STEERING VECTOR UNCERTAINTIES AND DIAGONAL LOADING

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ABSTRACT

In this paper, we study the performance of diagonally loaded beamformers in the presence of steering vector errors i.e. when there exists a mismatch between the actual steering vector of interest and the presumed one. We consider the problem of optimally selecting the loading level with a view to maximising the signal to interference plus noise ratio. Ignoring finite-sample effects we show that, in the presence of strong low-rank interference and broadband noise, the optimal loading level is negative. Numerical simulations attest to the validity of the analysis and show that diagonal loading with the optimal loading factor derived herein provides a performance close to optimum. It is also shown that negative diagonal loading is effective in finite-sample as long as the steering vector errors dominate over the finite-sample errors.

1. INTRODUCTION

It is a well-known fact that steering vector errors are detrimental to the performance of adaptive beamformers, especially when the signal of interest (SOI) is present in the measurements. In this case, the SOI is considered as an interference and thus tends to be eliminated by the adaptive beamformer [1]. However, mismatches between the actual steering vector and the presumed steering vector are inevitable in practice. They can be due e.g. to local scattering around the source, an inhomogeneous propagation medium, uncalibrated arrays or arrays undergoing deformations. Hence, designing robust adaptive beamformers that can maintain good signal to interference plus noise ratio (SINR) under these conditions is of utmost importance [1, 2]. Among the many robust adaptive beamformers proposed in the literature, diagonal loading [3] emerges as the most widely used due to its simplicity and its effectiveness in handling a wide variety of errors, including steering vector and finite-sample errors. Interestingly enough, it has also proved to be the solution to worst-case approaches proposed recently in [4–6]. The principle behind these approaches is to protect the beamformer's response for all steering vectors which lie in some ellipsoid centered around the nominal steering vector. It turns out that the solution to this problem amounts to generalized (indeed diagonal when the ellipsoid is a sphere) loading of the covariance matrix, with a loading level that is chosen adaptively, i.e. it depends on both the covariance matrix of the data and the ellipsoid.

A crucial issue is to select the loading level whether one considers fixed diagonal loading or the robust beamformers of [4–6] (in the latter case the problem is tantamount equivalent to choosing the size of the uncertainty ellipsoid). Indeed, the loading level enables to balance between a fully adaptive beamformer (no loading) and the conventional non-adaptive beamformer (infinite loading). Hence, its performance can vary quite significantly and finding an optimal loading level is of major interest. A good rule of thumb is to select the loading level some 5-10dBs above the noise level, see e.g. [1, Chapter 6]. Alternatively, a meaningful way of selecting the loading level is to fix the white noise gain (WNG), as suggested in [7], since diagonal loading corresponds to constraining the WNG. This is a physically appealing approach as the WNG enables to control the degree of adaptivity of the beamformer.

In this paper, we address the problem of finding the loading level which results in maximum SINR in the presence of steering vector errors. Towards this end, we provide an expression for the optimal loading level for any steering vector error. Since the optimal loading level—and thus the corresponding SINR—depends on the actual steering vector, we next consider random steering vector errors. The optimal loading level is then averaged with respect to (w.r.t) the probability density function (pdf) of the steering vector errors, resulting in a simple formula for the average optimal loading level.

2. DATA MODEL

Let the output of an array of m sensors be given by

$$\mathbf{r}_t = \mathbf{a} \mathbf{s}_t + \mathbf{n}_t \quad t = 1, \ldots, N$$

(1)

where $\mathbf{a}$ is the actual (unknown) steering vector of the source of interest. We assume that $\mathbf{n}$ differs from the nominal or presumed steering vector $\bar{\mathbf{a}}$ due e.g. to uncertainties about the direction of arrival (DOA), unknown gains and phases of the sensors, etc. $\mathbf{s}_t$ is the signal of interest waveform and is assumed to be a zero-mean random process with power $P = \mathbb{E} \{ |s_t|^2 \}$ while $\mathbf{n}_t$ is the noise contribution, including $K$ interferers and thermal noise. Hence, the covariance matrix $\mathbf{C} = \mathbb{E} \{ \mathbf{n}_t \mathbf{n}_t^H \} = \mathbf{A}^H \mathbf{P} \mathbf{A} + \sigma^2 \mathbf{I}$ where $\mathbf{A}$ stands for the $K$-dimensional interference subspace.

