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Eprints ID: 16007

To link to this article: DOI: 10.1016/j.ipm.2015.08.003
URL: http://dx.doi.org/10.1016/j.ipm.2015.08.003

To cite this version: Doumbouya, Mamadou Bilo and Kamsu-Foguem, Bernard and Kenfack, Hugues Argumentation and graph properties. (2016) Information Processing & Management, vol. 52 (n° 2). pp. 319-325. ISSN 0306-4573

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Argumentation and graph properties

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Abstract

Argumentation theory is an area of interdisciplinary research that is suitable to characterise several diverse situations of reasoning and judgement in real world practices and challenges. In the discipline of Artificial Intelligence, argumentation is formalised by reasoning models based on building and evaluation of interacting arguments. In this argumentation framework, the semantics of acceptance plays a fundamental role in the argument evaluation process. The determination of accepted arguments under a given semantics (admissible, preferred, stable, etc.) can be a time-consuming and tedious (in number of steps) process. In this work we try to overcome this substantial process by providing a method to compute accepted arguments from an argumentation framework. The principle of this method is to combine mathematical properties (e.g. symmetry, asymmetry, strong connectivity and irreflexivity) of graphs built from the argumentation system to compute sets of accepted arguments. In this work, we combine several graph properties to provide three main propositions; one for identifying accepted arguments under the admissible, preferred semantics and the other to easily identify stable extension. The proofs of the suggested propositions are detailed and this is part of an approach designed to increase collaborative decision-making by improving the effectiveness of reasoning processes.

1. Introduction

Argumentation, precisely abstract argumentation has been introduced by Dung (1995) in the 1990s. An argumentation system consists of a couple \((A, R)\), where \(A\) is a set of elements called also arguments and \(R\) a binary relation representing attack relation between arguments. Argumentation and acceptability are themes common to all our activities, and we deal with them in an interdisciplinary fashion. Today, the complex projects have become even more collaborative and online. In collaborative working environment, competencies such as information communication and knowledge sharing, providing a service as well as a product and the aptitude to reason in an interdisciplinary manner become decisive. Particularly, in remote collaborative activities (Doumbouya et al., 2015a; Doumbouya et al., 2015b; Doumbouya et al., 2015c) (like e-heath), professionals work regularly and methodically together in an interdisciplinary way to ensure that actions and practices are well-harmonised and of high quality. Argumentation systems have been used in several works to help in decisions making process for example in Doumbouya et al. (2015a), Amgoud and Prade (2009), Amgoud and Vesic (2014), Villata et al. (2013) and Cayrol and Lagasque-Schiex (2013). The process of decisions making consists of knowing arguments that should be accepted under a given semantics (admissible, preferred, stable, etc.) in an effort to consolidate the elements of decisions.

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http://dx.doi.org/10.1016/j.ipm.2015.08.003
In most of these published papers, before knowing the arguments that should be accepted under a given semantics, one has to build first extensions or labellings, because in the literature, there are several approaches (Baroni & Giacomin, 2009) to build argumentation semantics. The most common are:

- Extension-based approach is a theoretical reasoning in which the semantics specification concerns the generation of a set of extensions from an argumentation framework.
- Labelling-based approach is a theoretical reasoning in which the semantics specification concerns the generation of a set of labellings (e.g., possible alternative states of an argument) from an argumentation framework.

In the extension-based approach, there are several steps to follow for determining the acceptable arguments, namely:

1. Determination of conflict-free sets.
2. Determination of extensions:
   - admissible extensions,
   - preferred extensions;
   - complete extensions;
   - stable extensions...

We think that the process is too tedious, given that our previous works (Doumbouya et al., 2015a; Doumbouya et al., 2015b; Doumbouya et al., 2015c) uses the extension-based approach, it is why we propose this work to overcome this heavy process whenever it is possible. Our approach is based on the properties (strong connectivity, symmetric, asymmetric, irreflexive) of the graph theory, given that the argumentation systems use the visual representation framework with a directed graph modelling the attack relations between arguments. This process will permit to bypass the computation of extensions when it is possible. In other words by these properties of the graph we can compute easily the acceptable arguments under a given semantics (admissible, preferred, stable).

In the following, we firstly recall some basic notions of argumentation system and graph theory, we secondly explain our work in the materials and methods section followed by a discussion section, and finally we end this work by a conclusion with perspectives.

