Chapter 2

CONOCHAIN: Numerical method to predict combustion-generated noise in aero-engine

Abstract  This chapter presents the hybrid numerical approach named CONOCHAIN used to predict combustion noise levels emitted by an aero-engine. This method relies on (Fig. 2.1):

- compressible, turbulent and reactive Large-Eddy simulations within the combustion chamber to provide the accurate unsteady flow features at the exit of the combustion chamber,
- an actuator disk theory to mimic the acoustic behaviour of turbine stages within the turbine,
- a Helmholtz solver downstream of the exhaust nozzle to compute the acoustic propagation within the far-field.

The actuator disk theory is a two-dimensional acoustic modelling of the turbine proposed by Cumpsty and Marble [1977]. This approach allows computing both stator and rotor stages with subsonic and supersonic flows. Note that for the purpose of our application, this theory is complemented by a simple model to take into account the scattering of the convective entropy waves induced by their convection across blade rows. Finally, the Helmholtz solver AVSP-f used to compute the acoustic far-field is presented. The LES method is not presented in this chapter.
Introduction

Adequate evaluation of combustion noise emitted by a turboshaft engine requires methods able to capture the strong unsteadiness of the combustion chamber flow as well as the acoustic propagation and generation processes through the turbine stages and in the downstream jet flow. LES techniques have proven successful in providing accurate compressible simulations of complex turbulent flows because they provide temporally and spatially evolving set of fields representative of the spatially filtered governing system of Navier-Stokes equations [Leonard, 1974, Ferziger, 1997, Mason, 1994, Meneveau and Katz, 2000, Sagaut, 2000] thereby giving access to the most energetic flow motions of crucial importance in such configurations. LES of turbulent combustion have been validated on academic burners to retrieve mean statistical features but also unsteady flow characteristics [Menon and Patel, 2006, El-Asrag and Menon, 2007, Roux et al., 2005, Patel and Menon, 2008, Galpin et al., 2008, Albouze et al., 2009, Moureau et al., 2011]. These numerous academic works were followed by computations of turbulent and reactive flows within real burners despite the lack of experimental databases [Staffelbach et al., 2005, 2006, Boudier et al., 2008, Mahesh et al., 2006, Mare et al., 2004, Moin and Apte, 2006, Fureby, 2010, Kuenne et al., 2011, Gicquel et al., 2012]. Using LES compressible solvers, transient flows phenomena can be also addressed such as extinction and ignition sequences [Stow et al., 2011, Jones and Prasad, 2011, Boileau et al., 2008, Barré et al., 2014], thermo-acoustic instabilities [Staffelbach et al., 2009, Wolf et al., 2009, Hermeth et al., 2014].

Combustion-generated noise can be also investigated in real-scale burners and by use of LES. Direct combustion noise has been evaluated in free-space for turbulent and laminar flames [Talei et al., 2011, Ihme et al., 2009b, Silva, 2010, Bui et al., 2007, Flemming et al., 2007]. By construction, LES is able to capture vorticity and hot spots generation induced by a turbulent flame and therefore indirect combustion noise. For indirect noise to be produced, acceleration and deformation of these flow inhomogeneities through the turbine stages is needed. For this, LES of the combustor coupled with the complete turbine should be required but are still out of reach today in terms of computational cost. Instead, a hybrid approach is proposed in this work. Indirect combustion noise mechanisms within turbine stages were addressed analytically in the literature [Cumpsty and Marble, 1977, Pickett, 1975, Matta and Mani, 1979]. In the present work, LES of a realistic combustion chamber are coupled with the actuator disk theory proposed by Cumpsty and Marble [1977], validated by Moreau and co-workers [Duran et al., 2011, Duran and Moreau, 2012, 2013, Duran et al., 2013, Leyko et al., 2013] and called CHO-RUS. The LES of stabilized operating points of an engine will be used to provide numerous unsteady fields at the exit of the combustion over several planes where a wave decomposition is introduced. To separate acoustics, entropy and vorticity
waves are estimated issued by the flow. These waves are then injected in CHORUS to compute stage by stage propagation and generation of combustion noise. Discrimination of direct and indirect combustion noise is made thanks to this wave decomposition. Finally, using the Helmholtz solver AVSP-f, far-field propagation of the computed acoustic power at the turbine exit is simulated. The complete hybrid approach is summarized in the Figure 2.1.

This chapter presents the actuator disk theory of CHORUS used in CONOCHAIN followed by the description of the post-processing applied to the unsteady fields at the exit of the combustion chamber to extract the acoustic and convective waves. Finally, the free-stream propagation obtained using the Helmholtz solver AVSP-f is described.

2.1 The actuator disk theory : CHORUS

In real turbines, a fully analytic description of complex three-dimensional flows is out of reach but the low-frequency nature of combustion noise allows some simpli-
fications to deal with it theoretically. Similarly to one-dimensional cases addressed by Marble and Candel [1977] where the wavelengths of the perturbations longer than the axial blade chord, each blade row can be seen as a thin interface between two uniform regions where simple jump relations can be written between the inlet and outlet of a turbine row. In a real aeronautical turbine, the mean radius is close to 0.15 cm and the inter-wall spacing is close to 0.02 cm so that the compact assumption limits the studied frequency range of interest from 0 Hz to 2000 Hz approximately. Consequently, radial acoustic modes are not taken into account because their cut-on frequency is above the frequency limit of the compact assumption. As an example, the cut-off frequency computed with an analytical approach (C) of the first radial mode is close to 15 kHz for the engine well above the cut-off frequency of the first azimuthal mode (730 Hz) and out of the compact frequency limit. Thus, Cumpsty and Marble [1977] reduced the problem to two dimensions by unwrapping the turbine stages (Fig. 2.2) and proposed an analytical method to predict the generation and propagation of acoustic azimuthal waves through the blade rows. Between turbine blade rows, the flow is considered inviscid, isentropic, two-dimensional and uniform, characterized by the mean velocity vector \( \vec{w} \) (with modulus \( w \) and angle \( \theta \)), the mean Mach number \( M \), the mean pressure \( p \) and the mean density \( \rho \). Four dimensionless primitive fluctuations are considered:

- Entropy fluctuation \( s'/c_p \)
- Velocity modulus fluctuation \( w'/c \)
- Velocity angle fluctuation \( \theta' \)
- Pressure fluctuation \( p'/\gamma p \).

Assuming all these fluctuations are harmonic, it is more convenient to write the set of primitive variables in terms of waves \( \tilde{w} \) in the general form:

\[
\tilde{w} = \hat{\tilde{w}} \exp\left( i \left( \omega t - \vec{k} \cdot \vec{x} \right) \right),
\]

or \( \tilde{w} \exp\left( i \left( \omega t - k_x x - k_y y \right) \right) \),

or \( \tilde{w} \exp\left( i \left( \omega t - k \cdot (x \cos(\nu) + y \sin(\nu)) \right) \right), \tag{2.1} \)

where each wave is characterized by a wave vector \( \vec{k} \) (with modulus \( k \) and angle \( \nu \)). Thus, as detailed in this section, four waves (upstream-propagating and downstream-propagating acoustic waves, entropy and vorticity waves) are related to the fluctuating primitive variables.