In this paper, we consider the use of diagonal loading-based beamformers of the form

$$\mathbf{w} = (\mathbf{R} + \lambda \mathbf{I})^{-1} \bar{\mathbf{a}}$$

(2)

to handle the problem of steering vector errors. In (2), $\mathbf{R}$ denotes the covariance matrix, $\mathbf{I}$ is the identity matrix of size $m$ and $\lambda$ is a real weighting factor. Observe that we do not consider here finite-sample effects, i.e. we assume that the true covariance matrix is available. In order to consider both steering vector errors and finite-sample effects, one needs to assume that the two errors are of the same order of magnitude (typically $O(1/N)$), see [8]. However, this assumption may seem arbitrary since the errors are not likely to depend on $N$. Therefore, herein we consider that $N$ is large enough so that the steering vector errors dominate.
3. DERIVATION OF OPTIMAL LOADING LEVEL

3.1. Optimization for a given steering vector error

Let us first assume that \( \alpha \) is fixed and use the subscript \( i_a \) to emphasize it. Let

\[
w_{i_a} = (R_{i_a} + \lambda I)^{-1} \vec{a}
\]

denote the weight vector where \( R_{i_a} = P \alpha a H \) and \( C \) stands for the covariance matrix for a given \( \alpha \). The conditional SINR corresponding to the weight vector in (3) is thus given by

\[
\text{SINR}_{i_a} = \frac{P |w_{i_a}^H a|^2}{w_{i_a}^H C w_{i_a}}
\]

\[
= \frac{P |\vec{a}^H (R_{i_a} + \lambda I)^{-1} \vec{a}|^2}{\vec{a}^H (R_{i_a} + \lambda I)^{-1} C (R_{i_a} + \lambda I)^{-1} \vec{a}}
\]

Using Woodbury’s identity, and after some straightforward algebraic manipulations, it can be shown that

\[
\text{SINR}_{i_a} = \frac{1}{P (a - \gamma(a, \lambda) \vec{a})^H Z(\lambda) (a - \gamma(a, \lambda) \vec{a})}
\]

where

\[
\gamma(a, \lambda) = \frac{1 + P \alpha^H (C + \lambda I)^{-1} \vec{a}}{P \alpha^H (C + \lambda I)^{-1} \vec{a}}
\]

\[
Z(\lambda) = (C + \lambda I)^{-1} C (C + \lambda I)^{-1}
\]

For notational convenience, let

\[
f(\lambda) = (a - \gamma(a, \lambda) \vec{a})^H Z(\lambda) (a - \gamma(a, \lambda) \vec{a})
\]

be the function we wish to minimize w.r.t. \( \lambda \). Also, let

\[
C = U \Lambda U^H = U \Lambda_j \Lambda_j U^H + \sigma^2 U_n U_n^H
\]

be the eigen-decomposition of the interference plus noise covariance matrix \( C \) where \( U_j \) and \( \Lambda_j \) contain the \( K \) principal eigenvectors and eigenvalues respectively.

In the sequel we assume that for \( k = 1, \ldots, K \) and \( \ell = K + 1, \ldots, m \),

\[
\frac{|a^H u_k|^2}{\lambda_k} \ll \frac{|a^H u_{k+\ell}|^2}{\lambda_k} \ll \frac{|\vec{a}^H u_{k+\ell}|^2}{\sigma^2}
\]

This approximation is valid as soon as the eigenvalues corresponding to the interferences (\( \lambda_k \) for \( k = 1, \ldots, K \)) are large compared to the noise level \( \sigma^2 \)-high interference to noise ratio (INR)- or as soon as the projection of the steering vector of interest \( a \) (as well as the presumed steering vector) onto the interference subspace is small which amounts to consider that the interferences are outside the main beam of the array. It should be pointed out that in this type of situation that the use of a robust beamformer based on diagonal loading is advisable.