2. Background

2.1. Argumentation system

2.1.1. What is argumentation?

Argumentation is a reasoning model based on building and evaluation of interacting arguments. Argumentation theory is generally applied to non-monotonic reasoning, decisions making or for types of dialogue modelling such as negotiation. Most of developed models are based on Dung argumentation system (Dung, 1995). This methodological framework is composed of a set of arguments and binary relations emphasising potential conflicts between arguments.

2.1.2. Definitions and properties related to argumentation system

**Definition 1.** An (argumentation-based) decision framework $AF$ is a couple $(A, D)$ where:

- $A$ is a set of arguments
- $D$ is a set of actions, supposed to be mutually exclusive
- action: $A \rightarrow D$ is a function returning the action supported by an argument

**Definition 2.** From an argumentation-based decision framework $(A, D)$, an equivalent argumentation framework $AF = (A, R)$ is built where:

- $A$ is the same set of arguments
- $R \subseteq A \times A$ is a binary attack relation ($\alpha$ attacks $\beta$ is denoted $\alpha R \beta$ or $(\alpha, \beta) \in R$)

**Definition 3.** Let $AF = (A, R)$ be an argumentation framework, and let $B \subseteq A$

- $B$ is conflict-free if there are no $\alpha, \beta \in B$ such that $(\alpha, \beta) \in R$
- $B$ defends an argument $\alpha$ iff $\forall \beta \in A$, if $(\beta, \alpha) \in R$, then $\exists \gamma \in B$ such that $(\gamma, \beta) \in R$

**Definition 4 (Acceptability semantics).** Let $AF = (D, A, R)$ be a decision system, and $B$ be a conflict-free set of arguments.

- $B$ is admissible extension iff it defends any element in $B$.
- $B$ is a preferred extension iff $B$ is a maximal (w.r.t set $\subseteq$) admissible set.
- $B$ is a stable extension iff it is a preferred extension that defeats any argument in $A \setminus B$.

**Definition 5 (Argument status).** Let $AF = (D, A, R)$ be a decision system, and $\varepsilon_1, \ldots, \varepsilon_k$ its extensions under a given semantics. Let $a \in A$. 
• a is skeptically accepted iff \( a \in I_i \), \( \forall I_i \) with \( i = 1, \ldots, x \).
• a is credulously accepted iff \( \exists I_i \) such that \( a \in I_i \).
• a is rejected iff \( \exists I_i \) such that \( a \in I_i \).

**Property 1.** Let \( A^f = (D, A, R) \) be a decision system, and \( I_1, \ldots, I_x \) its extensions under a given semantics. Let \( a \in A \).

• a is skeptically accepted iff \( a \in \bigcap_{i=1}^{x} I_i \)
• a is rejected iff \( a \not\in \bigcup_{i=1}^{x} I_i \)

2.2. Recall of some definitions and properties on graphs

The graph theory (Carré, 1991) has been used since many years in several fields of science such as Artificial Intelligence (Levi & Sirovich, 1976; Sanfilippo, 2006), for problem solving in Artificial Intelligence domain. It has been used for several works in reasoning such as conceptual graphs (Kamsu-Foguem et al., 2013; Kamsu-Foguem & Noyes, 2013; Kamsu-Foguem et al., 2014; Sowa, 1992), and some works of argumentation theory (Baroni et al., 2005; Coste-Marquis et al., 2005). In this section we recall some basic notions of graphs theory including namely definitions.

**Definition 6.** A graph is a couple \((V, E)\) where :

• \( V \) is a set (finite) of objects. The elements of \( V \) are called vertex of the graph.
• \( E \) is a subset of \( V \times V \). The elements of \( E \) are called edges of the graph.

**Definition 7.**

• Two vertices \( x \) and \( y \) are adjacent if it exists the edge \((x, y)\) in \( E \). The vertices \( x \) and \( y \) are then called neighbours.
• An edge is incident to a vertex if \( x \) is one of its ends.
• The degree of a vertex \( x \) is the number of incident edges to \( x \). It is denoted \( d(x) \). For a simple graph, the degree of \( x \) corresponds to the number of adjacent vertices to \( x \). Thus a vertex with degree 0 is said isolated.

**Definition 8.** A graph is connected iff there exists a path between each pair of vertices.

**Definition 9.** An oriented graph is strongly connected if for any couple of vertices \( x, y \) there exists a path connecting \( x \) to \( y \).