This section presents the wave decomposition proposed for CHORUS followed by the linearised jump relations for fixed and rotating blade rows. The matrix formulation is also described to compute acoustic transfer functions of each turbine
As the flow within the turbine is considered isentropic, inviscid, homogeneous and two-dimensional, Euler equations can be used to described the mean flow. The axial and circumferential velocities $u$ and $v$ are respectively equal to $w \cos \theta$ and $w \sin \theta$ respectively. The perturbations are supposed to be small enough to not impact the mean flow which allows writing linearised Euler equations in terms of primitive fluctuating variables following [Cumpsty and Marble, 1977] for a mean steady homogeneous flow:

$$\frac{D}{Dt} \left( \rho' \right) + \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0,$$

(2.2)
\[
\frac{D}{Dt} (u') = -\frac{1}{\rho} \frac{\partial p'}{\partial x}, \quad (2.3)
\]
\[
\frac{D}{Dt} (v') = -\frac{1}{\rho} \frac{\partial p'}{\partial y}, \quad (2.4)
\]
\[
\frac{D}{Dt} \left( \frac{s'}{c_p} \right) = 0, \quad (2.5)
\]
where the material derivative is
\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{w} \cdot \nabla = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}, \quad (2.6)
\]
and the entropy \( s \) equals
\[
s = c_v \ln \frac{p}{\rho^\gamma}. \quad (2.7)
\]
The log-differentiation of Eq. 2.7 gives the fluctuating entropy, namely
\[
\frac{s'}{c_p} = \frac{p'}{\gamma p} - \frac{\rho'}{\rho}. \quad (2.8)
\]
By considering that pressure fluctuations are only related to acoustic waves, subtracting these fluctuations from the equation 2.8 removes the density fluctuations associated to the acoustic pressure variations from the entropy field. The vorticity fluctuation \( \xi' \) is
\[
\xi' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}. \quad (2.9)
\]
Deriving the azimuthal and axial linearised momentum equations over \( y \) (2.3,2.4), we obtain
\[
\frac{\partial}{\partial y} \left( \frac{D}{Dt} (u') \right) = -\frac{1}{\rho} \frac{\partial^2 p'}{\partial y \partial x} \quad (2.10)
\]
and
\[
\frac{\partial}{\partial x} \left( \frac{D}{Dt} (v') \right) = -\frac{1}{\rho} \frac{\partial^2 p'}{\partial x \partial y}. \quad (2.11)
\]
The Schwartz’s theorem allows removing pressure term by subtracting Eq. (2.10) from Eq. (2.11). The propagation equation of the vorticity is
\[
\frac{D}{Dt} (\xi') = 0. \quad (2.12)
\]
Furthermore, combining Eqs. 2.2, 2.3 and 2.4), a propagation equation for the pressure fluctuations can be obtained by eliminating density and velocity fluctuations. It reads
\[
\frac{D}{Dt}^2 \left( \frac{p'}{\gamma p} \right) - c^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left( \frac{p'}{\gamma p} \right) = 0. \quad (2.13)
\]
2.1.2 Wave classification

As mentioned above, any fluctuating primitive variable can be related to acoustic or convective waves. So, each wave $w$ can be written as

$$\hat{w} \exp \left( i (\omega t - k_x \cos(\nu) + y \sin(\nu)) \right). \quad (2.14)$$

The wave vector $\vec{k}$ is scaled using the mean sound speed $c$ and the pulsation $\omega$ to give $\vec{K} = \vec{k}c/\omega$. In CHORUS, a "mode-matching" assumption is applied to compute each dimensionless wave-number across the turbine stages. The azimuthal component $k_y = k \sin(\nu)$ of the wave number $\vec{k}$ is equal to $m/R$ ($m$ is the azimuthal mode number and $R$ is the mean radius) and conserved across each blade row.

2.1.2.1 Vorticity wave

The vorticity fluctuation is a pure hydrodynamic perturbation and does not generate pressure or entropy fluctuations. Using a wave formulation as specified in equation (2.1), a vorticity wave $w_v$ is

$$w_v = \xi' = \hat{w}_v \exp \left( i (\omega t - k_v (x \cos(\nu_v) + y \sin(\nu_v))) \right). \quad (2.15)$$

With equation (2.15) in equation (2.12), the dispersion relation of the vorticity reads

$$K_v M \cos(\nu_v - \theta) = 1. \quad (2.16)$$

The dispersion relation is used to compute the axial component of the dimensionless wave vector $\vec{K}_v$ knowing the azimuthal component. Combining the equations (2.2,2.5) and (2.9), dimensionless velocity fluctuations $u'/c$ and $v'/c$ can be related to the vorticity wave, namely

$$\frac{u'}{c} = -i\xi' \frac{\sin(\nu_v)}{\omega K_v}, \quad (2.17)$$

$$\frac{v'}{c} = i\xi' \frac{\cos(\nu_v)}{\omega K_v}. \quad (2.18)$$

In terms of velocity magnitude $w'$ and angle perturbations $\theta'$, we have

$$\frac{w'}{c} = -i\xi' \frac{\sin(\nu_v - \theta)}{\omega K_v}, \quad (2.19)$$

$$\theta' = i\xi' \frac{\cos(\nu_v - \theta)}{\omega M K_v}, \quad (2.20)$$

with

$$\frac{u'}{u} = \frac{w'}{w} - \theta' \tan \theta, \quad (2.21)$$

$$\frac{v'}{v} = \frac{w'}{w} + \theta' \frac{1}{\tan \theta}. \quad (2.22)$$
2.1.2.2 Entropy wave

Similarly to the vorticity wave, the entropy wave is defined as:

\[
\frac{w_s}{c_p} = \hat{w}_s \exp \left( i (\omega t - k_s \cdot (x \cos(\nu_s) + y \sin(\nu_s))) \right)
\]  

(2.23)

for which the entropy fluctuations are not associated to any pressure or velocity perturbations. Using the entropy conservation (2.5), the dispersion relation of the propagating entropy wave is

\[
K_s M \cos (\nu_s - \theta) = 1.
\]  

(2.24)

2.1.2.3 Acoustic waves

In CHORUS, the fluctuating pressure is purely acoustic, namely

\[
\frac{p'}{\gamma p} = w^+ + w^-
\]  

(2.25)

where \(w^+\) and \(w^-\) are the upstream and downstream propagating acoustic waves. Extracting the acoustic wave amplitudes from the pressure fluctuations requires to compute the wave vectors associated to each wave on the one hand and their velocity components on the other hand. Considering harmonic acoustic pressure fluctuations

\[
\frac{p'}{\gamma p} = \left( \frac{p'}{\gamma p} \right) \exp \left( i (\omega t - k \cdot (x \cos(\nu) + y \sin(\nu))) \right)
\]  

(2.26)

which satisfies the D’Alembertian of Eq. 2.13, we obtain a dispersion relation

\[
(1 - K M \cos(\nu - \theta))^2 - K^2 = 0
\]  

(2.27)

which is a second degree polynomial equation. Considering the axial component \(K_x = K \cos(\nu)\) and the azimuthal component \(K_y = K \sin(\nu)\) of the dimensionless wave vector \(\vec{K}\), the dispersion relation (2.27) is now

\[
(1 - K_x M \cos(\theta) - K_y M \cos(\theta))^2 - (K_x^2 + K_y^2) = 0.
\]  

(2.28)

The solutions of Eq. 2.28 can be either real or complex. The two roots of Eq. 2.28 are related to the upstream and downstream propagating acoustic waves \(w^+\) and \(w^-\). Solving for (2.28) yields the axial acoustic wave numbers \(K_x^+\) and \(K_x^-\) when \(1 - 2K_y M \sin \theta + K_y^2 (M^2 - 1)\) is positive. For the upstream-propagating acoustic wave \(w^+\), we get

\[
K_x^+ = \frac{M \cos \theta (1 - MK_y \sin \theta)}{(M \cos \theta)^2 - 1} - \frac{\sqrt{1 - 2K_y^2 M \sin \theta + K_y^2 (M^2 - 1)}}{(M \cos \theta)^2 - 1}
\]  

(2.29)
and for the downstream-propagating acoustic wave $w^-$,

$$K_x^- = \frac{M \cos \theta (1 - M K_y \sin \theta)}{(M \cos \theta)^2 - 1} + \frac{\sqrt{1 - 2K_y M \sin \theta + K_y^2 (M^2 - 1)}}{(M \cos \theta)^2 - 1}. \quad (2.30)$$

For azimuthal modes, the axial wave numbers defined in Eqs. 2.29 and 2.30 can have an imaginary part when $1 - 2K_y M \sin \theta + K_y^2 (M^2 - 1)$ is lower than 0. These cases correspond to evanescent waves and the axial wave numbers $K_x^+$ and $K_x^-$ are

$$K_x^+ = \frac{M \cos \theta (1 - M K_y \sin \theta)}{(M \cos \theta)^2 - 1} + i \frac{\sqrt{1 - 2K_y M \sin \theta + K_y^2 (M^2 - 1)}}{(M \cos \theta)^2 - 1} \quad (2.31)$$

and

$$K_x^- = \frac{M \cos \theta (1 - M K_y \sin \theta)}{(M \cos \theta)^2 - 1} - i \frac{\sqrt{1 - 2K_y M \sin \theta + K_y^2 (M^2 - 1)}}{(M \cos \theta)^2 - 1}. \quad (2.32)$$

