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Macro-scale modeling of two-phase flows within structured packings

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Structured packings as a structured porous medium

- A multi-scale process
- A complex interaction between gas and liquid
- A macro-scale model for the gas / liquid system?
Structured packings as a structured porous medium

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Structured packings as a structured porous medium

- A multi-scale process
- A complex interaction between gas and liquid

**A macro-scale model for the gas / liquid system?**
Up-scaling method

- Volume averaging method

\[
p_g \quad u_g \quad K^*_{\beta\gamma} \quad \langle u_g \rangle \quad \langle p_g \rangle^g \\
p_l \quad u_l \quad \langle u_l \rangle \quad \langle p_l \rangle^l
\]

- Example: Darcy equation

\[
\frac{\varepsilon}{t} \frac{\partial S_l}{\partial t} + \nabla \cdot \langle u_l \rangle = 0
\]

\[
\langle u_l \rangle = -\frac{K_0}{\mu_l} \cdot \left( \nabla \langle p_l \rangle^l - \rho_l g \right)
\]
Up-scaling method

- Volume averaging method

\[
p_g \quad u_g \quad \rightarrow \quad K^*_{\beta\gamma} \quad \langle u_g \rangle \quad \langle p_g \rangle^g \quad \langle u_l \rangle \quad \langle p_l \rangle^l
\]

- Example: Darcy equation

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\langle u_l \rangle = -\frac{K_0}{\mu_l} \cdot (\nabla \langle p_l \rangle^l - \rho_l g)
\]
Darcy’s laws for gas/liquid flows in packings?

- Darcy-generalised equation (single phase approximation) Soulaine et al., Pasquier et al.

\[
\langle u_g \rangle = -\frac{K_0}{\mu_g} \cdot (\nabla \langle p_g \rangle^g - \rho_g g) - F \cdot \langle u_g \rangle
\]

- Strong interaction between liquid and gas in the loading regime
  - pressure drop rise \( \nabla \langle p_g \rangle^g \)
  - liquid retention \( h_l \) increases

- Requires a coupled model at the macro-scale

Experimental data extracted from Brunazzi et al. (2002)
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Experimental data extracted from Brunazzi et al. (2002)
Up-scaling - Coupled model

- Viscous regime (Whitaker, Lasseux et al.)

\[
\begin{pmatrix}
\langle u_l \rangle \\
\langle u_g \rangle
\end{pmatrix} = - \begin{pmatrix}
\frac{K_{ll}}{\mu_l} & \frac{K_{lg}}{\mu_g} \\
\frac{K_{gl}}{\mu_l} & \frac{K_{gg}}{\mu_g}
\end{pmatrix}
\begin{pmatrix}
\nabla \langle p_l \rangle_l \\
\nabla \langle p_g \rangle_g
\end{pmatrix}
\]

- Application: Two-phase flow in particle beds

- 1D resolution of the coupled system (OpenFoam)

- Closures of $K_{ll}^*$, $K_{gg}^*$, $K_{lg}^*$, $K_{gl}^*$ from experimental results

Calide experiment - IRSN - see Chikhi et al. 2016
Numerical resolution : IMPES Method

- IMplicit Pressure Explicit Saturation
- Sequential resolution of an equation on pressure and on saturation

\[
\frac{\partial}{\partial t} \begin{pmatrix} S_l \\ S_g \end{pmatrix} + \nabla \cdot \begin{pmatrix} \langle u_l \rangle \\ \langle u_g \rangle \end{pmatrix} = 0
\]

\[
\begin{pmatrix} \langle u_l \rangle \\ \langle u_g \rangle \end{pmatrix} = - \begin{pmatrix} \frac{K^*_l}{\mu_l} & \frac{K^*_l}{\mu_l} \\ \frac{K^*_g}{\mu_g} & \frac{K^*_g}{\mu_g} \end{pmatrix} \begin{pmatrix} \nabla \langle p_l \rangle^I \\ \nabla \langle p_g \rangle^g \end{pmatrix}
\]

- Finite volume method
- Scalar or tensorial effective parameters \( K^* \)
**Up-scaling - Coupled model**

- **Dimensionless pressure drop in the viscous regime**

\[
- \frac{|\nabla \langle p_g \rangle^g|}{\rho_l g}
\]

![Image showing experimental results and numerical modeling](image-url)

