Multi-robot Cooperative Systems for Exploration

Advances in dealing with constrained communication environments

Facundo Benavides[1], Pablo Monzón[2], Caroline P. Carvalho Chanel[3] and Eduardo Grampín[1]

***Abstract*— In the present document, the authors introduce the Cooperative Exploration problem as well as the most relevant approaches in order to show the most common drawbacks and opportunities to improve the state of art solutions. Subsequently, a preliminary version of a multi-robot exploration proposal is described. The first results obtained in simulated scenarios support the underlying ideas are feasible and promising. They show that is possible to cope with real communication constraints (always present in practice), being more fault tolerant and still having good performance regarding the total exploration time. Next steps to fully implement a more reliable and robust system are discussed.**

***Keywords:* Multi-robot systems, Cooperative systems, Exploration tasks.**

I. INTRODUCTION

The exploration problem is considered as one of the fundamental problems in autonomous mobile robotics. The exploration task refers to achieve the complete coverage of a previously unknown environment [7]. Currently, there are several real scenarios where achieving the whole exploring of a zone is one of the main parts of the mission. Some of them are: planetary exploration, reconnaissance, rescue, agriculture, cleaning or the exploration of dangerous places as mined lands and radioactive zones [2]. Due to their inner qualities (mainly efficiency and robustness), in many cases a multi-robot system is chosen to carry this task out [31]. Even so, it is not just concerning to add more and more robots. Therefore, in order to conceive a powerful solution it is necessary to deal with coordination strategies and possibly to consider: the environment characteristics and model, member heterogeneity (shape, size, motor and sensory capabilities, etc), task assignment algorithm, mapping approach and last but not least, the underlying communication system.

Outline

The document is organized as follows. Section II briefly presents a general description of the cooperative exploration problem and the main contribution/drawbacks from a set of surveyed proposals. Next, in Section III a problem formalization is presented as well as the main characteristics of a first version of a dual role based approach. Preliminary experiments and results are shown and discussed in Section IV. Finally, conclusions and future works are presented in Sections V and VI.

[1]Facundo Benavides and Eduardo Grampín are with the Computer Science Institute, Faculty of Engineering, Universidad de la República, Montevideo, Uruguay {fbenavid,grampin}@fing.edu.uy

[2]Pablo Monzón is with the Electrical Engineering Institute, Faculty of Engineering, Universidad de la República, Montevideo, Uruguay monzon@fing.edu.uy

[3]Caroline P. Carvalho Chanel is with the Design and Control of Aerospace Vehicles Department, ISAE-SUPAERO, Toulouse, France caroline.chanel@isae-supaero.fr

II. RELATED WORK

A. General formulation

Schematically, the exploration of an environment can be seen as the composition of Mapping and Motion Planning tasks. As a matter of fact, a map is needed in order to plan new motions. On the other hand, to choose a correct motion sequence is needed to optimally expand the knowledge about the environment -represented by the map. As a consequence, *mapping* is constantly interleaved with *motion planning*, and vice versa during the whole process [28], [7], [20]. In Fig. 1 the interaction between Mapping, Motion Planning and Localization tasks can be seen.

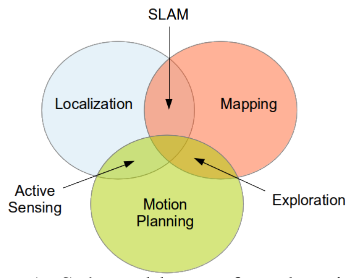


Fig. 1: Sub-problems of exploration.

Additionally, the *cooperative* exploration problem is commonly defined as the full coverage of a previously unknown environment, using a robot team and regarding some optimal criteria [7]. Typically, the overall exploration time is the most commonly used criterion to measure and to compare the proposals quality.

Thus, in the context of multi-robot systems, both mapping and motion planning imply cooperative actions. Obviously, each robot must be able to solve both of them individually. However, they only will be able to take the advantages of working in group if they are also able to keep coordinated during the process. Due to this, in many approaches explicit communication turns into a very important aspect.