Under this assumption, we first show that \( \gamma(a, \lambda) \) is a linear function of \( \lambda \). Indeed,

\[
a^H (C + \lambda I)^{-1} a = \sum_{k=1}^m \frac{|a^H u_k|^2}{\lambda_k} \approx \sum_{k=K+1}^m \frac{|a^H u_k|^2}{\lambda_k + \sigma^2}
\]

\[
= \frac{a^H U_n U_n^H \vec{a}}{\lambda + \sigma^2} = \frac{a^H \vec{a}_n}{\lambda + \sigma^2}
\]

where \( U_n = [u_{K+1} \ldots u_m] \) is the sub-dominant subspace of \( C \) and \( \vec{a}_n = U_n^H \vec{a} \). Similarly,

\[
a^H (C + \lambda I)^{-1} \vec{a} = \sum_{k=1}^m \frac{a^H u_k u_k^H \vec{a}}{\lambda + \lambda_k} 
\]

\[
\approx \sum_{k=K+1}^m \frac{a^H u_k u_k^H \vec{a}}{\lambda + \sigma^2}
\]

\[
= \frac{a^H U_n U_n^H \vec{a}}{\lambda + \sigma^2} = \frac{a^H \vec{a}_n}{\lambda + \sigma^2}
\]

\[
(11)
\]

\[
\vec{a}_n = U_n^H \vec{a}.
\]

Using (10)-(11) along with the expression (6) for \( \gamma(a, \lambda) \), it follows that

\[
\gamma(a, \lambda) \approx \frac{\sigma^2 + P \alpha^H \vec{a}_n}{P \alpha^H \vec{a}_n} + \frac{\lambda}{P \alpha^H \vec{a}_n} 
\]

\[
\gamma_0 + \gamma_0 \lambda
\]

\[
(12)
\]

Therefore

\[
f(\lambda) \approx \sum_{k=1}^m \frac{\lambda_k |a - \gamma(a, \lambda) \vec{a}|^2}{(\lambda + \lambda_k)^2} 
\]

\[
\approx \sum_{k=K+1}^m \frac{\sigma^2 |a - \gamma(a, \lambda) \vec{a}|^2}{(\lambda + \sigma^2)^2}
\]

\[
= \frac{\sigma^2}{(\lambda + \sigma^2)^2} \left| U_n^H (a - \gamma(a, \lambda) \vec{a}) \right|^2
\]

\[
= \frac{\sigma^2}{(\lambda + \sigma^2)^2} \left| \alpha + \alpha' \vec{a} \right|^2
\]

\[
(13)
\]

with \( \alpha \Delta U_n^H (a - \gamma_0 \vec{a}) \) and \( \alpha' = -\gamma_0 U_n^H \vec{a} \). The bottom right-hand side of (13) is only an approximation of the true \( f(\lambda) \) in (7) which holds under the hypothesis (9). The expression in (13) is likely not to be very accurate for large values of \( \lambda \) as the ratios \( \lambda_k |a^H u_k|^2 / (\lambda + \lambda_k)^2 \) and \( \sigma^2 |a^H u_k|^2 / (\lambda + \sigma^2)^2 \) are involved whereas the hypotheses consider \( |a^H u_k|^2 / \sigma^2 \) and \( |a^H u_k|^2 / (\lambda + \sigma^2)^2 \), respectively. However, choosing a large value for \( \lambda \) is not advisable since it would be tantamount to using a non-adaptive beamformer whose performance is likely to be very poor compared to that of an adaptive beamformer. Observe that it is sensible to choose a \( \lambda \) whose magnitude is a few decibels below the noise level (otherwise the interferences would be buried in the artificial noise). For \( \lambda \)'s close to the optimal value, we will show in the next section that (13) closely matches the true \( f(\lambda) \).

Differentiating (13) and setting the result to zero, it holds that

\[
\frac{\partial f}{\partial \lambda} = 0 \Rightarrow \lambda_{\text{opt}}^a = \frac{\| \alpha \|^2 - \sigma^2 \text{Re} [\alpha^H \alpha']}{\sigma^2 \| \alpha' \|^2 - \text{Re} [\alpha^H \alpha']}
\]

\[
(14)
\]

Furthermore, using the expressions of \( \gamma_0 \) and \( \gamma_0 \) in (12), we have

\[
\| \alpha \|^2 = \| \alpha_n \|^2 - 2 \sigma^2 + \frac{\sigma^2 + P \| \alpha_n \|^2}{P^2} \| \vec{a}_n \|^2 
\]

\[
(15)
\]

\[
\| \alpha' \|^2 = \| \vec{a}_n \|^2 - 2 \gamma_0^2 \| \vec{a}_n \|^2 
\]

\[
(16)
\]

\[
\text{Re} [\alpha^H \alpha'] = -\frac{1}{P} + \frac{\sigma^2 + P \| \alpha_n \|^2}{P^2} \| \vec{a}_n \|^2 
\]

\[
(17)
\]
Inserting these expressions into (14) yields, after some straightforward derivations, the following simple expression of the optimal loading level for a given error on the steering vector:

\[ \lambda_{oa}^{\text{opt}} = -(\sigma^2 + P ||a_n||^2) \]  