Once the acoustic wave vectors $K_x^+$ and $K_x^-$ are known, the associated velocity components $w'_c$ and $\theta'$ of the acoustic pressure fluctuations $p'/\gamma p$ can be computed by combining Eqs. 2.3, 2.4 and 2.26. Noting that $K_x^{\pm} = K_x^{(1)\pm} + iK_x^{(2)\pm}$, velocity magnitude and angle fluctuations induced by the fluctuating pressure $p'/\gamma p$ associated to the upstream-propagating wave $w_+$ are

$$\frac{w'}{c} = \frac{p'}{\gamma p} \frac{K^+ \cos (\theta - \nu^+) + iK_{x}^{(2)+} \cos \theta}{1 - K^+ M \cos (\nu^+ - \theta) - iK_{x}^{(2)+} M \cos \theta}, \quad (2.33)$$

$$\theta' = \frac{p'}{\gamma p} \frac{K^+ \sin (\nu^+ - \theta) + iK_{x}^{(2)-} \sin \theta}{M 1 - K^+ M \cos (\nu^+ - \theta) - iK_{x}^{(2)-} M \cos \theta}, \quad (2.34)$$

and

$$\frac{w'}{c} = \frac{p'}{\gamma p} \frac{K^- \cos (\theta - \nu^-) - iK_{x}^{(2)-} \cos \theta}{1 - K^- M \cos (\nu^- - \theta) + iK_{x}^{(2)-} M \cos \theta}, \quad (2.35)$$

$$\theta' = \frac{p'}{\gamma p} \frac{K^- \sin (\nu^- - \theta) - iK_{x}^{(2)-} \sin \theta}{M 1 - K^- M \cos (\nu^- - \theta) + iK_{x}^{(2)-} M \cos \theta}, \quad (2.36)$$

for the fluctuating pressure $p'/\gamma p$ associated to the downstream-propagating wave $w_-$. 

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2.1.2.4 Matrix formulation

Acoustic, entropy and vorticity waves can be expressed in terms of fluctuating primitive variables through

\[
\begin{pmatrix}
\frac{s'}{c_p} \\
\frac{w'}{c} \\
\frac{p'}{\gamma p} \\
\theta'
\end{pmatrix} = [\mathbf{M}]
\begin{pmatrix}
w_s \\
w_v \\
w^+ \\
w^-
\end{pmatrix}
\]

(2.37)

where the matrix \([\mathbf{M}]\) is

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -i \sin(\nu - \theta) & \frac{K^+ \cos(\theta - \nu^+) + i K_x^{(2)+} \cos \theta}{1 - K^+ M \cos(\nu^+ - \theta) - i K_x^{(2)+} M \cos \theta} & \frac{K^- \cos(\theta - \nu^-) - i K_x^{(2)-} \cos \theta}{1 - K^- M \cos(\nu^- - \theta) + i K_x^{(2)-} M \cos \theta} \\
0 & i \cos(\nu - \theta) & \frac{K^+ \sin(\nu^+ - \theta) + i K_x^{(2)+} \sin \theta}{1 - K^+ M \cos(\nu^+ - \theta) - i K_x^{(2)+} M \cos \theta} & \frac{K^- \sin(\nu^- - \theta) - i K_x^{(2)-} \sin \theta}{1 - K^- M \cos(\nu^- - \theta) + i K_x^{(2)-} M \cos \theta}
\end{pmatrix}
\]

(2.38)

the wave vector \(\mathbf{W}\) is

\[
\mathbf{W} = \begin{pmatrix}
w_s \\
w_v \\
w^+ \\
w^-
\end{pmatrix}
\]

(2.39)

and the primitive vector \(\mathbf{P}\) is

\[
\mathbf{P} = \begin{pmatrix}
\frac{s'}{c_p} \\
\frac{w'}{c} \\
\frac{p'}{\gamma p} \\
\theta'
\end{pmatrix}
\]

(2.40)

This wave formulation allows computing the convective and acoustic waves at the interface between the homogeneous steady mean flow and the turbine blade. Jump relations are however required to get the transfer functions between blade rows in terms of primitive variables. In the following description, \([\mathbf{M}]\) will be used in two states \(u\) and \(d\) corresponding to upstream and downstream conditions. \([\mathbf{M}^{(u)}]\) and \([\mathbf{M}^{(d)}]\) are obtained using \(\theta, \nu\) and \(K\) for these two conditions.

2.1.3 Jump relations across a blade row

Using a similar approach as proposed by Marble and Candel [1977], conservation laws are written across a blade row and linearised to link fluctuating primitive upstream and downstream variables. The two-dimensional problem imposes to
find in this case four jump relations that will differ depending on the blade row type (stator or rotor) and the sonic state of the flow at the trailing edge of the blade as specified in Table 2.1. In a stator, the flow expansion is isentropic and

<table>
<thead>
<tr>
<th>Flow</th>
<th>Stator</th>
<th>Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsonic</td>
<td>• Mass conservation</td>
<td>• Mass conservation</td>
</tr>
<tr>
<td></td>
<td>• Enthalpy conservation</td>
<td>• Rothalpy conservation</td>
</tr>
<tr>
<td></td>
<td>• Entropy conservation</td>
<td>• Entropy conservation</td>
</tr>
<tr>
<td></td>
<td>• Kutta condition</td>
<td>• Kutta condition</td>
</tr>
<tr>
<td>Supersonic</td>
<td>• 1-D Mass conservation</td>
<td>• Mass conservation</td>
</tr>
<tr>
<td></td>
<td>• Enthalpy conservation</td>
<td>• Rothalpy conservation</td>
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<tr>
<td></td>
<td>• Entropy conservation</td>
<td>• Entropy conservation</td>
</tr>
<tr>
<td></td>
<td>• 1-D mass flow rate in a choked nozzle</td>
<td>• 1-D mass flow rate in a choked nozzle</td>
</tr>
</tbody>
</table>

Table 2.1: Relations used to build linearised jump conditions across a blade row.

leads to an enthalpy conservation contrary to the rotor in which the mean flow provides mechanical work to the stage, so, rothalpy conservation is used. The entropy conservation through a blade row is an assumption made by Cumpsty and Marble [1977] but Leyko et al. [2010] and Duran and Moreau [2011] showed that the distortion of the mean streamlines within the blade rows induced a scattering effect of the entropy waves. An attenuation model is hence proposed in this work to take into account this phenomenon which is then validated on the two-dimensional simulations [Leyko et al., 2010, Duran and Moreau, 2011]. Finally, a Kutta condition is applied at the trailing edge of the blade for subsonic flows in the moving reference frame. Similarly, for a supersonic flow at the exit of a stage, a mass flow rate relation for choked flows in one-dimensional nozzle is used.

The following section details the linearisation of the relations summarized in the Table 2.1.

2.1.3.1 Mass conservation

The conservation of the axial mass flow rate $\dot{m}$ across a blade row gives:

$$\dot{m} = \rho w \cos \theta. \quad (2.41)$$

From a logarithmic differentiation of (2.41):

$$\frac{\dot{m}'}{\dot{m}} = \frac{\rho'}{\rho} + \frac{1}{M^2} \frac{w'}{c} - \theta' \tan \theta \quad \text{where} \quad M = \frac{w}{c}. \quad (2.42)$$
Substitute (2.5) into (2.42):
\[
\frac{\dot{m}'}{\dot{m}} = \frac{p'}{\gamma p} - \frac{s'}{c_p} + \frac{1}{M} \frac{w'}{c} - \theta' \tan \theta.
\] (2.43)

### 2.1.3.2 Total enthalpy conservation

The total enthalpy conservation through a blade row is equivalent to the stagnation temperature \(T_t\) conservation:
\[
T_t = T \left(1 + \frac{\gamma - 1}{2} M^2\right).
\] (2.44)

Linearising equation (2.44) yields:
\[
\frac{T'_t}{T_t} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2} \left(\gamma - 1\right) \frac{p'}{\gamma p} + \frac{s'}{c_p} + \left(\gamma - 1\right) M \frac{w'}{c}.
\] (2.45)
2.1.3.3 Rothalpy conservation