B. Tasks assignment task

When multiple robots are involved in an exploration task, avoiding several of them moving to the same place seems to be clever. The task assignment task concerns the choice of new places[1] to visit in a coordinated way. To reach this purpose, is quite often to split the task up into two steps or

[1]Those singular places will be referred along this text as tasks or targets indistinctly.

faces. The first one, called *Task Identification*, concerns the points of interest identification and it strongly depends on both the sensory robot capabilities and on the underlying environment representation. The second one, called *Task Allocation*, concerns the search of a distribution of tasks to robots that maximizes the overall system utility and minimizes the amount of overlapped information obtained by all of them [7].

1) Task Identification methods : The most widely used representation for this purpose is the *Occupancy Grid* structure. Based on it, in 1998 a method to identify interesting points[2] in an environment was proposed by *Yamauchi* [29]. Since then, the majority of proposals have adopted this proposal leading to a family of solutions which are well known as *Frontier Points* or *Frontier Region* based approaches [32], [19].

2) Task Allocation methods: There are a wide variety of essayed solutions. Even so, the majority of methods are centralized and usually compute an objective function in order to estimate the utility of reaching each one of the previously identified targets. This function enables a robot to locally prioritize the targets in its scope and potentially enable the whole team to search for the best global distribution [10].

One of the most popular centralized method is based on the notion of *Auctions*[3]. Like in [7], [20], the allocation decision is processed centrally by a greedy algorithm which considers the bids made by the robots. Those bids are based on local prioritization of the targets (typically, regarding the distance between the current position and every target). Although, this method owes his popularity to be easy to understand and to implement, falling in local minima is its major shortcoming [8].

Thanks to its well known search properties, other authors have used *Genetic Algorithms* [12]. The main purpose was to avoid some drawbacks present in other approaches without loosing performance. However, this is also a centralized approach that additionally requires that the number of robots during the whole exploration process remains invariant.

Far from meta-heuristics, in [28] an operative research based approach is presented. This method combines an environmental segmentation technique [4] with the centralized task allocation method proposed by Kuhn in 1955 [11]. The working hypothesis is that in highly structured environments is more convenient to perform the exploration after having divided the environment into disjoint segments. This way, it is expected to achieve full exploration decreasing the sensory overlapping between agents as much as possible.

Recently, a novel approach was fully presented in [2]. It works in a very simple and decentralized way attempting to distribute the robots over the unexplored locations as much as possible. The underlying idea is that if this could be done over time until the end of exploration, the exploration time would be smaller. In practice, the main contribution of this approach consists in providing a better distribution of robots on the terrain decreasing the overall cost of exploration for a big set of practical scenarios.

C. Cooperative mapping

It concerns the ability to build a single global map from the "local" maps that are built by each of the team members separately. Consequently, as well as having the ability to build a map, in order to share information each robot must also be able to communicate with the other ones. However, this possibility could be not always present or, even if it is, it must be defined which are the appropriate moments to exchange information [26].

D. Communication

Despite the lack of realism, most of the proposals assume ideal communication conditions[5]. This way the authors can put all attention on higher level problems. However, the resultant algorithms are often either so theoretic or really applicable just on a few set of controlled environments. On the contrary, the team is forced to be close enough in order to be fully communicated all time (for instance, not spreading further than the limits of communication ranges) [22], [13], [25]. In real scenarios many things may put the ideal working assumption at risk. In open environments - or simply large ones- the distances among robots could be easily bigger than the scope of communication devices. Depending on the terrain, the robots could be likely to get stuck. When both conditions are present in one environment, the exploration strategy should take it into account in order to prevent robots moving away from the rest for long periods. If it happens and a robot get stuck, all gathered information had been lost, many resources had been wasted (e.g. time and battery) and the zone will need to be explored again by another robot. Thus, some proposals are starting to tackle the exploration problem without assuming the existence of ideal communication [23], [9], [14], [16].

III. PROPOSAL

Taking into account the aspects presented above (described in Section II), a specific instance of the cooperative exploration problem is defined in this section. The main objective of the section is to very well define the boundaries of an instance which permits to work on more realistic scenarios as well as the characteristics of a solution proposal. As a consequence several definitions will be given and some real communication constraints will be considered.

