuple \( \text{MAS} = \langle A \cup A_n, \text{R}_{\text{att}} \cup R_n \rangle \) is the meta-argumentation system associated with \( \text{BAS} \) (it is a Dung’s AS).

### 3. Recursive interactions

Recursive interactions were explored in [3] for recursive attacks (the AFRA) and in [8] for recursive supports plus attacks (the ASAF). The idea is to model that the validity of an interaction may depend on other interactions (e.g., because of preferences [11]). Here we focus on the ASAF [8], where arguments interact with other arguments or interactions:

**Def. 5 (ASAF)** An Attack-Support Argumentation Framework (ASAF) is a tuple \( \langle A, \text{R}_{\text{att}}, \text{R}_{\text{sup}} \rangle \) where \( A \) is a set of arguments, \( \text{R}_{\text{att}} \subseteq A \times (A \cup \text{R}_{\text{att}} \cup \text{R}_{\text{sup}}) \) is an attack relation, and \( \text{R}_{\text{sup}} \subseteq A \times (A \cup \text{R}_{\text{att}} \cup \text{R}_{\text{sup}}) \) is a necessary support relation. Note that \( \text{R}_{\text{sup}} \) is assumed to be irreflexive and transitive. We assume that \( \text{R}_{\text{att}} \cap \text{R}_{\text{sup}} = \emptyset \).

[8] translates an ASAF into an AS in two steps: first, the ASAF is turned into a BAS with necessary support; then, this BAS is turned into an AS by adding extended attacks. For the first step, the following schemas describe the translation of the 4 basic cases:

<table>
<thead>
<tr>
<th>ASAF</th>
<th>Associated BAS</th>
<th>ASAF</th>
<th>Associated BAS</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image2.png" alt="Case 2 Diagram" /></td>
<td><img src="image2.png" alt="Case 2 Bas Diagram" /></td>
<td><img src="image3.png" alt="Case 3 Diagram" /></td>
<td><img src="image3.png" alt="Case 3 Bas Diagram" /></td>
</tr>
</tbody>
</table>

Given the BAS associated with the ASAF, the second step followed in [8] is to create an AS by adding complex attacks, namely Case 2 - extended attacks (see Def. 3):

**Def. 6 (AS associated with BAS and with ASAF)** Let \( \text{BAS} = \langle A, \text{R}_{\text{att}}, \text{R}_{\text{sup}} \rangle \) be the BAS associated with ASAF. The pair \( \text{AS}' = \langle A', \text{R}' \rangle \), where \( A' = A \) and \( \text{R}' = \text{R}_{\text{att}} \cup \{ (a, b) \mid \text{there is a sequence } a_1 \text{R}_{\text{att}} a_2 \text{R}_{\text{sup}} \ldots \text{R}_{\text{sup}} a_n, n \geq 3, \text{ with } a_1 = a, a_n = b \} \) is the AS associated with \( \text{BAS} \) and ASAF.

### 4. Encoding recursive interactions in MAS

In this section, we propose to use the MAS (see Sect. 2) for encoding recursive interactions, addressing the following issues:

- distinguish between labelled and unlabelled interactions, *i.e.* between interactions that may be involved in a recursion (either as a target, or as targeting another interaction) and the other interactions;
- encode labelled interactions, *i.e.* the ability to reason about them; and
- encode recursive interactions, *i.e.* the impact of an interaction on another one.
For this purpose, we need to formalize the notion of labelled interaction. So we propose a slightly modified version of the ASAF, which we call the labelled ASAF.

**Def. 7 (Labelled ASAF)** A labelled ASAF is a 5-uple \( \langle A, R_{att}, R_{sup}, V, \mathcal{L} \rangle \) where \( A \) is a set of arguments, \( R_{att} \subseteq A \times (A \cup R_{att} \cup R_{sup}) \) is an attack relation, \( R_{sup} \subseteq A \times (A \cup R_{att} \cup R_{sup}) \) is a necessary support relation, \( V \) is a set of labels (denoted by greek letters) and \( \mathcal{L} \) is a bijection from \( R \subseteq (R_{att} \cup R_{sup}) \) to \( V \). We still assume that \( R_{att} \cap R_{sup} = \emptyset \).