(18)

The following comments are in order. The first important thing to be noted is that \( \lambda_{oa}^{\text{opt}} \) is always negative which is quite an unexpected result as usually a positive loading level is always considered. However, this seemingly surprising should be re-examined under the following grounds. It has been noted recently that negative diagonal loading may outcome as a possible solution to the doubly constrained robust Capon beamformer of [6]. Hence, negative diagonal loading might not be such an unexpected result. As will be illustrated in the next section, positive diagonal loading is also able to compensate for steering vector errors but does not manage to provide as high a SINR as negative loading with \( \lambda_{oa}^{\text{opt}} \).

Note also that \( \lambda_{oa}^{\text{opt}} \) depends on the noise level, the source power and the squared norm of the projection of the steering vector onto the subspace orthogonal to the interference subspace. Since these parameters are not known (even if \( \sigma^2 \) can be accurately estimated), \( \lambda_{oa}^{\text{opt}} \) cannot be computed for any given \( \alpha \). However, it provides a rough order of magnitude of the optimal loading level. Moreover, some further approximations can be made - see next section - yielding an even simpler expression. It should be observed that, in contrast to positive diagonal loading, \( R + \lambda_{oa}^{\text{opt}} I \) can be rank-deficient (and hence non invertible) if \(-\lambda_{oa}^{\text{opt}}\) coincides with an eigenvalue of the covariance matrix. Hence, care should be taken in order to avoid this potential source of problem.

Reporting (18) in (13), it is straightforward to show that the SINR corresponding to \( \lambda_{oa}^{\text{opt}} \) is approximately

\[ \text{SINR}_{oa} (\lambda_{oa}^{\text{opt}}) = \frac{P ||a_n||^2}{\sigma^2} \]  

(19)

This is to be compared with the optimal performance obtained with a (hypothetical) clairvoyant beamformer which would know \( a \) and is thus given by

\[ w_{oa}^{\text{opt}} = C^{-1} a \]  

(20)

The SINR corresponding to \( w_{oa}^{\text{opt}} \) is

\[ \text{SINR}_{oa}^{\text{opt}} = P a^H C^{-1} a \]

\[ = \frac{P ||a_n||^2}{\sigma^2} + P \left| \Lambda_{1/2}^{-1/2} a \right|^2 \]

(21)

where \( a_j = U_j^H a \). Comparing (19) with (21), it can be conjectured that diagonal loading with the optimal loading level will have a performance very close to the optimum since the second term in the right-hand side of (21) is small under the hypothesis (9). More precisely, the difference between the two SINRs is likely to be small in the case of high INR or interferences outside the main beam. This fact will be validated in the next section by numerical simulations.

3.2. Optimization for random steering vector errors

Since the optimal loading level \( \lambda_{oa}^{\text{opt}} \) depends on the actual steering vector which is unknown, we propose to characterize it "on average". Towards this end, we assume that the steering vector \( a \) is random with correlation matrix \( R_a = E_a \{ aa^H \} \) where \( E_a \{ \cdot \} \) stands for the statistical expectation with respect to the pdf of \( a \). Under the stated assumptions, it is straightforward to see that

\[ \tilde{X}_{oa}^{\text{opt}} \triangleq E_a \left\{ \lambda_{oa}^{\text{opt}} \right\} = -\left( \sigma^2 + P \text{Tr} \left\{ U_a U_a^H \{ aa^H \} \right\} \right) \]

\[ = -\left( \sigma^2 + P \text{Tr} \left\{ U_a U_a^H R_a \right\} \right) \]  

(22)

A further simplification can be made by noting that, under the stated hypotheses, the projection of the steering vector onto the interference subspace is small so that \( ||a_n||^2 \) can be replaced by \( ||a||^2 \) in (18). Taking the expectation with this modification, we end up with the following very simple expression

\[ \tilde{X}_{oa}^{\text{approx}} = -(\sigma^2 + P \text{Tr} \{ R_a \}) \]  

(23)