A. Robot model

Given a robot team $R = \{R_1, R_2, \ldots, R_M\}$ consisting of *M* homogeneous circular rigid differential driven mobile robots, such that every robot is defined by a traditional representation: $R_i = (x_i, y_i, \theta_i, r_i, s_i, c_i)$ where $i \in [1..M]$ and x_i, y_i, θ_i represent the configuration of robot over time (position of his center and heading on W), r_i represents the radius of robot body, s_i, c_i represent the sensory capabilities

[2]Points that lies just on the borderline between known and unknown regions.

[3]Although, there is a decentralized version, it is not largely used.

[4]Mostly based on *Voronoi* diagrams [5], [27], [17].

[5]Without errors nor losses, with unlimited bandwidth and scope.

as maximum radius of sensing and maximum range of communication, respectively.

1) Body model: For each robot a body configuration function is defined as follows $bod_i : \mathbb{R} \rightarrow \{\mathbb{R}^2\}$such that:

$$d_i(x, y) = \sqrt{(x - x_i(t))^2 + (y - y_i(t))^2} \quad (1)$$

$$bod_i(t) = \{(x, y) \mid r_i \geq d_i(x, y)\} \quad (2)$$

2) Sensory model: For each robot a sensing function is defined as follows $sen_i : \mathbb{R} \rightarrow \{\mathbb{R}^2\}$ such that:

$$sen_i(t) = \{(x, y) \mid s_i \geq d_i(x, y)\} \quad (3)$$

3) Communication model: For each robot a strength signal function is defined as follows $com_i : \mathbb{R} \rightarrow \mathbb{R}$ such that[6]:

$$com_i(d_j) = -10PLlog_{10}(d_j/c_i) - \begin{cases} nW * WAF & nW < C \\ C * WAF & nW \geq C \end{cases} \quad (4)$$

where *PL* represents the path loss rate, d_j represents the distance between two robot locations: the transmitter (R_i) and receiver (R_j), *nW* represents the number of walls present between transmitter and receiver, *C* represents the maximum number of walls considered to model the signal attenuation caused by walls and *WAF* represents the wall attenuation factor.

In Fig.2 the shape of the function $com_i(d_j)$ can be seen as well as the attenuation effect caused by both the distance between transmitter and receiver and the wall interference.

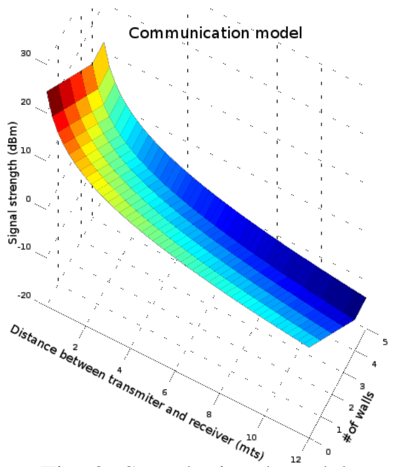


Fig. 2: Stregth signal model.

This way, it is expected to better represent the communication constraints that are widely present in a variety of real scenarios, particularly indoor (e.g. office-like scenarios).

B. Environment model

Given a 2D bounded previously unknown environment $W \in R^2$. The environment *W* will be represented by an occupancy grid structure where each cell can belong to three different probabilistic states {*free, occupied, unknown*}. Typically, $p(cell ==' free') = 1 - p(cell ==' occupied')$ and whether $p(cell ==' free') == 0.5$ the cell is labeled as *unknown*. Those states represent all possible theoretic situations in which a point of the environment can be classified over time. In Fig.3 the cell states and possible transitions are shown.

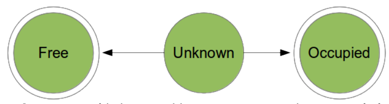


Fig. 3: Possible cell states and transitions.