The above definition enables to distinguish interactions that are not involved in a recursion; they may be considered as always “valid” and will be called “basic” in the following. Since the aim of labels is to enable reasoning about interactions and encode recursive interactions, basic interactions do not require labels. Moreover, each label corresponds to a unique labelled interaction and vice-versa. The main difference between Def. 7 and Def. 5 is the explicit integration of labels into the ASAF. In the following, a labelled interaction will be confused with its label. In order to define the MAS associated with a labelled ASAF, next we explain the encoding of each component of the ASAF.

**Encoding basic attacks/supports.** Such interactions correspond to unlabelled interactions and can be directly encoded using the MAS given in [7] (see Def. 4).

**Encoding labelled interactions.** In order to reason about an interaction that is attacked or supported we must be able to refer to it; hence, it must be labelled and its label will be used as a “meta-argument”. A labelled interaction \( \alpha = (a, b) \) encompasses two types of links. One link relates \( \alpha \) to \( b \), representing the role of \( \alpha \) (either an attack or a support), and will be called the *effect-link*. The other link relates \( \alpha \) to its source \( a \), representing the grounding of \( \alpha \), and will be called the *ground-link*. The idea of “grounded” interaction is close to the notion of evidential argumentation in the work of [14,15]. It means that “an interaction makes sense only if its source argument is accepted”.

These two links suggest two kinds of validity for the interaction. We reserve the term *validity* for the effect-link. For instance, in a graph containing only \( \alpha \) attacked by \( b \) (through an attack \( \beta = (b, \alpha) \)), \( \alpha \) is not valid. Similarly, if \( \alpha \) is supported by \( c \) (with a support \( \gamma = (c, \alpha) \)) and \( c \) is attacked by \( d \), then \( \alpha \) is not valid. Concerning the ground-link, we use the term *grounded*. For instance, \( \alpha = (a, b) \) is not grounded if \( a \) is attacked and not defended. Note that a support can be valid even though its source is not accepted. So interactions may be valid and not grounded, or grounded and not valid. We call *active* an interaction which is both valid and grounded. Intuitively, if \( \alpha \) is only attacked by a non-active interaction (whatever the origin of this non-activation), then \( \alpha \) should be valid. If \( \alpha \) is supported by an interaction \( \beta \) which is valid but not grounded, then \( \alpha \) should not be valid. However, if \( \beta \) is not valid, the validity of \( \alpha \) cannot be affected by \( \beta \).

The following table synthesizes the above notions for a labelled interaction \( \alpha = (a, b) \).

<table>
<thead>
<tr>
<th>Type of link</th>
<th>Meaning of the link</th>
<th>Corresponding Notions</th>
</tr>
</thead>
<tbody>
<tr>
<td>effect-link</td>
<td>describes the role of ( \alpha ) wrt ( b ) (is affected by interactions on ( \alpha ))</td>
<td>validity</td>
</tr>
<tr>
<td>ground-link</td>
<td>describes the existence of ( \alpha ) wrt ( a ) (takes into account only the source of ( \alpha ))</td>
<td>groundness</td>
</tr>
</tbody>
</table>

If an attack \( \alpha = (a, b) \) is active, then \( a \) and \( b \) cannot belong to the same extension. And if a support \( \alpha = (a, b) \) is active, then if \( b \) is accepted then \( a \) must be also accepted.