The total rothalpy $I$ can be seen as an enthalpy conservation in the moving reference frame of the rotor. It is defined as:

$$ I = h_t - U w \sin \theta $$

(2.46)

and corresponds to the difference between the total enthalpy of the flow and the work provided to the rotor ($U$ is the azimuthal velocity of the row). The linearisation of the equation (2.46) gives:

$$ \frac{I'}{T} = \frac{h'_t - U v'}{h_t - U v} = \frac{h'_t - \frac{U w'}{h_t v}}{1 - \frac{U w'}{h_t}}, $$

(2.47)

defining the parameter $\psi$ as

$$ \psi = \frac{U v}{h_t}. $$

(2.48)

Combining equations (2.45), (2.22) with (2.47), the linearised rothalpy equation becomes

$$ \frac{I'}{T} = \frac{1}{1 - \psi} \left[ T'_t - \psi \left( \frac{1}{M} \frac{w'}{c} + \frac{\theta'}{\tan \theta} \right) \right]. $$

(2.49)

As mentioned above, the rothalpy can be seen as a total enthalpy conservation and for an azimuthal rotor velocity $U$ equal to zero, the linearised equation (2.49) tends to the linearised total temperature relation (2.45). In CHORUS, the total enthalpy $h_t$ is written as:

$$ h_t = \frac{w^2}{2} + c_p T = c^2 \left( \frac{M^2}{2} + \frac{1}{\gamma - 1} \right), $$

(2.50)

where the parameter $\psi$ equals

$$ \psi = \frac{UM \sin \theta}{c \left( \frac{M^2}{2} + \frac{1}{\gamma - 1} \right)}. $$

(2.51)

2.1.3.4 Entropy conservation

Cumpsty and Marble [1977] supposed that the entropy fluctuation across a blade row is conserved, namely

$$ \frac{s'}{c_p|_{(u)}} = \frac{s'}{c_p|_{(d)}}. $$

(2.52)

Furthermore, the entropy is not necessarily conserved mode-by-mode across a blade row according to two-dimensional numerical simulations of turbine stages [Leyko et al., 2010, Duran and Moreau, 2012, Duran et al., 2013].
Convection of an entropy wave through a blade row

Cumpsty and Marble [1977] assessed that the entropy fluctuation is not altered by convection through the turbine stages. However, entropy-to-acoustic transfer functions were found to depend on the scattering of the normal entropy wave to higher-order modes through the turbine stages because of a non-uniform convective velocity through the blade rows [Leyko et al., 2010, Duran and Moreau, 2012, Duran et al., 2013]. As the entropy wave is carried by the mean flow, the streamlines distortion through a blade vane leads to a time delay \( \tau_d \) between the entropy ‘particles’ and the entropy wave front. This delay can be evaluated using a very crude model described below: assume that the axial velocity profile across a blade vane is parabolic as shown in Fig. 2.4. The dimensionless pitch length \( y \) ranging from 0 to 1 for each blade vane is build using the dimensionless perimeter \( r \) also ranging from 0 to 1, namely

\[
y = N_{Vanes}.r - (i_{Vanes})
\]  

\( (2.53) \)
where \( N_{\text{Vanes}} \) is the number of blade vanes and \( i_{\text{Vanes}} \) is the \( i^{th} \) considered blade vane. The dimensionless axial velocity profile for the complete row is

\[
\frac{U_{x}^{\text{axial}}(r)}{U_{\text{axial}}^{\text{max}}} = 4y(r)(1 - y(r)).
\]  

(2.54)

To ensure mass conservation across the row, the integration over the dimensionless perimeter of the velocity profile (2.54) gives the maximum velocity \( U_{\text{axial}}^{\text{max}} \) according to the mean velocity \( U_{\text{axial}}^{\text{mean}} \),

\[
U_{\text{axial}}^{\text{max}} = \frac{U_{\text{axial}}^{\text{mean}}}{\int_0^1 U_{x}^{\text{axial}}(r)\,dr}.
\]  

(2.55)

The delay \( \tau_d \) is computed by using a log-differentiation of the expression

\[
U_{x}^{\text{axial}}(r) = \frac{L_x}{\tau(r)}
\]  

(2.56)

where \( \tau(r) \) is the convective time across the blade row, and leads to

\[
\frac{dU_{x}^{\text{axial}}(r)}{U_{\text{axial}}^{\text{mean}}} = -\frac{\tau_d}{\tau_{\text{mean}}}
\]  

(2.57)

where \( \tau_{\text{mean}} = L_x/U_{\text{axial}}^{\text{mean}} \), and

\[
\tau_d(r) = -\frac{L_x}{U_{\text{axial}}^{\text{mean}}} \frac{U_{x}^{\text{axial}}(r) - U_{x}^{\text{axial}}}{U_{\text{axial}}^{\text{mean}}}.
\]  

(2.58)

Using the delay \( \tau_d \), a entropy-to-entropy transfer function \( D_m(f) \) through a blade row can be computed in the frequency domain for the convective azimuthal wave where the mode-index is \( m \), namely

\[
\frac{s'/c_p}{s'/c_p} = D_m(f) = \int_0^1 \exp(i2\pi (f\tau_d(r) + mr))\,dr.
\]  

(2.59)

This formulation corresponds to the phase-shift induced by the streamlines distortion between the ‘entropy particles’ and the entropy wave front for the frequency \( f \) at the exit of the blade row.

This model is compared with the numerical results from Leyko et al. [2010], Duran and Moreau [2012], Duran et al. [2013] for the stator as well as the complete turbine stages in Fig. 2.5 for a planar entropy wave (ie \( m = 0 \)). A very good agreement is found between the analytical transfer functions and the numerical one for stator and turbine stage cases indicating that the blade row acts like a low-pass filter.
The attenuation of the planar entropy wave through a blade row does not correspond to a loss of entropy. Indeed, the entropy signal power is transferred into the higher azimuthal modes for which the mode-index is a multiple of the number of blade vanes. To assess this conservation of the entropy signal through a blade row, the energy signal carried by the planar mode and the first higher azimuthal modes are displayed in Fig. 2.6 where the sum of the modal power tends to 1. As the index of these higher modes is a multiple of the number of blade vanes which induces that the azimuthal modes for which their cut-on frequency are lower than the frequency limit of the compact assumption are not fed by the entropy wave scattering.

So, in CHORUS, there is no entropy transfer from the planar mode to the higher modes. It is interesting to note that Sattelmayer [2003] and Morgans et al. [2013] found similar attenuation functions of a convective scalar with analytical approaches and a DNS of a turbulent flow in a pipe in which an entropy wave was injected.

As the two-dimensional actuator disk theory is based on four primitive variables to describe the fluctuating field, a additional equation is required to close the problem. Contrary to the jump relations, the last equation uses a Kutta’s condition for a subsonic stage and the linearised relation of mass flow rate within a one-dimensional choked isentropic nozzle when the stage is choked.
Kutta’s condition for unchoked flow

For subsonic flows, Kutta’s condition imposes the flow deviation at the trailing edge of the blade. Cumpsty and Marble [1977] proposed to set to zero the fluctuating velocity angle at the trailing which yields

$$\theta'_1 = \beta \theta'_2,$$

(2.60)

where $\beta$ is a numerical parameter set to an infinitesimal quantity (not zero to avoid singularity in matrix products). To extend the Kutta’s condition to the rotor case, equation (2.60) has to be written in the relative frame. To do so, the angle flow tangent function is log-derived and gives

$$\frac{(\tan \theta_r)'}{(\tan \theta_r)} = \frac{v'}{v - U} - \frac{u'}{u'},$$

(2.61)
and using some algebra and the equations (2.21) and (2.22), we have

$$\theta'_r = \left( \frac{(w \cos \theta)^2}{w^2 - 2wU \sin \theta + U^2} \right) \left[ \frac{U}{w \cos \theta M c} \frac{w'}{c} + \theta' \left( 1 + (\tan \theta)^2 - \frac{U}{w \cos \theta \tan \theta} \right) \right].$$

(2.62)

The fluctuating flow angle $\theta'_r$ in the relative frame is equal to $\theta'$ for $U = 0$.