Finally, on *W* it is possible to define the set of previously unknown static rigid obstacles O_j, such that:

$$O_j = \{(x, y) \in W \mid state(x, y) =' occupied'\} \quad (5)$$

C. Global Objective

The objective of exploration will be to achieve the full coverage of an indoor environment, a priori totally unknown, with a team of potentially heterogeneous robots, in minimal time. Equation 6 express the condition that must be reached in order to complete the task.

$$(W - \bigcup_j O_j) \subseteq \bigcup_{i,t} sen_i(t) \quad (6)$$

where *W* represents the environment, O_j a set of static obstacles and sen_i the information sensed by each robot *i* over time.

D. Utilities and Costs

In order to better adapt the making decision process to real environments, where the system objectives could oppose one another (exploration vs connectivity), a task utility function is defined. This function takes into account both the traveling cost and connectivity utility and its purpose is to find a good balance between them.

1) Path cost: It measures the path cost for a robot to reach a target from its current configuration. A function is defined as follows $pathCost_i : T \rightarrow \mathbb{R}$ such that:

$$pathCost_i(t_j) = d_j \quad (7)$$

where d_j is the minimal distance needed to robot R_i to travel from its current configuration (x_i, y_i, θ_i) to the target t_j. If $d_j == \infty$ means that the target is unreachable for the robot. This way, the team heterogeneity (regarding size aspect) is taken into account and the system would be able to deal with scenarios where some regions could be inaccessible to some robots.

2) Connectivity Utility: It measures the connectivity utility of a place in terms of how connected would be the robot with the rest of its team members. A function is defined as follows $connectivity_i : \{R\} \rightarrow \mathbb{R}$ such that[7]:

$$connectivity_i(\bigcup_{k \neq i} \{R_k\}) = \sum_{p=1}^{P} \lambda^p com_i(d_p) \quad (8)$$

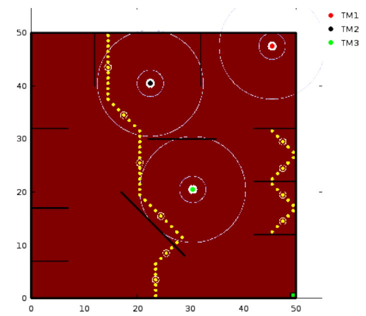
where $0 \leq \lambda \leq 1$ is a tuning parameter. If $\lambda == 0$, then the system does not care about connectivity at all: like in ideal communication scenarios. On the contrary, if $\lambda == 1$ then the system would penalize targets that could break the connectivity of the team. Otherwise, the system will consider the connectivity aspect depending on whether λ is closer to *0*

[6] Adapted from [1].

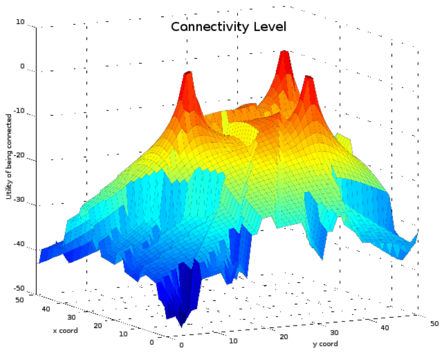
[7] Inspired from [18].

or closer to *1*. $0 <= P <= |\{R_k\}|$, is the amount of robots in a subgroup. $\{d_1, d_2, \ldots, d_P\}$ is an increasing ordered set of distances between the R_i robot location and every other sub-group member R_k.

The Fig.4b shows the result of applying the $connectivity_i$ function on the experimental environment shown in Fig.4a. On it there are three other robots (located in the corresponding peak positions) and several walls. In order to appreciate the behavior of the function, the location of the robot *i* is set on every single possible position of the environment.



(a) Experimental environment.



(b) Connectivity utility evaluation.

Fig. 4: Connectivity utility function.

3) Task utility: In order to guide the optimal task distribution search a task utility function is defined as follows $utility_i : T \rightarrow [0..1]$ such that:

$$utility_i\,(T_j) = -\alpha pathCost_i\,(T_j) + \beta connectivity_{T_j}\left(\bigcup_{k \neq i} \{R_k\}\right) \quad (9)$$

where R_k represents every potential sub-group member surrounding T_j location and $\alpha, \beta \in [0..1]$ are tuning parameters that permit to adjust the kind of solutions the system will search for. If $\alpha == \beta == 0$, then the system will behave completely random. Yet, if $\alpha == 1$ and $\beta == 0$ then the system will show a greedy behaviour. On the contrary, if $\alpha == 0$ and $\beta = 1$ then the system will search for solutions according to λ parameter (see Eq. 8). Otherwise, the system will try to balance path costs and connectivity utility depending on the values of α, β and λ.