The ground-link is a necessary support between the meta-argument and the source argument; thus, an interaction \( \alpha = (a, b) \) will be “grounded” only if \( a \) is accepted. This

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Note that if all interactions are unlabelled (\( \mathcal{V} = \emptyset \)), the labelled ASAF is reduced to a simple BAS.
support is basic since it is not defeasible. The effect-link of an attack (resp. a support) \( \alpha = (a, b) \) is a basic attack (resp. support) from \( \alpha \) to \( b \). We encode a labelled interaction \( \alpha \) with basic interactions since an attack (or a support) to \( \alpha \) will be encoded by attacks (supports) to the meta-arguments that are introduced. So, a labelled attack \( \alpha = (a, b) \) is encoded by \( a \rightarrow \alpha \rightarrow b \) and a labelled support \( \beta = (c, d) \) is encoded by \( c \rightarrow \beta \rightarrow d \).

That is, a labelled interaction is encoded in two steps: first, a meta-argument is introduced with a basic support from its source (ground-link) and a basic attack or support to its target (effect-link); then, the basic supports are encoded in MAS (see Sect. 2). So, \( \alpha \) (resp. \( \beta \)) is encoded in the MAS by \( a \rightarrow N_{\alpha a} \rightarrow \alpha \rightarrow b \) (resp. \( c \rightarrow N_{\beta c} \rightarrow \beta \rightarrow N_{\beta b} \rightarrow d \)).

**Encoding recursive interactions.**

Our aim is to encode an attack (resp. a support) on a labelled interaction through attacks (resp. supports) on the meta-arguments associated with it (the labels and the \( N_{ij} \), see the previous paragraph). However, every meta-argument does not play the same role and a deeper analysis is needed in order to identify the meta-arguments that will be affected by the recursive interaction. Let \( \alpha = (a, b) \) and \( \beta = (c, \alpha) \) be two labelled interactions. We discuss their encoding on two cases, with two sub-cases each, considering the intuitively desirable preferred extension of the MAS denoted by \( E \). All these cases will be synthetized in Def. 8.

**Case 1:** \( \alpha \) is an attack. Encoding \( \alpha \) produces the meta-arguments \( \alpha \) and \( N_{\alpha a} \).

- **Case 1.1:** \( \alpha \) is attacked by \( \beta \). In this case, \( E \) should be \( \{a, c, \beta, b\} \). So \( a \) and \( c \) are accepted, \( \beta \) is active, \( \alpha \) is not active (it is grounded but not valid) and \( b \) can be accepted; this result holds whatever the status of \( a \). Now, if \( c \) is attacked by \( d \) (with a basic attack) \( E \) should be \( \{a, d, N_{c\beta}, \alpha\} \), which corresponds to the set \( \{a, d, \alpha\} \) after removing the meta-argument \( N_{c\beta} \). Since \( c \) is not accepted, \( \beta \) is not grounded nor active, and \( \alpha \) can be valid. Also, since \( a \) is accepted \( \alpha \) is grounded, thus active. So \( a \) and \( b \) cannot belong to the same extension.

- **Case 1.2:** \( \alpha \) is supported by \( \beta \). \( E \) should be \( \{a, c, \beta, \alpha\} \). So \( a \) and \( c \) are accepted, \( \beta \) and \( \alpha \) are active and thus, \( b \) cannot be accepted. Now, if \( c \) is attacked by \( d \) (with a basic attack), \( E \) should be \( \{a, d, N_{c\beta}, N_{\beta a}, b\} \), which corresponds to the set \( \{a, d, b\} \) after removing the meta-arguments \( N_{c\beta} \) and \( N_{\beta a} \). Since \( c \) is not accepted, \( \beta \) is not grounded. Furthermore, since \( \beta \) is valid, \( \alpha \) is not valid nor active. Thus, \( a \) and \( b \) can belong to the same extension (whatever the status of \( a \)).

**Case 2:** \( \alpha \) is a support. Encoding \( \alpha \) produces the meta-arguments \( \alpha, N_{\alpha a} \) and \( N_{ab} \).