**Definition 10.** A graph is said symmetric if for any vertices \( x \) and \( y \) if \((x, y) \in E \) then \((y, x) \in E \).

**Definition 11.** Some additional definitions:

• Reflexive graph: \( \forall x \in V, (x, x) \in E \)
• Irreflexive graph: \( \forall x \in V, (x, x) \not\in E \)
• Asymmetric graph: \( \forall x, y \in V, (x, y) \in E \Rightarrow (y, x) \not\in E \) (If \( G \) is asymmetric then \( G \) is irreflexive).
• Antisymmetric graph: \( \forall x, y \in V, (x, y) \in E \) and \((y, x) \in E \Rightarrow x = y \) (If \( G \) is asymmetric then \( G \) is antisymmetric).
• Transitive graph: \( \forall x, y, z \in V, (x, y) \in E \) and \((y, z) \in E \Rightarrow (x, z) \in E \).

3. Materials and methods

In the following \( V = A \) and \( E = R \) (refer to the definitions and properties above on the argumentation theory). In our research, we are working with structured argumentation system wherein nodes have internal structure. Bourguet in (Bourguet, 2011) has proposed an algorithm for building graphs of attack in such argumentation system and has demonstrated that the generated graph is irreflexive and may be neither symmetrical, nor be transitive. So, it is not an obligation that a graph to be symmetrical or transitive.

The method we propose aims to reduce the computational steps of acceptable arguments under the admissible and preferred semantics. This method is based on the combination of some graph properties namely irreflexivity, symmetry and strong connectivity. When the graph of attacks is built, then it is checked if this graph satisfies the properties that are relevant and interesting to us. This process is depicted in Fig. 1.

This process is applied to the resulted graph of attacks. The process checks if the graph satisfies the specified properties in our proposed Algorithm 1:

• if yes, it computes to know if there are acceptable arguments under a given semantics;
• otherwise, the process continues with the extension-based approach, which is one of the most used approach, which is also used in our previous works (Doumbouya et al., 2015a; Doumbouya et al., 2015b; Doumbouya et al., 2015c).

In this work we focused on three main graph properties (see Proposition 1). So when the resulted graph of attacks satisfies these properties then all its edges (arguments) are extracted and returned as acceptable arguments under the admissible, preferred semantics.
Algorithm 1: Algorithm for retrieving credulously accepted arguments under the admissible and preferred semantics.

**Data:** Graph of attacks $G$, all arguments of the set $S$

**Result:** Return a set of arguments that are credulously accepted under the admissible and preferred semantics

/* The method isStronglyConnected() checks if the graph $G$ is strongly connected, isSymmetric() verifies if it is symmetric and isIrreflexive() to check if the graph $G$ is irreflexive */

if $G$ isStronglyConnected() && $G$ isSymmetric() && $G$ isIrreflexive() then
    return $S$
end

return null;

3.1. Propositions and proofs

**Proposition 1.** If the graph of attacks is strongly connected, symmetric et irreflexive then all the arguments are credulously accepted under admissible, preferred semantics.

See Fig. 2 for an illustration of such type of graph.

**Proof.**

(1) All the arguments taken one by one represent conflict-free sets due to the irreflexive property of the graph.

- if $u$ and $v$ are adjacent: this means that $u$ et $v$ attack each other since the graph is symmetric and thus {$u$} and {$v$} are admissible.
• if \( u \) and \( v \) are not adjacent and the \( \{u, v\} \) is conflict-free. Suppose that there is an argument \( z \) such that \( u \) and \( z \) attack each other and \( z \) and \( v \) attack each other too.
  • Let us demonstrate that \( \{u, v\} \) defends each of its elements. \( z \) attacks \( u \) and \( v \in \{u, v\} \) attacks \( z \), therefore \( \{u, v\} \) defends \( u \). Given that the graph is symmetric, one demonstrates in the same manner that \( \{u, v\} \) defends \( v \). Thus \( \{u, v\} \) without conflict and defending each of its elements, \( \{u, v\} \) est therefore admissible.
  • So by applying this demonstration in an iterative manner on conflict-free sets, one will demonstrate their possible admissibility.
  • Given that any preferred extension is an admissible one in the way of maximal inclusion, then the arguments of admissible extensions are necessarily included in one of the preferred extensions.
  • An argument is credulously accepted under a given semantics if it is included at least in one extension of this semantics. One concludes that all the arguments are credulously accepted under admissible, preferred semantics. □

Proof.