E. Coordination method

1) Task Identification method: As was mentioned above, a very helpful definition is the notion of *task*. A *task* is commonly defined as a location where a robot wants to go to perform his work (in this case, to explore: to sense unknown regions).

Then, a set of tasks $T = \{T_1, T_2, \ldots, T_N\}$ is defined to represent at each moment, the set of targets that the robot team could be interested in.

Moreover, it is easy to see that in any case, as much closer to the frontier -between known and unknown regions- the tasks are identified as much information the system can gain. Therefore, a *task* represents a location where there is at least one neighbour point that is unknown yet. Thus, the set of tasks T at any moment is defined as follows:

$$FP = \{(x,y) \mid (x,y) \in W_{known} \wedge (neighbour\,(x,y) \bigcap W_{unknown} \neq \emptyset)\} \quad (10)$$

$$FR = \{(x,y) \mid (x,y) \in FP \wedge \textit{is located in the center of FP.}\} \quad (11)$$

$$T = \{T_1, T_2, \ldots, T_N\} = \{(x,y) \mid (x,y) \in FR\} \quad (12)$$

As a consequence, *free* cells (defined in Sec.III-B) could be over labeled as *frontier point (FP)* or *frontier region (FR)* depending on whether it represents just a single location or the center of a group of *frontier point* cells.

2) Task Allocation method: In order to take advantage of the individual computing power of the robots, a decentralized approach is followed. Typically, estimation of costs and utilities as well as local maps building and localization are the tasks chosen to be made by themselves. However, to achieve a cooperative behaviour both the local map and localization information must be shared among teammates. In addition, trying to increase the fault tolerance of the system, loneliness situations will be avoided as much as possible. Thus, every moment the system will intend to explore preserving communication networks as big as possible.

Depending on the relation between $|T|$ and $|R|$ two quite different scenarios could be considered. On the one hand, if $|T| < |R|$, it will be necessary less robots than the total. To tackle this case a dual role approach is planned but has not been implemented yet. On the other hand, if $|T| \geq |R|$, all robots will be necessary in order to reach the maximal amount of targets. In that case, the goal is to choose the distribution of tasks to robots which reports the maximal utility implying the minimal cost. The proposal consists in employing the same allocation criterion as in the *minPos* approach, but using the task utility function defined in Eq.9 instead of just the path cost. A pseudo-code of the allocation algorithm can be seen in [4].

IV. EXPERIMENTAL RESULTS

In order to have a quantitative measure of the system performance, a comparative study was conducted. The first preliminary experiments concerned the comparison between the results obtained with an own implementation of *minPos* approach and with this proposal. A team composed by two robots was put to explore a simulated environment with the goal of reaching 95% of coverage. The robots were set up to be 0.25m of radius, having 3m and 6m of sensing radius and communication range, respectively. The total amount of time was measured taking into account the quantity of movements made by the robots. As well as, the connectivity level between team members was measured during the whole process

using the function presented in Eq.8. This experiment intents to confirm the hypothesis that introducing the connectivity notion into the making decision process can help to become the system more tolerant to real conditions, particularly non-ideal communication conditions. In Fig.5 the environment[8] used as benchmark is shown. While the red zone represents the free region, dark red zones represent walls and the green cell represents the base from where both robots start the process.

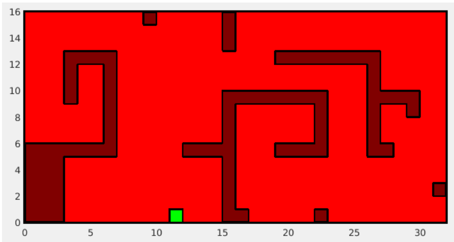


Fig. 5: Simulated environment.

In Tab.I some data that describes the overall performance of both approaches are presented.