- **Case 2.1:** \( \alpha \) is attacked by \( \beta \). If \( \beta \) is active, then \( \alpha \) is not valid; this is captured by an attack from the meta-argument \( \beta \) to the meta-argument \( N_{ab} \). Thus, \( \alpha \) is not active, captured by an attack from \( \beta \) to \( \alpha \) in MAS. If \( \beta \) is not active (e.g., if \( c \) is attacked) and \( \beta \) is the only interaction that impacts \( \alpha \), then \( \alpha \) is valid; hence, \( b \) is accepted implies \( a \) is accepted. If \( a \) is attacked by \( e \) (with a basic attack), \( E \) should be \( \{e, c, N_{\alpha a}, \beta, b\} \), which corresponds to the set \( \{e, c, \beta, b\} \). Since \( c \) is accepted, \( \beta \) is grounded. Moreover, \( \beta \) is valid and so active. Therefore, \( \alpha \) is not valid nor active and \( b \) (not being attacked) can be accepted even though \( a \) is not accepted. Note that the presence of \( N_{\alpha a} \) in \( E \) means that \( \alpha \) is not grounded. Moreover, if \( c \) is attacked by \( d \) (with a basic attack), \( E \) should be \( \{e, d, N_{\alpha a}, N_{\beta c}, N_{ab}\} \), which corresponds to the set \( \{e, d\} \). Now, \( c \) is not accepted; so \( \beta \) is not grounded nor active, and \( \alpha \) is valid. However, since \( a \) is not accepted, \( \alpha \) is not grounded nor active and \( b \) cannot be accepted. Lastly, if we remove the attack on \( a \) by \( e \), \( E \) should be \( \{a, d, \alpha, N_{c\beta}, b\} \), which corresponds to \( \{a, d, \alpha, b\} \). \( c \) is not accepted, \( \beta \) is

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5This set could be considered as the extension of the labelled ASAF; however, since this paper reports only a preliminary study, the expected outcomes of the framework following our approach are not yet defined.
not grounded nor active, and $\alpha$ is valid. Also, $\alpha$ is grounded (thus active), so $a$ and $b$ are accepted.

- **Case 2.2**: $\alpha$ is supported by $\beta$. If $\beta$ is valid and not grounded, then $\alpha$ is not valid nor active; this is captured by attacks from $N_{\beta\alpha}$ to $N_{ab}$ and $\alpha$ in MAS. If $a$ is attacked by $e$ and $c$ by $d$ (with basic attacks), $E$ should be $\{e, d, N_{\alpha\alpha}, N_{c\beta}, N_{\beta\alpha}, b\}$, which corresponds to the set $\{e, d, b\}$. Since $c$ is not accepted, $\beta$ is not grounded nor active. Then, $\alpha$ is not valid (nor active) and $b$ can be accepted even though $a$ is not accepted.

**Def. 8 (Attacked or supported attacks/supports in MAS)** The following schemas describe the encoding of an attacked (resp. supported) attack/support in a MAS.

<table>
<thead>
<tr>
<th>Labelled ASAF</th>
<th>Associated BAS</th>
<th>Associated MAS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1.1</strong>: $a$ $\xrightarrow{\alpha} b$ $\beta$</td>
<td>$a$ $\xrightarrow{\alpha} b$ $\beta$</td>
<td>$a$ $\xrightarrow{\alpha} b$ $\beta$</td>
</tr>
<tr>
<td><strong>Case 1.2</strong>: $a$ $\xrightarrow{\alpha} b$ $\beta$</td>
<td>$a$ $\xrightarrow{\alpha} b$ $\beta$</td>
<td>$a$ $\xrightarrow{\alpha} b$ $\beta$ $\xrightarrow{\beta}$ $N_{\beta\alpha}$</td>
</tr>
<tr>
<td><strong>Case 2.1</strong>: $a$ $\xrightarrow{\alpha} b$ $\beta$</td>
<td>$a$ $\xrightarrow{\alpha} b$ $\beta$</td>
<td>$a$ $\xrightarrow{\alpha} b$ $\beta$ $\xrightarrow{N_{\alpha\alpha}}$ $N_{ab}$</td>
</tr>
<tr>
<td><strong>Case 2.2</strong>: $a$ $\xrightarrow{\alpha} b$ $\beta$</td>
<td>$a$ $\xrightarrow{\alpha} b$ $\beta$</td>
<td>$a$ $\xrightarrow{\alpha} b$ $\beta$ $\xrightarrow{N_{\beta\alpha}}$ $N_{\beta\alpha}$</td>
</tr>
</tbody>
</table>