**Mass flow rate for choked vanes**

For a supersonic stage often encountered in the first stator of high-pressurized turbines, Kutta’s condition is replaced by the specific expression of the mass flow rate through a choked nozzle. Indeed, the mass flow rate is proportional to

$$\frac{p}{\sqrt{\gamma}} \left( 1 + \frac{\gamma - 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

(2.63)

and the linearisation of Eq. 2.63, after some algebras, gives

$$\frac{p'}{\gamma p} \frac{\gamma - 1}{2} + \frac{1}{2} \frac{s'}{c_p} - \frac{w'}{c} \frac{1}{M} + \theta' \frac{1}{1 - M^2} \tan \theta = 0,$$

(2.64)

which is considered in the upstream region of the blade. For the rotor case, Eq. 2.64 has to written in the relative frame especially for the relative Mach number $M_r$. 71
and yields
\[
p' \gamma p \left( \frac{(\gamma - 1)}{2} \right) + \frac{1}{C_p} \frac{w'}{c}
\]
\[
+ \left( \frac{\gamma + 1}{2} \frac{M_r^2}{1 - M_r^2} \right) f_u \frac{1}{M} \left( 1 - \frac{U}{w} \sin \theta \right) - \frac{\mu}{M(1 - M_r^2)} \right)
\]
\[
+ \theta' \left[ \tan \theta - \left( f_u \frac{U}{w} \cos \theta \right) \left( \frac{(\gamma + 1) M_r^2}{\mu} \right) \right] \frac{\mu}{1 - M_r^2} \]
\]
(2.65)

where
\[
M_r = \frac{\sqrt{(w \sin \theta - U)^2 + (w \cos \theta)^2}}{c},
\]
(2.66)
\[
f_u = \frac{1}{1 + \left( \frac{U}{w} \right)^2 - 2 \frac{U}{w} \sin \theta},
\]
(2.67)
and
\[
\frac{M_r'}{M_r} = w' c \left[ f_u \frac{1}{M} \left( 1 - \frac{U}{w} \sin \theta \right) \right] - \theta' \left( f_u \frac{U}{w} \cos \theta \right) - \frac{\gamma - 1}{2} \frac{p'}{\gamma p} - \frac{1}{2} \frac{s'}{C_p}.
\]
(2.68)

Similarly to the Kutta’s condition in the relative frame (2.62), the linearised expression of the mass flow rate in a supersonic vane (2.65) is equal to the relation (2.64) when the rotor velocity \( U \) is equal to 0.

Matrix formulation of the acoustic transfer functions

The jump relations written across a blade row can be represented through a matrix product between upstream and downstream fluctuating primitive variables:

\[
[B]^{(u)} \left( \begin{array}{c}
s' / c_p \\
w' / c \\
p' / \gamma p \\
\theta'
\end{array} \right)^{(u)} = [B]^{(d)} \left( \begin{array}{c}
s' / c_p \\
w' / c \\
p' / \gamma p \\
\theta'
\end{array} \right)^{(d)}
\]
(2.69)

where the subscript \( u \) represents the upstream region of the blade row and \( d \) the downstream region. As the jump relations written for the stator case are equivalent with the rotor case when \( U \) is set to 0, the matrix \( [B] \) is specified for subsonic or supersonic rotating blade rows.
A subsonic rotor: the matrix $[B]$ is

$$
\begin{pmatrix}
-1 & 0 & 0 \\
\frac{1}{1-\xi} & \frac{1}{1-\xi} \left(\frac{\gamma-1}{\mu} M \right) & \frac{1}{1-\xi} \left(\frac{\gamma-1}{\mu} \right) \\
0 & f_w \left(1 + (\tan \theta)^2 - \frac{U \tan \theta}{w \cos \theta}\right) & 0
\end{pmatrix}
$$

(2.70)

where

$$
f_w = \frac{(w \cos \theta)^2}{w^2 - 2w U \sin \theta + U^2}
$$

(2.71)

for the upstream zone and

$$
\begin{pmatrix}
-1 & 0 & 0 \\
\frac{1}{1-\xi} & \frac{1}{1-\xi} \left(\frac{\gamma-1}{\mu} M \right) & \frac{1}{1-\xi} \left(\frac{\gamma-1}{\mu} \right) \\
0 & f_w \left(1 + (\tan \theta)^2 - \frac{U \tan \theta}{w \cos \theta}\right) & 0
\end{pmatrix}
$$

(2.72)

for the downstream zone.

A supersonic rotor: similarly to the subsonic rotor, only the Kutta’s condition is removed in the matrix $B$ to be replaced with the linearized equation of the mass flow rate within a one-dimensional isentropic nozzle, namely

$$
\begin{pmatrix}
-1 & 0 & 0 \\
\frac{1}{1-\xi} \left(\frac{\gamma-1}{\mu} M \right) & \frac{1}{1-\xi} \left(\frac{\gamma-1}{\mu} \right) & \frac{1}{1-\xi} \left(\frac{\gamma}{\mu} \right) \\
0 & 0 & 0
\end{pmatrix}
$$

(2.73)

where

$$
C_w = \left(\frac{\gamma + 1}{2} \frac{M_r^2}{1 - M_r^2} \left[f_u \frac{1}{M} \left(1 - \frac{U}{w \sin \theta}\right)\right] - \frac{\mu}{M(1 - M_r^2)}\right)
$$

(2.74)

and

$$
C_\theta = \left[\tan \theta - \left(f_u \frac{U}{w \cos \theta}\right) \left(\frac{\gamma + 1}{2} \frac{M_r^2}{\mu}\right)\right] - \frac{\mu}{1 - M_r^2}
$$

(2.75)

for the upstream region. For the downstream zone, the matrix $[B]$ becomes

$$
\begin{pmatrix}
-1 & 0 & 0 \\
\frac{1}{1-\xi} \left(\frac{\gamma-1}{\mu} M \right) & \frac{1}{1-\xi} \left(\frac{\gamma-1}{\mu} \right) & \frac{1}{1-\xi} \left(\frac{\gamma}{\mu} \right) \\
0 & 0 & 0
\end{pmatrix}
$$

(2.76)

Combining these matrix with matrix equations (2.38), acoustic transfer functions across turbine stages are available.
2.1.4 Computation of a turbine stage

Combining the wave formulation defined in (2.1) and the jump relations matrix (2.69), the transfer function of a single blade row is

\[
\begin{bmatrix}
B^{(u)} \\
M^{(u)} \\
A^{(u)}
\end{bmatrix}
\begin{bmatrix}
W^{(u)}
\end{bmatrix} =
\begin{bmatrix}
M^{(d)} \\
B^{(d)} \\
A^{(d)}
\end{bmatrix}
\begin{bmatrix}
W^{(d)}
\end{bmatrix},
\]

(2.77)

where \( W \) is the wave vector (2.39) and \([M]\) is the matrix to convert primitive variables into waves (2.37). The matrix \([B]\) is detailed in section 2.1.3.4. This matrix product allows calculating azimuthal mode by mode the transfer functions which corresponds to a "mode-matching" assumption. For a complete turbine stage, it is necessary to take into account the phase-shift of the waves between the leading edge of a first row and the leading edge of the next row. Using the wave formulation defined in Eq. 2.1, the phase-shift becomes a function of the axial blade chord of the row, the axial spacing blade spacing and the axial wave number.

Noting \( W^{(d)}_{(i)} \) the wave vector defined in Eq. 2.39 at the trailing of a \( i^{th} \) row and \( W^{(u)}_{(i+1)} \) the wave vector at the leading edge of the \((i + 1)^{th}\) row, the phase-shift matrix product is:

\[
W^{(u)}_{(i+1)} = [T] \cdot W^{(d)}_{(i)},
\]

(2.78)

where \([T]\) is a diagonal matrix for which the diagonal components are equal to \( \exp\left(-i k_{x}\cdot(L_{x} + L_{x}^{blade})\right) \) for which \( L_{x} \) and \( L_{x}^{blade} \) are the axial spacing between two successive rows and the axial chord of a blade respectively. For a turbine stage composed of \( N_{s} \) blade rows, the matrix product becomes

\[
\prod^{N_{s}-1}_{i=1} \left( [A]^{(u)}_{(i+1)} [T_{(i)}] [A]^{-1}_{(i)} \right) [A]^{u}_{(1)} \cdot W^{1}_{(u)} = [A]^{(d)}_{(N_{s})} W^{(d)}_{(N_{s})}.
\]

(2.79)

This system (2.79) has to be modified to solve the acoustic propagation through the turbine. Indeed, the four unknowns are the three downstream propagating waves at the exit of the turbine \( (w_{s}, w_{v}, \text{and} \, w^{+}) \) and the reflected acoustic wave in the combustion chamber \( w^{-} \). The non-reflecting boundary condition at the exit of the turbine imposes \( w^{-} = 0 \) in the wave vector \( W^{(d)} \). The three upstream-propagating waves are provided by the LES. Contrarily, for the computation of the wave vector between turbine stages, the matrix problem requires an additional step. Once the upstream and downstream wave vectors of the complete turbine solved, we use the matrix product (2.79) up to the considered turbine stage.