(2) Given that the graph is symmetric, then any attacked argument defends itself and thus belongs to the admissible extension uniquely formed by this argument due to the irreflexive property of the graph. The property strong connectivity of the graph of attacks guarantees that there is no isolated argument (argument outside the system of interactions through the attack relation). In conclusion all the arguments forming this type of graph are credulously accepted under the admissible semantics and therefore under the preferred semantics too. □

Proposition 2. In a symmetrical graph two adjacent elements are never included in the same extension.

Proof. The proof is obvious, because between two adjacent elements there is always an attack relation. These two elements cannot be in the same conflict-free set and thus can never belong to the same extension. □

Proposition 3. If the graph of attack is asymmetric and irreflexive and if there is an argument that attacks all the others then there is a unique stable extension formed by this argument.

Proof.

• The irreflexive property of the graph guarantees that an argument does not attack itself and thus the set formed by this argument is conflict-free.
  • Let \( \alpha \) such that \( \forall x \in A, (\alpha, x) \in R \), this means that \( \alpha \) always attacks and is never attacked in return because of the asymmetric property of the graph. Thus \( \{\alpha\} \) is an admissible extension.
  • And thus as any argument outside the singleton \( \{\alpha\} \) is attacked by \( \alpha \) then \( \{\alpha\} \) is a stable extension.
  • Uniqueness demonstration:
    • Let assume that there exist two stable extensions \( \{\alpha\} \) and \( \{\beta\} \) this means that \( \alpha \) attacks any element out of \( \{\alpha\} \) whose \( \beta \) and also that \( \{\beta\} \) attacks any element out of \( \{\beta\} \) whose \( \alpha \), which is contradictory given the attack graph is asymmetric.
    Therefore there is a unique stable extension. □

3.2. Algorithmic implementation of Proposition 1

Proposition 1 is the main contribution of this paper, that is why we find it interesting to propose an algorithmic implementation of this proposition.

The provided Algorithm 1 takes as inputs a graph \( G \) and a set \( S \) composed of all the edges (here called arguments) of \( G \). And after it checks if the \( G \) verifies the properties of strong connectivity, symmetry and irreflexivity. Two cases will be triggered:

• Either \( G \) satisfies all the properties, and then \( S \) is returned.
• Either \( G \) does not satisfy the condition, and the algorithm returns null.

We assume that, there already exits algorithms to check the different properties of a graph depending on its abstract structure. Computational complexity: Here, the complexity of our algorithm depends on those of the conditional test (strong connectivity, symmetric and irreflexivity algorithms), the first possible processing (if the condition is true) and the second possible processing (if the condition is false). Since it is a conditional treatment then the computational complexity of Algorithm 1 is determined as follows:

\[
\text{complexityOf(Algorithm 1)} = \text{complexityOf(isStronglyConnected())} + \text{complexityOf(isSymmetric())} + \text{complexityOf(isIrreflexive())} + O(1)
\]

4. Discussions

In the literature, the two most common approaches to define argumentation semantics (Baroni & Giacomin, 2009), are:

• Extension-based
• Labelling-based
Table 1
Comparison of the different methods.

<table>
<thead>
<tr>
<th>Our proposal</th>
<th>Extension-based approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Checking if the graph satisfies the properties of strong connectivity, symmetry and irreflexivity. Here it is verified.</td>
<td>1. Determination of conflict-free sets: {\emptyset}, {\alpha}, {\beta}, {\gamma}, {\delta}, {\beta, \gamma}, {\beta, \delta}, {\gamma, \delta}, {\beta, \gamma, \delta}.</td>
</tr>
<tr>
<td></td>
<td>2. Determination of admissible extensions: (e_1=\emptyset), (e_2={\alpha}), (e_3={\beta, \gamma}), (e_4={\beta, \delta}), (e_5={\gamma, \delta}), (e_6={\beta, \gamma, \delta}).</td>
</tr>
<tr>
<td></td>
<td>3. Determination of preferred extensions: (e_3={\alpha}) and (e_6={\beta, \gamma, \delta}).</td>
</tr>
<tr>
<td>Accepted arguments: (\alpha, \beta, \gamma, \delta) are credulously accepted under the preferred semantics</td>
<td></td>
</tr>
</tbody>
</table>

It is clearly observed that the expressiveness of the two approaches is equivalent in terms of definitions and formulations. The extension-based approach is the most popular since it has been used in several works deal with argumentation and decision making process. This approach is also recalled by Baroni and Giacomin (2009). It generally consists to determine some extensions and build acceptable arguments under a given semantics.