Given a labelled interaction $\alpha = (a, b)$ and an extension $E$, the following cases can occur:

<table>
<thead>
<tr>
<th>Attack $\alpha$</th>
<th>Support $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha \in E$: $\alpha$ is active and in that case $N_{\alpha\alpha} \notin E$ and $N_{ab} \notin E$</td>
<td></td>
</tr>
<tr>
<td>$\alpha \notin E$ and $N_{\alpha\alpha} \notin E$: $\alpha$ is grounded but not active, so it is not valid</td>
<td></td>
</tr>
<tr>
<td>$\alpha \notin E$, $N_{\alpha\alpha} \in E$ and $N_{ab} \in E$: $\alpha$ is not active, not grounded, but valid</td>
<td></td>
</tr>
<tr>
<td>$\alpha \notin E$, $N_{\alpha\alpha} \in E$ and $N_{ab} \notin E$: $\alpha$ is not active, not grounded and not valid</td>
<td></td>
</tr>
</tbody>
</table>

5. **Comparison with ASAF**

Let us compare the ASAF and MAS approaches for encoding labelled and recursive interactions. Both approaches follow two steps. The first step produces a BAS in both cases; however, they differ in the encoding of supports. Moreover, the second step is quite different. Let us first consider the encoding of a labelled attack.

**Prop. 1** Let $\alpha = (a, b)$ be a labelled attack. The translation of $\alpha$ given in Sect. 3 is exactly the same as the one given in Sect. 4: $a \xrightarrow{\alpha} b$ becomes $a \Rightarrow \alpha \Rightarrow b$ where $\alpha$ denotes a meta-argument associated with the attack $(a, b)$.

Let us now consider a labelled support $\alpha = (a, b)$. The first step of the ASAF approach (Sect. 3) produces $a \Rightarrow \alpha^+ \Rightarrow \alpha^- \Rightarrow b$, where two meta-arguments are used for representing the support $\alpha$. In contrast, the first step of the MAS approach (Sect. 4)
produces \( a \rightarrow \alpha \rightarrow b \), where only one meta-argument is created for representing \( \alpha \). However, when encoding \( \alpha \rightarrow b \), the second step of the MAS approach will produce \( \alpha \rightarrow N_{ab} \rightarrow b \). So \( \alpha \) (resp. \( N_{ab} \)) in the MAS plays the role of \( \alpha^+ \) (resp. \( \alpha^- \)) in the associated BAS of the ASAF. Indeed, they mainly differ in the encoding of the ground-link.

The second step produces different AS. The ASAF approach handles the remaining supports through extended attacks. Thus, since by Def. 6 no extended attacks are added, \( a \rightarrow \alpha^+ \rightarrow \alpha^- \rightarrow b \) is turned into: \( a \rightarrow N_{\alpha a} \rightarrow \alpha \rightarrow N_{ab} \rightarrow b \) and the preferred extension of the AS is \( \{ a, \alpha^+, \beta \} \), which corresponds to \( \{ a, \alpha, b \} \) in the ASAF. In contrast, the MAS approach handles both supports by creating meta-arguments (according to Def. 4). So \( a \rightarrow \alpha \rightarrow b \) is turned into: \( a \rightarrow N_{\alpha a} \rightarrow \alpha \rightarrow N_{ab} \rightarrow b \) and the preferred extension of MAS (and BAS) is \( \{ a, \alpha, b \} \).

Note that, in the ASAF approach, the resulting AS has no connection between the source \( a \) and the meta-arguments associated with \( \alpha \). Differently, the MAS links \( a \) and \( \alpha \) with a sequence of attacks going through the meta-argument \( N_{\alpha a} \). This is because the ASAF approach does not treat all supports in the same way, whereas the MAS approach provides a unified handling through the addition of meta-arguments.