A last remark can be made about the matrix resolution. Although supersonic flow can be considered at the exit of turbine stages, the matrix problem (2.79) does not allow computing supersonic wave propagation in the steady mean flow.
Indeed, the problem is only valid for subsonic axial Mach number and the mean Mach number has to be limited to $1/\cos\theta$ which is always true in real engines.

Using the acoustic waves computed anywhere in the turbine, the acoustic power is finally obtained with a formulation to take into account the convective terms [Bretherton and Garrett, 1968]. So, the acoustic intensity in the $i^{th}$ turbine stage is equal to

$$I^{(i)} = \left[ \left( M \cos \theta + \cos \nu_+ \right) (1 + M \cos(\theta - \nu_-)) \right]^{(i)} \left( w_+^{(i)} \right)^2 + \left[ \left( M \cos \theta + \cos \nu_- \right) (1 + M \cos(\theta - \nu_-)) \right]^{(i)} \left( w_-^{(i)} \right)^2 \left( \gamma pc \right)^{(i)}. \tag{2.80}$$

### 2.1.5 Selection of higher azimuthal modes computed with CHORUS

The theory presented in this chapter is valid below a frequency limit corresponding to the compact assumption. Furthermore, considering azimuthal modes in noise computation implies an additional condition. Indeed, the compact theory imposes to ignore blading details where the blade pitch-chord ratio is assumed to be low. Mishra and Bodony [2013] show that this theory failed to capture noise generation for local entropy disturbances smaller than the pitch length convected through a blade vane. More precisely, predicted acoustic powers are over-estimated at the turbine stage outlet. The consequence is that azimuthal modes for which the azimuthal wavelength is similar to the blade pitch $L_y$ can not be properly dealt with CHORUS. So, only the first azimuthal modes have to be considered in noise computations.

To find the mode-index of the selected higher azimuthal mode, a simple criterion must be derived. Similarly to the compact frequency limit in which the ratio between the axial blade chord and the axial wavelength is used, this criterion is based on the ratio between the pitch length $L_y$ of a blade row and the azimuthal wavelength $\lambda_y$. We will assume that CHORUS can be applied safely for azimuthal modes as long as $\frac{L_y}{\lambda_y}$ does not exceed 0.1 (the mode wavelength must be larger than 10 times the blade pitch length). So, considering a turbine row composed of $N_{\text{blades}}$ blades with a mean radius $R$, the azimuthal wavelength $\lambda_y$ of the mode $m$ is

$$\lambda_y = \frac{2\pi R}{m} \tag{2.81}$$

and the pitch length of the blade row $L_y$ is

$$L_y = \frac{2\pi R}{N_{\text{blades}} - 1}. \tag{2.82}$$
This criterion leads to

\[
\frac{L_y}{\lambda_y} = \frac{m}{N_{\text{blades}} - 1}.
\]  

(2.83)

In the TEENI turbine stages, the number of blades is roughly 20 which imposes to use only the first two azimuthal modes at the combustor exit to perform noise computations presented in chapter 7.

2.1.6 Literature test cases

To validate the implementation, simple tests are presented on the basis of acoustic transfer functions of a subsonic stator computed by Kaji and Okazaki [1970a] at very low-frequency and using a semi-actuator disk theory. These test cases were also computed by Cumpsty and Marble [1977]. To further validate the coding, the two-dimensional approach is also compared with the one-dimensional compact theory of Marble and Candel [1977] for a subsonic nozzle. Finally, entropy-to-acoustic transfer functions computed by Cumpsty and Marble [1977] as a function of the inverse of the azimuthal wave number \( K_y \) are compared with our results.

2.1.6.1 Purely acoustic Kaji’s test cases

The first test corresponds to a subsonic blade row with no flow deviation (Fig. 2.8). Results of the acoustic transmission and reflection coefficients for an incident upstream-propagating acoustic wave according to the wave angle \( \nu \) are plotted in Figure 2.9. In the second test case shown in Fig. 2.10, the Mach number is set to 0.5 and the results are presented in Figure 2.11. Although the Kaji’s theory
Figure 2.9: Acoustic transmission and reflexion coefficients for an incident upstream-propagating acoustic wave according to the wave angle $\nu$ computed with CHORUS (—) and compared with Kaji’s results (●) - $M = 0.1$ and $\theta = 60^\circ$ (Fig. 2.8).

Figure 2.10: Sketch of a subsonic blade row with no deviation (second test case).

is based on an actuator disk theory taking into account a finite chord length, we found a very good agreement between the compact theory and these results. When the angle of the incident acoustic wave is equal to the blade angle (Figs. 2.9(b) and 2.11(b)), the acoustic transmission coefficient is unitary and there is no reflection (Figs. 2.9(a) and 2.11(a)). Kaji’s results taken in this comparison correspond
Figure 2.11: Acoustic transmission and reflexion coefficients for an incident upstream-propagating acoustic wave according to the wave angle $\nu$ computed with CHORUS (−) and compared with Kaji’s results (●) - $M = 0.5$ and $\theta = 60^\circ$ (Fig. 2.10).

to their most compact case where $L_{x,\text{blade}}/\lambda = \pi/16$.

Figure 2.12: Sketch of a subsonic blade row with deviation (third test case).

A more realistic stator was also computed by both Kaji and Okazaki [1970a] and Cumpsty and Marble [1977] (Fig. 2.12). For the transmission coefficients (Fig. 2.13(b) ), a very good agreement is found but some differences appear for
the reflection case (Fig. 2.13(a)) where Chorus coefficient is very similar to the Kaji’s result. The reflection coefficient found by Cumpsty is higher than our results for small incidence angles (Fig. 2.13(a)) while Kaji has shown that the reflection coefficient decreases with the compactness of the blade rows.

![Graphs showing acoustic transmission and reflection coefficients](image)

Figure 2.13: Acoustic transmission and reflection coefficients for an incident downstream-propagating acoustic wave according to the wave angle \( \nu \) computed with CHORUS (—) and compared with Kaji’s results (●) and Cumspt’s results (□) (Fig. 2.12).

### 2.1.6.2 Entropy-to-acoustic transfer function of a subsonic nozzle

The entropy-to-acoustic transfer functions computed with CHORUS are compared with the analytical one-dimensional approach of Marble and Candel [1977] for a subsonic nozzle (i.e., a stator vane without flow deviation) with an inlet Mach number equal to 0.1 and exit Mach number varying from 0.1 to 0.9. As expected, the transmission and reflection coefficients for an incident acoustic wave (Fig. 2.15) and an entropy wave (Fig. 2.16) perfectly match with the coefficients computed with CHORUS.
Figure 2.14: Sketch of a subsonic nozzle.