TABLE I: Comparative analysis.

	minPos approach	Author's approach
Duration	89(movs)	149(movs)
Coverage %	95.409	96.607
Connection time %	42.697	83.221

Regarding these data it is possible to notice that exists a conflict of interests between both objectives: minimizing the total amount of exploration time and keeping the connectivity between teammates during the whole process. As long as the *minPos* approach has been faster, on the other hand, the approach presented here could ensure connectivity during more than 4/5 of the total exploration time.

In Fig.6 the connectivity level between both teammates during the *minPos*-based exploration process can be seen.

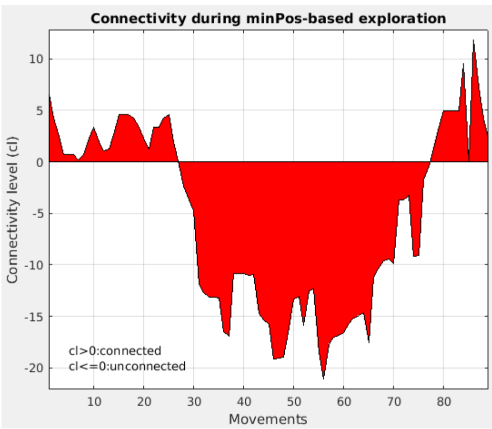


Fig. 6: Connectivity level during *minPos*-based exploration.

This chart shows that after a first stage where the robots were connected, they loss the connectivity until almost the end. Actually, they stay connected just 43% of the total time.

[8]The same environment was used to test the *minPos* approach in [3]

In this case is easy to see that if, during the unconnected period, one of them had suffered an irrecoverable failure, the other robot would be forced to explore back a portion of the environment. This would cause the waste of addition resources and the delay on the termination time.

On the other hand, there are two chart where the behavior of this proposal may be analyzed. Firstly, in Fig.7 the connectivity level reached during the exploration is akin to the one obtained with the *minPos* approach. This result is totally expected since the *alpha* value used is very close to 1.0.

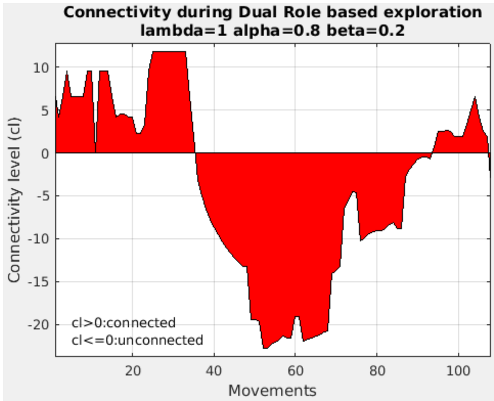


Fig. 7: Connectivity level during this-proposal-based exploration.

However, in Fig.8 the results show a better connected process. Setting the *alpha* value up to 0.6 caused the balance between objectives was put in favour of connectivity.

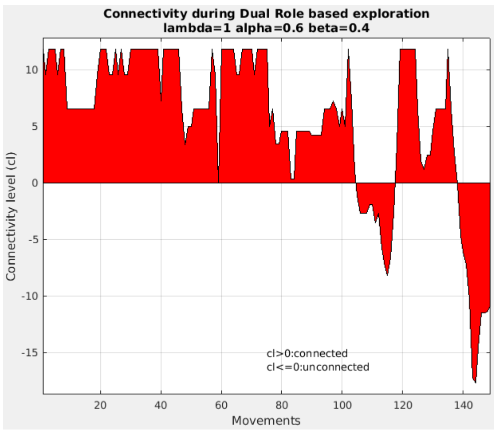


Fig. 8: Connectivity level during this-proposal-based exploration.

V. CONCLUSIONS

The first results obtained in simulated scenarios support the underlying ideas presented in this proposal are feasible and promising. They also show that is possible to cope with real communication constraints (always present in practice), being more fault tolerant and still having good performance regarding the total exploration time.