Let us now consider a labelled ASAF represented by \( c \beta \rightarrow a \alpha \rightarrow b \). On the one hand, following the ASAF approach, the associated BAS is \( c \beta \rightarrow a \alpha \rightarrow \alpha^+ \rightarrow \alpha^- \rightarrow b \). By Def. 6, there is an extended attack from \( \beta \) to \( \alpha^+ \). So the resulting AS is: \( c \beta \rightarrow a \alpha^+ \rightarrow \alpha^- \rightarrow b \) and the preferred extension of the AS is \( \{ c, \beta, \alpha^- \} \), which is mapped into \( \{ c, \beta, \alpha \} \) in the ASAF.

Since the ASAF approach deems all interactions as labelled, the resulting AS can be uselessly complex. To address this issue, the MAS approach offers two alternatives:

- If interactions are always labelled (even though they are not involved in a recursion) the associated BAS is \( c \beta \rightarrow a \alpha \rightarrow \alpha^+ \rightarrow \alpha^- \rightarrow b \), and the resulting AS is:

\[
\begin{array}{cccccc}
\vdots & \beta & \rightarrow & a & \rightarrow & \alpha^+ \\
\rightarrow & N_{\beta a} & \rightarrow & \beta & \rightarrow & \alpha
\end{array}
\]

\{ \{ c, \beta, N_{\beta a}, N_{ab} \} \} is the preferred extension of MAS, corresponding to \{ \{ c, \beta \} \} in BAS.

- If labels are only used for reasoning about interactions involved in a recursion, we can directly apply Def. 4 to obtain a simpler system: \( c \rightarrow a \rightarrow N_{ab} \rightarrow b \)

Here, the preferred extension is \( \{ c, N_{ab} \} \), corresponding to the extension \{ \{ c \} \} of the BAS.

A main difference between both approaches regards the presence of interactions in their extensions. Every interaction in an extension of a MAS is active (grounded and valid). In contrast, the meaning ascribed by the ASAF approach differs in the case of attacks and supports. As in the MAS approach, the presence of an attack in the extension obtained by the ASAF approach means that this attack is active. This is because the ASAF approach condenses the validity and groundness of an attack \( \alpha \) through the meta-argument \( \alpha \). However, by combining these features, it does not allow to easily identify situations in which \( \alpha \) is not grounded but valid (or vice-versa). On the other hand, if a support \( \beta \) belongs to an extension of the ASAF, we can only assure that it is valid. This is because \( \beta \) is represented by meta-arguments \( \beta^+/\beta^- \) in the associated AS, which also capture the groundness (resp. non-groundness) of the support. As a result, the MAS approach is more flexible than the ASAF because of handling the different features of an interaction separately. Also, it ascribes the same meaning to the presence of every interaction in its extensions, in contrast with the ASAF.
6. Conclusion and future works

We have introduced a new framework for handling recursive interactions in bipolar argumentation systems, extending the work of [8]. Our study addresses the following issue: “How can the validity of an interaction be affected if this interaction is attacked or supported by another one?”. Drawing on examples, we identified different kinds of validity of interactions (namely the notions of “grounded interaction”, “valid interaction” and “active interaction”). Then, we proposed a new method for flattening an ASAF using meta-arguments. The comparison with the original approach of [8] highlights the similarities and differences between the frameworks, and confirms the choices given in [8].

Encodings of attacks and supports through meta-arguments can be found in [18] for the purpose of representing second-order attacks. Our work goes further since the MAS enables to represent attack and support to both attack and support relations, at any level.

Our study has been essentially carried out from examples. So it opens several lines for future work: give a formal proof of the intuitions behind the meaning of the meta-arguments; formally define the expected outcomes of our framework; and compare more deeply our proposal with the existing works, particularly in terms of outcomes.

References