We stress the fact that this is a very simple expression which depends in a simple way on the noise level, the source power and the steering vector correlation matrix \( R_a \). Observe that equation (23) is still simpler in the case of DOA uncertainties. Indeed, assume that \( \alpha = \alpha (\hat{\theta}) \) where \( \hat{\theta} \) is a random variable with mean \( \theta \) and some a priori pdf \( p (\theta) \). Whatever \( p (\theta) \), since \( ||a||^2 = m \), we necessarily have \( \text{Tr} \{ R_a \} = m \) and thus

\[ \tilde{X}_{oa}^{\text{approx}} = -(\sigma^2 + P m) \]  

in the case of DOA uncertainties or pointing errors. As will be illustrated next, the use of \( X^{\text{approx}} \) or \( \tilde{X}_{oa}^{\text{approx}} \) enables to obtain a SINR comparable with that of the clairvoyant beamformer in most situations.

4. NUMERICAL ILLUSTRATIONS

In this section, we assess the validity of the analysis presented above for both fixed and random steering vector errors. In all simulations, we consider an uniform linear array of \( m = 10 \) sensors spaced a half-wavelength apart. In a first series of simulation, we consider a fixed steering vector error and validate the expression (18). Towards this end, we consider the case of pointing errors, i.e. the source of interest impinges from \( \Delta \theta \) while its DOA is assumed to be \( \theta^0 \). In addition to the signal of interest, two interferences are present whose DOAs are \(-20^\circ, 30^\circ\) and whose powers are 20dB and 30dB above the white noise level, respectively. Figure 1 displays the exact SINR - given by (4) - and the approximated \( \text{SINR}^{\text{approx}} \) computed from (13) versus the loading level \( \lambda \), for various values of \( \Delta \theta \), ranging from a fifth to half the null-to-null beamwidth (\( BW_{NN} \)) of the array. The vertical line corresponds to \( \lambda_{oa}^{\text{opt}} \). The horizontal (upper) line corresponds to the clairvoyant beamformer and thus to the optimal SINR. From inspection of this figure, it can be seen that the approximation in (13) is very accurate, at least for not too large values of \( \lambda \). The optimal SINR is always obtained for a negative value of \( \lambda \) and this value is very close to \( \lambda_{oa}^{\text{opt}} \). This assesses the validity of our analysis. Additionally, the SINR obtained with \( \lambda_{oa}^{\text{opt}} \) is very close to the optimum SINR. Observe also that a positive loading level also enables to compensate for the pointing errors but it does not provide a SINR as large as that obtained with the optimal negative loading level. The difference is more pronounced when \( \Delta \theta \) increases. Finally, in contrast to positive diagonal loading where a large range of values for \( \lambda \) roughly provide the same SINR, the SINR varies more significantly around
the maximum for negative loading levels. Hence, selecting a negative \( \lambda \) may be more delicate.

For completeness, we now vary \( \Delta \theta \) and, for each value of \( \Delta \theta \), we look for the loading level that results in the largest SINR. In Figure 2, this optimal loading level is compared with \( \lambda_{\text{opt}}^{\text{true}} \) and with \(- (\sigma^2 + Pm)\). As can be seen, \( \lambda_{\text{opt}}^{\text{true}} \) really provides the optimal level, except for very small pointing errors. However, in the latter case, despite the fact that \( \lambda_{\text{opt}}^{\text{true}} \) is not exactly the optimal value, the SINR loss is negligible. In addition, observe that for small pointing errors diagonal loading is not really useful. Also, notice that \( \lambda_{\text{opt}}^{\text{true}} \) is rather close to \(- (\sigma^2 + Pm)\) as \(||a||^2 \approx ||\tilde{a}||^2\); hence this latter value may be used as a further approximation without too much penalizing performance.

In a second series of simulation, we consider random steering vector errors and \( a \) is varied randomly in each Monte-Carlo run. More precisely, the true DOA of the source of interest is uniformly distributed on \([-\Delta \theta, \Delta \theta] \) while the assumed DOA is \( 0^\circ \) and \( \tilde{a} = a(0^\circ) \). The SINR will be plotted versus \( \Delta \theta \) and the latter is normalized to the array null-to-null bandwidth. The signal to noise ratio (SNR) is defined as

\[
\text{SNR} = 10 \log_{10} \left( \frac{P \text{Tr} \{ R_a \}}{\sigma^2} \right)
\]

and corresponds to the array SNR. Similarly to the fixed error case, two interferences are present with the same characteristics as previously. In all simulations, the SINR is evaluated as follows. \( N_e = 500 \) Monte-Carlo simulations are run with a different random \( a \) and, for a given weight vector \( w \), the average SINR is computed as

\[
\text{SINR}(w) = \frac{1}{N_e} \sum_{n=1}^{N_e} P \left| w^H a(n) \right|^2
\]