**Experimental results Clavier et al. (2015)**

**Numerical modeling of the coupled system (OpenFoam)**
Up-scaling - Coupled model

- Inertial regime (Lasseux et al., 2008)

\[
\begin{pmatrix}
\langle u_l \rangle \\
\langle u_g \rangle
\end{pmatrix} = -\begin{pmatrix}
\frac{K^*_l}{\mu_l} & \frac{K^*_g}{\mu_g} \\
\frac{K^*_g}{\mu_l} & \frac{K^*_g}{\mu_g}
\end{pmatrix}
\begin{pmatrix}
\nabla \langle p_l \rangle^l \\
\nabla \langle p_g \rangle^g
\end{pmatrix} - \begin{pmatrix}
F^*_l & F^*_g \\
F^*_g & F^*_g
\end{pmatrix}
\begin{pmatrix}
\langle u_l \rangle \\
\langle u_g \rangle
\end{pmatrix}
\]

\[
|\nabla \langle p_g \rangle^g| = \frac{\rho_l g}{\rho_l g}
\]

\[
Re_g = 0
\]

\[
Re_l = 0
\]

\[
Stokes Re_l = 128
\]

\[
Inertia Re_l = 128
\]
Application to structured packings

- **1D resolution of a air / water system**

- Comparison of a Darcy model and coupled models (viscous and inertial)

- Closures:
  - Viscous: analogy to a liquid film in a cylinder tube
  - Inertial: based on the closures from the Calide experiment

\[
K_{gg} = K_0 (1 - S_l)^2 ; \quad K_{ll} = K_0 S_l^3 \\
K_{lg} = \frac{\mu_g}{\mu_l} \frac{S_l^2}{(1 - S_l)} ; \quad K_{gl} = \frac{(1 - S_l)}{S_l}
\]
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Application to structured packings

- Analysis of the pressure drop $\nabla \langle p_g \rangle^g$ and retention $h_l$

![Graph 1](image1.png)

- The inertial coupled model reflects the increase in retention and pressure drop
- Comparison from Suess and Spiegel (1991) for illustration

![Graph 2](image2.png)
Modeling of the dispersion of liquid? (1)

- Tomography visualisation

  Fourati et al. (2012)

- A two-equation model for the liquid phase

  Mahr and Mewes (1999)

  Schug et al. (2015)
Modeling of the dispersion of liquid? (1)

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Modeling of the dispersion of liquid? (1)

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- A two-equation model for the liquid phase

  Mahr and Mewes (1999)

  Schug et al. (2015)
Modeling of the dispersion of liquid? (2)

- Two-equation model for the liquid phase (Mahr and Mewes, Soulaie et al.)

\[
\langle u_i \rangle = -\frac{K^*_i}{\mu_i} \cdot (\nabla \langle p_i \rangle - \rho_i g) ; \quad \langle u_2 \rangle = -\frac{K^*_2}{\mu_2} \cdot (\nabla \langle p_2 \rangle - \rho_2 g)
\]

\[
K^*_i = K^*_i \left( \begin{array}{ccc}
0 & 0 & 0 \\
0 & \cos^2 \theta^* \sin^2 \theta^* & \pm \cos \theta^* \sin \theta^* \\
0 & \pm \cos \theta^* \sin \theta^* & \cos^2 \theta^* \sin^2 \theta^*
\end{array} \right)
\]

\(i = 1, 2\)

- Liquid exchange at the contact points?

\[
\frac{\partial S_{i1}}{\partial t} + \nabla \cdot \langle u_{i1} \rangle = \dot{m} \quad \quad \quad \quad \frac{\partial S_{i2}}{\partial t} + \nabla \cdot \langle u_{i2} \rangle = -\dot{m}
\]
Modeling of the dispersion of liquid? (2)

- Two-equation model for the liquid phase (Mahr and Mewes, Soulaine et al.)

\[
\langle u_{l_1} \rangle = - \frac{K_{l_1}^*}{\mu_l} \cdot \left( \nabla \langle p_{l_1} \rangle - \rho_l \mathbf{g} \right) \quad ; \quad \langle u_{l_2} \rangle = - \frac{K_{l_2}^*}{\mu_l} \cdot \left( \nabla \langle p_{l_2} \rangle - \rho_l \mathbf{g} \right)
\]

\[
K_{l_i}^* = K_{l_i}^* \begin{pmatrix}
0 & 0 & 0 \\
0 & \cos^2 \theta^* \sin^2 \theta^* & \pm \cos \theta^* \sin \theta^* \\
0 & \pm \cos \theta^* \sin \theta^* & \cos^2 \theta^* \sin^2 \theta^*
\end{pmatrix}
\]
i = 1, 2