In Modgil and Caminada (2009), the authors proposed a method to build labellings. Their method is based on labelling and their aim is to provide an easy and an intuitive account of formal argumentation. We think that their method is not easy as they assume. We consider that our proposal is more easier because many people are familiar with the graph theory and visual reasoning used for manipulating the graphs.

In this work contrary to the previous ones (Doumbouya et al., 2015a; Doumbouya et al., 2015b; Doumbouya et al., 2015c), we intend to bypass the generation of extensions for computing acceptable arguments. In other words, we proposed a method that is based on the graph properties to compute the acceptable arguments under a given semantics from an argumentation system.

The graph of Fig. 2 illustrates a graph of attacks that respects the properties mentioned in the proposition above: \(\alpha, \beta, \gamma\) and \(\delta\) represent the edges of the graph called arguments.

Table 1 above shows the comparison between our proposal and the extension-based approach where arguments are accepted under the preferred semantics. In this comparison, one can see that when the properties are satisfied, one needs only one step to perform the acceptable arguments in our proposal while in the extension-based approach, one needs three steps to get the accepted arguments.

Even if several works e.g. Coste-Marquis et al. (2005) (symmetry) and Baroni et al. (2005) (strong connectivity) have been achieved in the field of argumentation and graphs theory, and also through complexity issues in argumentation (Baumann, 2011; Coste-Marquis et al., 2005; Dunne, 2007; Dvořák et al., 2012), however, most of these works deal with one graph property at a time while in this contribution we try to combine several graphs properties. But unlike our work they do not use graph properties in their reasoning process for the generation of acceptable arguments. The work proposed by Charwat et al. (2015) is for solving reasoning problems in argumentation systems by the use of algorithms (for instance algorithm for deciding skeptical acceptance under preferred semantics, for computing preferred labellings, etc.). Several of these studies are focused on one graph property i.e. symmetry for Coste-Marquis et al. (2005) even if they combine the property of symmetry and irreflexivity and strong connectivity for Baroni et al. (2005) while our proposal combines several graph properties such as symmetry, irreflexivity and strong connectivity in Proposition 1 and asymmetry and irreflexivity in Proposition 3. Indeed, in addition to the work of Coste-Marquis et al. (2005), we added a new graph property (strong connectivity) for proposing a new proposition namely Proposition 1 in order to bring out potential acceptable arguments under admissible, preferred semantics as our main work is to provide a rigorous decision making framework.

![Fig. 2. Graph of attacks.](image-url)
5. Conclusion

All the works concerning argumentation are based on the argumentation semantics formalising the argument evaluation process which can be done by a method (either procedural or declarative) using an extension-based approach or a labelling-based approach (Baroni & Giacomin, 2009). In this work, we try to bypass the computation of these extensions by providing a method based on the graph properties to know the arguments that should be accepted under a given semantics.

Here we focus on the properties of strong connectivity, symmetry and irreflexivity and the semantics of admissibility, preferability and stability. We believe that this work will reduce the computation steps to know arguments that should be accepted under a given semantics.

In perspective, we will investigate other properties of the graph theory to know their impact on the final acceptance of an argument or a set of arguments and at the same time make an implementation of one of the reduction-based approaches exploiting current effective software tools established for other goals (Charwat et al., 2015) (for instance, the satisfiability in propositional logic (SAT)-based systems, constraint-satisfaction problems (CSP)-based approach or answer-set programming (ASP)-based approach). However, we think that the ASP-based approach is more suitable for us, because it has commonalities with our previous works (Doumbouya et al., 2015a; Doumbouya et al., 2015b; Doumbouya et al., 2015c), namely:

• the support of query-based implementation;
• the argumentation framework used as an input database.

These two commonalities are essential in the target applications (e.g. collaborative reasoning and scientific expertise), since the argumentation framework can be used as an input database, this means that data can be retrieved by queries also from remote databases and then could facilitate the decision making process in remote collaboration activities.

References