The average SINR obtained with the clairvoyant beamer (20) is, cf (21)

\[
\text{SINR}^{\text{opt}} = \mathcal{E}_a \left\{ \text{SINR}_{\hat{a}}^{\text{opt}} \right\} = P \text{Tr} \left\{ C^{-1} R_a \right\} \quad (25)
\]

For comparison purposes, we also display the performance of the minimum variance distortionless response (MVDR) beamformer which is given by

\[
w_{\text{MVDR}} = \arg \max_w \frac{\mathcal{E}\left\{ |w^H a s_n|^2 \right\}}{\mathcal{E}\left\{ |w^H n|^2 \right\}} = \arg \max_w \frac{w^H R_a w}{w^H C w} = \mathcal{P} \left\{ C^{-1} R_a \right\} \quad (26)
\]

where \( \mathcal{P} \{ \cdot \} \) stands for the principal eigenvector of the matrix between braces. The average SINR associated with the MVDR beamformer is readily obtained as

\[
\text{SINR}_{\text{MVDR}} = \mathcal{P} \lambda_{\max} \left\{ C^{-1} R_a \right\} \quad (27)
\]

where \( \lambda_{\max} \{ \cdot \} \) corresponds to the maximum eigenvalue. Figure 3 compares the performances of the clairvoyant beamformer, the MVDR beamformer and the diagonally loaded beamformers with \( \lambda = \lambda_{\text{opt}}^{\text{true}} \) and \( \lambda = \lambda_{\text{approx}}^{\text{true}} \). This figure plots the average SINR versus \( \Delta \theta \). The following observations can be made. First, the diagonally loaded beamformer, either with \( \lambda = \lambda_{\text{opt}}^{\text{true}} \) or \( \lambda = \lambda_{\text{approx}}^{\text{true}} \), performs as well as the clairvoyant beamformer up to half the null-to-null bandwidth. Furthermore, using \( \lambda = \lambda_{\text{approx}}^{\text{true}} \) instead of \( \lambda = \lambda_{\text{opt}}^{\text{true}} \) results in a marginal degradation. Finally, both diagonally loaded beamformers outperform the MVDR beamformer.

Finally, in a last simulation, we study how the previous results, derived under the assumption of a known covariance matrix, transpose in the finite-sample case. Towards this end, we again consider the case of a fixed pointing error \( \Delta \theta \) and we analyze the performance obtained with

\[
w = \left( \hat{R} + \lambda I \right)^{-1} \tilde{a}
\]

where \( \hat{R} = N^{-1} \sum_{n=1}^{N} x_i x_i^H = N^{-1} X X^H \) is the sample covariance matrix estimated from \( N \) snapshots. More precisely, 500 Monte-Carlo simulations are run with a different \( X \) (while \( a \) is
fixed) and the SINR corresponding to (28) is averaged over the 500 runs. In Figure 4, we display the ratio of the average SINR obtained with $\lambda_{\text{eq}}$ to the average SINR obtained with a positive loading level which is set to 10dB above the noise level. It can be observed that the larger $\alpha - \overline{\alpha}$, the smaller is the number of snapshots $N$ above which negative diagonal loading performs better than positive loading. Therefore, negative diagonal loading is an effective solution even in finite-sample, especially when the steering vector errors are large. In contrast, positive loading proves to be more performant when $N$ and $\alpha - \overline{\alpha}$ are small.

5. CONCLUSIONS

This paper considered the use of diagonal loading to compensate for steering vector errors and dealt with the problem of optimally selecting the loading level. We considered the case of steering vectors errors only (i.e. no finite-sample effects) and the case of weak signal detection in the presence of strong interferences. Within this framework, it was shown that there exists a value of the loading level which results in maximal SINR, and that this optimal level is negative. A simple and closed-form expression for the average - with respect to the pdf of the steering vector - optimal loading level was also derived. Numerical simulations attested to the validity of the analysis and showed that diagonal loading with optimal selection of the loading level can provide a performance very close to that of a clairvoyant beamer. It was also shown that negative diagonal loading is effective in finite-sample whenever the finite-sample errors are small compared to the steering vector errors.

6. REFERENCES