VI. FUTURE WORK

From the particular formulation of the cooperative exploration problem given in Sec. III, a full development of a Dual Role based approach will be carried out. Two roles will be considered: explorer and relay robots. While *explorer* robots will be in charge of gathering as much information as they can (and simultaneously serving as communication relay), *relay* robots must mainly intend to reduce the loss of connectivity among team members (though simultaneously sharing their local maps, they will not be in charge of exploring new places). Another important aspect to be solved is the algorithm to compute the candidates places where the relay-robots could locate itself. Some references have been already found on this matter [21], [6]. This way is expected the system might be even more flexible (being more adaptive to a bigger number of scenarios) and for that reason, more efficient (avoiding the presence of idle robots or decreasing the completion time of exploration). After that several new experiments will be conducted. They will consist of both simulated and real environments regarding the metrics suggested in [30] and comparing the results with other approaches (not only with *minPos*). The exploration will be carried out in a 2D bounded office-like environment. Robots could be identical or not. The teams could be composed by *IRobot* or *KheperaIII* units. Artificial potential field approach could be used in order to conduct motion planning tasks [24], [4], [15]. The connectivity will be limited in range and bandwidth. Additionally, during the simulated experiments information loss could be simulated -under a very strong control- in order to test robustness.

REFERENCES

[1] Paramvir Bahl and Venkata N Andpadmanabhan. RADAR: An In-Building RF-based User Location and Tracking System. *Proc. Ninet. Annu. Jt. Conf. IEEE Comput. Commun. Soc.*, 00(c):775–784, 2000.

[2] Antoine Bautin. *Stratégie d'exploration multirobot fondée sur le calcul de champs de potentiels*. PhD thesis, Département de formation doctorale en informatique, École doctorale IAEM, 2013.

[3] Antoine Bautin and Olivier Simonin. MinPos : a Novel Frontier Allocation Algorithm for Multi-robot Exploration. *ICIRA*, 7507, 2012.

[4] Antoine Bautin, Olivier Simonin, and François Charpillet. Stratégie d'exploration multirobot fondée sur les champs de potentiels artificiels. *Revue d'intelligence artificielle*, 26(5):523–542, October 2012.

[5] Priyadarshi Bhattacharya and Marina L. Gavrilova. Roadmap-Based Path Planning: Using the Voronoi Diagram for a Clearance-Based Shortest Path. *IEEE Robotics & Automation Magazine*, 15(2):58–66, June 2008.

[6] P. Bose, P. Morin, I. Stojmenovic, , and J. Urrutia. Routing with guaranteed delivery in ad hoc wireless networks. *Workshop on Discrete Algorithms and methods for mobile computing and communications*, 7:48–55, 1999.

[7] W. Burgard, M. Moors, C. Stachniss, and F.E. Schneider. Coordinated multi-robot exploration. *IEEE Transactions on Robotics*, 21(3):376–386, June 2005.

[8] Rodolfo C. Cavalcante, Thiago F. Noronha, and Luiz Chaimowicz. Improving combinatorial auctions for multi-robot exploration. *2013 16th International Conference on Advanced Robotics (ICAR)*, pages 1–6, 2013.

[9] Micael S. Couceiro, Carlos M. Figueiredo, Rui P. Rocha, and Nuno M.F. Ferreira. Darwinian swarm exploration under communication constraints: Initial deployment and fault-tolerance assessment. *Robotics and Autonomous Systems*, 62(4):528–544, April 2014.

[10] G Ayorkor Korsah, Anthony Stentz, and M Bernardine Dias. A comprehensive taxonomy for multi-robot task allocation. *The International Journal of Robotics Research*, 32(12):1495–1512, 2013.

[11] H. W. Kuhn. The Hungarian method for the assignment problem. *Naval Research Logistics*, 2:83–97, 1955.

[12] Xin Ma, Qin Zhang, and Yibin Li. Genetic algorithm-based multi-robot cooperative exploration. In *2007 IEEE International Conference on Control and Automation, ICCA*, volume 00, pages 1018–1023, 2007.

[13] Alejandro R. Mosteo, Luis Montano, and Michail G. Lagoudakis. Multi-robot routing under limited communication range. *2008 IEEE International Conference on Robotics and Automation*, pages 1531–1536, May 2008.