- Liquid exchange at the contact points?

\[
\frac{\partial S_{l_1}}{\partial t} + \nabla \cdot \langle u_{l_1} \rangle = \dot{m} \quad ; \quad \frac{\partial S_{l_2}}{\partial t} + \nabla \cdot \langle u_{l_2} \rangle = -\dot{m}
\]
Modeling of the dispersion of liquid? (3)

- Closure of the liquid transfer at contact points:

\[ \dot{m} = h_1 \left( \langle p_{l_2} \rangle^{l_2} - \langle p_{l_1} \rangle^{l_1} \right) \]

- Assuming capillary pressure effects at contact points

\[ P_{c_i} (S_i) = \langle p_g \rangle^g - \langle p_{l_i} \rangle^{l_i} \]

\[ \dot{m} = h_1 (P_{c_2} (S_{l_2}) - P_{c_1} (S_{l_1})) \]

- Brooks and Corey relation

\[ P_{c_i} = P_{c_0} S_i^{0.5} \]
Test case (1)

- Variation of the liquid exchange coefficient

| $h_1$ | 0.1 | 1 | 10 |

- 9 injection points

\[
K_{ll_i}^+ = K_{ll_i}^* \begin{pmatrix}
0 & 0 & 0 \\
0 & \cos^2 \theta^*_i \sin^2 \theta^*_i & \pm \cos \theta^*_i \sin \theta^*_i \\
0 & \pm \cos \theta^*_i \sin \theta^*_i & \cos^2 \theta^*_i \sin^2 \theta^*_i
\end{pmatrix} \quad i = 1, 2
\]

\[
K_{ll_i}^* = K_{ll_i} \begin{pmatrix}
\cos^2 \theta^*_i \sin^2 \theta^*_i & 0 & \pm \cos \theta^*_i \sin \theta^*_i \\
0 & 0 & 0 \\
\pm \cos \theta^*_i \sin \theta^*_i & 0 & \cos^2 \theta^*_i \sin^2 \theta^*_i
\end{pmatrix} \quad i = 1, 2
\]
Test case (2) - Regimes of dispersion
Perspectives

■ Accurate closure of the effective parameters
  
  • Numerical modeling at the local-scale (VOF)

■ Impact of the coupled system on the dispersion?

■ Closure of the liquid exchange term $\hat{m}$
  
  • Numerical modeling at the micro-scale (quantification of the liquid exchange)
Perspectives

- Accurate closure of the effective parameters
  - Numerical modeling at the local-scale (VOF)

- Impact of the coupled system on the dispersion?

- Closure of the liquid exchange term $\dot{m}$
  - Numerical modeling at the micro-scale (quantification of the liquid exchange)
Perspectives

- Accurate closure of the effective parameters
  - Numerical modeling at the local-scale (VOF)

- Impact of the coupled system on the dispersion?

- Closure of the liquid exchange term $\hat{m}$
  - Numerical modeling at the micro-scale (quantification of the liquid exchange)
Annexe - Calide experiment - Retention

(a) Gas retention $S_g$ as a function of the gas phase velocity
Closures for the viscous terms - based on experimental results

\[
K_{ll} = K_0 S_l^3 \quad ; \quad K_{lg} = 5.5 \frac{\mu_g}{\mu_l} \frac{S_l^2}{(1 - S_l)} \\
K_{gg} = K_0 (1 - S_l)^4 \quad ; \quad K_{gl} = \frac{\mu_l}{\mu_g} \frac{K_{gg} K_{lg}}{K_{ll}}
\]

Closures for the viscous terms - based on experimental results

\[
F_{ll} = \frac{\rho_l}{\mu_l} K \langle u_l \rangle \quad ; \quad F_{lg} = K_{lg} \frac{(1 - S_l)^3}{(1 - S_l)^3 + S_l^n} \\
F_{gg} = \frac{\rho_g}{\mu_g} K \langle u_g \rangle \quad ; \quad F_{gl} = f_{lg} (1 - S_l)^6
\]
Annexe - Calide experiment - Effective parameters (1)

(a) Intrinsic and couples permeabilities - Experimental results (Clavier)
Annexe - Calide experiment - Effective parameters (2)

(b) \( \langle u_L \rangle = 0.02 \ (Re = 128) \)
Annexe - air / water system in packings - Effective parameters (1)

(a) Viscous permeabilities $K^*$
Annexe - air / water system in packings - Effective parameters (2)

(b) Inertial correction terms $F$