Figure 2.15: Acoustic transmission and reflection coefficients according to Mach number computed with CHORUS (—) and compared with one-dimensional theory (●) for an incident acoustic wave through a subsonic nozzle (Fig. 2.14).
Figure 2.16: Acoustic transmission and reflection coefficients according to Mach number computed with CHORUS (—) and compared with one-dimensional theory (●) for an incident entropy wave through a subsonic nozzle (Fig. 2.14).
2.1.6.3 Entropy-to-acoustic transfer function within a subsonic blade row

In this case, the entropy-to-acoustic transfer functions are computed through the subsonic stator displayed in the Figure 2.12 used by Cumpsty and Marble [1977]. The deviation of the mean flow in the downstream requires to distinguish the transfer functions between the azimuthal waves for which the mode index $m$ are positive or negative. The transfer functions are plotted according to the inverse of the azimuthal wave number $K_y$ and tend to the compact values for a planar entropy wave when $1/K_y$ tends to $+\infty$. The cut-off frequency of the evanescent azimuthal waves corresponding to $K_y = 0.98$ is well-computed and a good agreement is found between CHORUS and Cumpsty’s results. Attention was drawn in the slight frequency shift of the acoustic peak found for the right downstream-propagating wave coefficient in Fig. 2.17(b). As mentioned by Cumpsty, their computation should be inaccurate close to this wave number because of an analytical singularity.
Figure 2.17: Entropy-to-acoustic transfer functions for an incident azimuthal entropy wave according to the inverse azimuthal wave number $K_y$ computed with CHORUS (—) and compared with Cumpsty’s results (●) for a subsonic blade row (Fig. 2.12).
2.2 Post-processing the waves at the end of a combustion chamber simulation

This section presents the different steps required to convert a set of numerous instantaneous LES fields into acoustic and convective waves. The analytical method CHORUS computes the acoustic generation within turbine stages in terms of acoustic downstream and upstream going acoustic waves, entropy and vorticity waves in the frequency domain of longitudinal and azimuthal modes. To do so, as seen in the Fig. 2.18, numerous instantaneous LES fields at the exit of the combustion chamber are interpolated on a structured grid composed of successive Cartesian planes where the interpolating time step has to be small enough to avoid aliasing effects and set the maximum computed frequency ($F_{\text{max}} = 1/\Delta t_{\text{interp}}$). The duration of the numerical simulation sets the minimal frequency computed ($f_{\text{min}} = 1/t_{\text{LES}}$). Note that here the plane normal vectors are collinear to the motor-axis while the diagnostic planes are located at the leading edge of the high-pressurized stator. The velocity magnitude $w$ and angle $\theta$, the pressure $p$ and
the entropy $s$ are the primitive variables required to build the wave vector. The methodology is described below.

**Computation of the temporal 2-D primitive variable vector:** Using the LES fields interpolated on two-dimensional structured planes, the Cartesian velocity vector $(u_x, u_y, u_z)$ is expressed in the polar coordinate system of the engine. The four primitive CHORUS variables are the fluctuations of dimensionless pressure $p'/\gamma p$, the velocity magnitude $w'/c$, the velocity angle $\theta'$ and the entropy $s'/c_p$. A radial averaging over the planes of the primitive variables is performed and it is equivalent to a radial mode decomposition where only the fundamental mode is kept. Each fluctuation is obtained by subtraction of temporal and spatial averaging of the set of solutions from the instantaneous fields which gives the primitive vector $\mathbf{P}(x, \alpha, t)$ at each node of the planes or equivalently as a function of the axial $x$, angular $\alpha$ positions and time, namely

$$
\mathbf{P}(x, \alpha, t) = \begin{pmatrix}
\frac{s'}{c_p} \\
w'/c \\
\frac{p'}{\gamma p} \\
\theta'
\end{pmatrix}.
$$

(2.84)

**Temporal and angular Fourier Transforms:** A temporal Fourier transform is used to express the primitive vector $\mathbf{P}$ in the frequency domain. This FFT is based on the Welsh method which splits the time signals into $N$ segments with an overlap to get smoothed averaged power spectral densities. A spatial Fourier transform is then applied to get the primitive vector $\mathbf{P}(x, \alpha, \omega)$ in terms of azimuthal modes, namely

$$
\mathbf{P}(x, m, \omega) = \frac{1}{\alpha_{max} - \alpha_{min}} \int_{\alpha_{min}}^{\alpha_{max}} \mathbf{P}(x, \alpha, \omega)e^{im\alpha} d\alpha
$$

(2.85)

where $\alpha_{min}$ and $\alpha_{max}$ are the minimum and the maximum azimuthal angles of the computational domain and $m$ is the azimuthal index indicating the mode number (ie $m = 0$ corresponds to the planar mode).

**From the primitive vector to the wave vector** The wave vectors $\mathbf{W}(x, m, \omega)$ are provided by the wave decomposition described in the equation(2.37),

$$
\mathbf{P}(x, m, \omega) = [\mathbf{M}]\mathbf{W}(x, m, \omega).
$$

(2.86)

Although these wave vectors can be used in CHORUS, the proposed wave decomposition does not distinguish the acoustic pressure from the hydrodynamic field. A specific treatment is thus introduced as detailed hereafter.
Hydrodynamic filtering First, presuming that acoustics at low-frequency is essentially two-dimensional, a radial averaging allows removing a large part of hydrodynamic perturbations. Likewise, according to a filtering method proposed by Polifke et al. [2003], the acoustic and convective waves can be combined using the wave velocity and the planes axial spacing to improve the hydrodynamic filtering. This specific property applied to a wave $w$ is written as,

$$\hat{w}(m, \omega) = \frac{1}{N_x} \sum_{i=1}^{N_x} w(x_i, m, \omega) \exp(-ik_x x_i) \quad (2.87)$$

where $N_x$ is the number of axial planes and $k_x$ the axial component of the corresponding wave number.

The wave decomposition allows computing the acoustic power generated by the combustion chamber with the tool CHORUS (Chapter 7) as well as a modal description of the unsteady fields at the exit of the combustor (Chapters 5 and 6).

2.3 A Helmholtz solver: AVSP-f

The last step of the CONOCHAIN methodology is the computation of the acoustic propagation through the exit jet of the engine as well as through the far-field. Ideally to compare the numerical results with experimental measurements provided by microphones located up to 20 meters away from the engine, the numerical simulation of a spherical domain around the engine up to the sensors is needed. For a turboshaft engine, the exit jet has some remarkable properties: a low-Mach number (close to 0.15) which limits the effect of the shear layers on the acoustic directivity and the absence of co-axial jet. These thermal shear layers generates temperature gradients which can impact the acoustic propagation. These characteristics justify the use of the no-Mach number non-isothermal Helmholtz solver AVSP-f to compute the acoustic propagation in the TEENI’s context.

2.3.1 Solving the Phillips’ equation

AVSP-f is based on a formulation of the Phillips’ equation taking into account non-homogeneous sound velocity. Similarly to the Lighthill’s analogy, the Phillips
equation can be seen as a wave operator for reacting flows, namely

\[
\frac{D^2 \pi}{Dt^2} - \frac{\partial}{\partial x_i} \left( c_s^2 \frac{\partial \pi}{\partial x_i} \right) = \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j} - \frac{\partial}{\partial x_i} \left( \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} \right) + \frac{D}{Dt} \left[ \frac{\gamma - 1}{\rho c^2} \left( \dot{\omega}_T + \sum_k h_k \frac{\partial J_k}{\partial x_i} - \frac{\partial q_i}{\partial x_i} + \tau_{ij} \frac{\partial u_i}{\partial x_j} + \dot{Q} \right) \right] + \frac{D^2}{Dt^2} (\ln r) \]

(2.88)

where \( \pi = p'/\gamma p \). The equation (2.88), introduced by Chiu and Summerfield [1974] and Kotake [1975], accounts for some acoustic-flow interactions, since gradients of the sound velocity \( c \) are included in the acoustic operator. Contrary to the Lighthill’s analogy, mean temperature, sound velocity and density are not assumed homogeneous in space as is the case for turboshift engine exit flows. The zero-Mach number assumption leads to some simplifications in the equation (2.88). Moreover, the species diffusion flux \( \frac{\partial J_k}{\partial x_i} \), the external energy source \( \dot{Q} \), the heat fluxes \( \frac{\partial q_i}{\partial x_i} \) and the changes in the molecular weight of the mixture \( \frac{D}{Dt} (\ln r) \) are neglected. Finally, the last source term is the monopolar source of noise \( \frac{\gamma - 1}{\rho c^2} \dot{\omega}_T \) induced by the combustion process and the simplified Philips’ equation (2.89) is

\[
\frac{\partial^2 \pi}{\partial t^2} - \frac{\partial}{\partial x_i} \left( c_s^2 \frac{\partial \pi}{\partial x_i} \right) = \frac{\partial}{\partial t} \left( \frac{\gamma - 1}{\rho c^2} \dot{\omega}_T \right). \tag{2.89}
\]

For far-field computation, no combustion process is considered and \( \dot{\omega}_T = 0 \). Considering harmonic fluctuations (\( \pi = \pi e^{i\omega t} \)), the equation (2.89) becomes

\[
\omega^2 \hat{\pi} + \frac{\partial}{\partial x_i} \left( c_s^2 \frac{\partial \hat{\pi}}{\partial x_i} \right) = 0. \tag{2.90}
\]

### 2.3.2 AVSP forced Helmholtz solver

AVSP forced version is devoted to the prediction of the acoustic field propagating in non-homogeneous media and generated by source terms or injection of acoustic waves across boundary conditions. To take into account three-dimensional geometries, unstructured meshes are used and this imposes some restrictions in terms of numerical methods. A finite volume discretization technique is used. As the finite volume methods have been conceived under the conservative philosophy, conservation equations are accurately solved. However, developments of higher order schemes are very complex under the finite volume approach which limits to second-order the AVSP numerical schemes coupled with a cell-vertex method.
After the discretization of the equation (2.90) over unstructured grids, a matrix system is obtained and solved using a Generalized Minimal RESidual (GMRES) method [Saad and Schultz, 1986]. An acoustic flux is injected across a boundary condition to solve the acoustic field. Injecting acoustic waves from CHORUS in the AVSP-f domain needs however some attention. Typically, AVSP-f solves the acoustic field in terms of acoustic pressure and velocity which are provided by the downstream-propagating wave computed by CHORUS at the exit of the engine.