[14] Jyh-Ching Pham, Viet-cuong and Juang. A multi-robot, cooperative, and active slam algorithm for exploration. *Int. Journal of Innovative Computing, Information and Control*, 9(6):2567–2583, 2013.

[15] Alessandro Renzaglia and Agostino Martinelli. Potential field based approach for coordinate exploration with a multi-robot team. In *2010 IEEE Safety Security and Rescue Robotics*, pages 1–6. IEEE, jul 2010.

[16] Martijn N. Rooker and Andreas Birk. Multi-robot exploration under the constraints of wireless networking. *Control Engineering Practice*, 15(4):435–445, April 2007.

[17] Waldir L. Roque and Dionsio Doering. Trajectory planning for lab robots based on global vision and Voronoi roadmaps. *Robotica*, 23(4):467–477, 2005.

[18] Weihua Sheng and Ning Xi. Multi-robot area exploration with limited-range communications. In *2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS) (IEEE Cat. No.04CH37566)*, volume 2, pages 1414–1419. Ieee, 2004.

[19] Weihua Sheng, Qingyan Yang, Jindong Tan, and Ning Xi. Distributed multi-robot coordination in area exploration. *Robotics and Autonomous Systems*, 54(12):945–955, 2006.

[20] Reid Simmons, David Apfelbaum, Wolfram Burgard, Dieter Fox, Mark Moors, Sebastian Thrun, and Håkan Younes. Coordination for Multi-Robot Exploration and Mapping. In *Aaai*, pages 852–858. Menlo Park, CA; Cambridge, MA; London; AAAI Press; MIT Press; 1999, 2000.

[21] I. Stojmenovic and X. Lin. Power-aware localized routing in wireless networks. *IEEE Int. Paral lel and Distributed Processing Symp.*, pages 48–55, 2000.

[22] Gurkan Tuna, Kayhan Gulez, and V. Cagri Gungor. The effects of exploration strategies and communication models on the performance of cooperative exploration. *Ad Hoc Networks*, 11(7):1931–1941, September 2013.

[23] Luis Valentin, Rafael Murrieta-Cid, Lourdes Muñoz-Gómez, Rigoberto López-Padilla, and Moises Alencastre-Miranda. Motion strategies for exploration and map building under uncertainty with multiple heterogeneous robots. *Advanced Robotics*, 28(17):1133–1149, 2014.

[24] Joan Vallvé and Juan Andrade-Cetto. Potential information fields for mobile robot exploration. *Robotics and Autonomous Systems*, September 2014.

[25] Jose Vazquez and Chris Malcolm. Distributed Multirobot Exploration Maintaining a Mobile Network. *IEEE International Conference on Intelligent Systems*, (June), 2004.

[26] Teresa a. Vidal-Calleja, Cyrille Berger, and Simon Lacroix. Event-driven loop closure in multi-robot mapping. In *2009 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 1535–1540. Ieee, October 2009.

[27] Ron Wein, Jur P. van den Berg, and Dan Halperin. The visibility–Voronoi complex and its applications. *Computational Geometry: Theory and Applications*, 36(1):66–87, 2007.

[28] K.M. Wurm, C. Stachniss, and W. Burgard. Coordinated multi-robot exploration using a segmentation of the environment. *2008 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 1160–1165, September 2008.

[29] Brian Yamauchi. Frontier-based exploration using multiple robots. *Proceedings of the second international conference on Autonomous agents AGENTS 98*, (May):47–53, 1998.

[30] Zhi Yan, Luc Fabresse, Jannik Laval, and Noury Bouraqadi. Metrics for performance benchmarking of multi-robot exploration. *IEEE International Conference on Intelligent Robots and Systems*, 2015-December:3407–3414, 2015.

[31] Zhi Yan, Nicolas Jouandeau, and Arab Ali. A Survey and Analysis of Multi-Robot Coordination. *International Journal of Advanced Robotic Systems*, 10:1, 2013.

[32] Jing Yuan, Yalou Huang, Tong Tao, and Fengchi Sun. A cooperative approach for multi-robot area exploration. *IEEE/RSJ 2010 International Conference on Intelligent Robots and Systems, IROS 2010 - Conference Proceedings*, pages 1390–1395, 2010.