2.3.3 Analytical description of a low-Mach number hot jet

A simple description of the temperature field at the exit of a turboshaft engine is proposed in this section for low-Mach number jets. Although no velocity fields are taken into account, a qualitative modelling of the temperature patterns is required because of the major impact of temperature gradients in the refraction of sound waves as analytically addressed in the appendix B. A low-Mach number jet flow can be described into three main zones. As shown in Fig. 2.19, a potential core composes the first zone of the jet where the mean velocity is constant up to the X-coordinate \( x_p \). A transient zone is located between the end of the potential core the fully developed region. A modelling of a low-Mach number jet flow is proposed in this section where a strong assumption is made: the mean temperature is proportional to the mean velocity. Using scaling velocity laws for the different regions of the jet flow, the dimensionless temperature field is built. The mean pressure everywhere in the jet is equal to the atmospheric pressure.

**Axial velocity variation:** In the potential core, the velocity is constant and equal to the mean inlet value. In the fully-developed zone, the modified axial Reynolds number is constant, namely

\[
Re_x = U_{inlet} R_{nozzle} = r_{12}(x) U_x
\]

(2.91)

where \( U_{inlet} \) is the mean inlet velocity, \( R_{nozzle} \) the nozzle radius, \( U_x \) the center-line axial velocity and \( r_{12}(x) \) is the full width at half maximum of the velocity Gaussian profile. The axial velocity is given by

\[
U_x(x) = U_{inlet} \frac{2R_{nozzle} B}{x} \text{ where } B = 6.
\]

(2.92)

In the transient zone, we assume that the axial velocity decreases linearly from the end of the potential core to the full-developed region.

**Radial velocity profile in the potential core:** Zaman [1998] proposed an expression of \( x_p \) according to the mean jet flow density \( \rho_{jet} \) and pressure \( P_{jet} \), the
Atmosphere \((P_a, \rho_a, T_a)\)

Nozzle

Radius = \(R_{\text{nozzle}}\)

\(x = 0\) \hspace{1cm} x = x_p \hspace{1cm} x = x_t\)

\begin{align*}
\text{Potential core} & \quad \text{Transient zone} & \quad \text{Fully developed region} \\
P_{\text{jet}} &= P_a \\
T_{\text{jet}} &= T_{\text{inlet}}
\end{align*}

Gaussian radial temperature profiles

Figure 2.19: Sketch of a low-Mach number jet flow

atmospheric density \(\rho_{atmo}\), pressure \(P_{atmo}\) and the mean velocity \(U_{\text{inlet}}\), namely

\[ \frac{x_p}{D_j} = \sqrt{\frac{\rho_j U_j^2 + P_j - P_a}{K_u U_j \sqrt{\rho_a}}} \quad \text{and} \quad K_u = 0, 16. \quad (2.93) \]

To smooth the boundary between the potential core and the quiescent medium, a Gaussian law is applied where the full width at half maximum increases linearly from the nozzle lips to \(x_p\). So, the velocity profile is equal to

\[
\begin{cases}
1 \text{ if } R < R_{\text{core}} \\
\exp\left(\frac{R}{2R_{\text{nozzle}}} \frac{2}{2\sigma^2} \right) \text{ if } R > R_{\text{core}}
\end{cases}
\]

(2.94)

where

\[
\sigma = \frac{1}{2 \sqrt{2ln2} x_p}. \quad (2.95)
\]

So, the radial velocity is a hat close to the nozzle and tends to a Gaussian profile when \(x\) tends to \(x_p\) as seen in Fig. 2.20.
Radial velocity profile in the transient and fully-developed zones: In these zones, the radial velocity profiles are Gaussian functions but the amplitudes and the full width at half maximum differs. In the transient zone, the axial velocity is continuous at the end of potential core and the beginning of the fully developed region is defined by the equation (2.92) [Pope, 2000]. So, the radial velocity profile is

\[
A \exp \left( -\left( \frac{r}{2R_{\text{nozzle}}} \right)^2 \frac{1}{2\sigma^2} \right) \quad (2.98)
\]

where

\[
\sigma = \frac{Re}{2R_{\text{nozzle}}U_{\text{inlet}}} \frac{1}{\sqrt{2\ln 2}}. \quad (2.97)
\]

In the fully-developed region, the equation (2.92) provides the axial velocity. Knowing consequently \( r_{12} \), the profile becomes

\[
A \exp \left( -\left( \frac{r}{2R_{\text{nozzle}}} \right)^2 \frac{1}{2\sigma^2} \right) \quad (2.98)
\]
with

\[ A = \frac{1}{\sigma \sqrt{2\pi}}. \quad (2.99) \]

**Temperature field** Finally, the temperature field is plotted in Fig. 2.21 where the three main zones described above are visible. Up to 4\(D\), the potential core induces strong temperature gradients around the jet. The transient zone is located from 4\(D\) to 7\(D\) and the fully-developed zone above 7\(D\) smooths the temperature profiles to reach a maximum close to 0.35 at 15\(D\).

![Dimensionless temperature profiles](image)

**Figure 2.21:** Dimensionless temperature profiles \(T_x\) according to dimensionless radius \(R/D\) and axial coordinate \(x/D - D\) the nozzle diameter.

induces strong temperature gradients around the jet. The transient zone is located from 4\(D\) to 7\(D\) and the fully-developed zone above 7\(D\) smooths the temperature profiles to reach a maximum close to 0.35 at 15\(D\).

## 2.4 Conclusion

This chapter presented the numerical methodology CONOCHAIN used in this thesis to compute the direct and indirect combustion noise within a real aero-
Part II

Large-Eddy simulations of the TEENI combustion chamber dedicated to the combustion noise evaluation
The part presents the Large-Eddy simulations of the TEENI annular combustion chamber to provide unsteady fields at the exit of the combustor and apply the CONOCHAIN methodology described in chapter 2. To do so, numerical simulations correspond to stabilized operating points performed during the TEENI project in terms of mean features but also in terms of acoustics. This part is composed of two main chapters:

- the first one presents the LES method and solver AVBP used to perform the numerical simulations,

- the second chapter describes the simulated experimental operating points and the numerical set-up of the LES,

- the third one presents the simulations of a single sector to apply CONOCHAIN at two experimental operating points, 344kW and 904kW,

- the full-scale simulation of the TEENI combustion chamber at 904kW is presented and compared with the single sector simulation in the last chapter.
Chapter 3

Numerical tools for the Large-Eddy Simulations

3.1 Large Eddy Simulation with AVBP

Large Eddy Simulation is dedicated to solve the turbulent compressible Navier-Stokes equations and is an intermediate numerical concept between Direct Numerical Simulations and the Reynold Averaged Navier-Stokes methodology. In RANS concept, the governing equations are based on an ensemble average over a set of realizations [Pope, 2000] which removes the temporal dependence of the Navier-Stokes equations but yields unclosed terms. These terms are modelled to be representative of the physics taking place over the entire range of frequencies present in the ensemble of realizations. Contrarily, DNS consists of solving the compressible Navier-Stokes equations without any approximation, where the large scales of the turbulent flow are resolved as well as the small ones. Although this approach allows capturing complex physics and flow interactions, the computational cost limits this technique to canonical cases when the Reynolds number grows. Large Eddy simulation [Sagaut, 1998] is an original approach based on the energy cascade in turbulent flows where the kinetic energy is convected by the large turbulent scales and dissipated at the small ones. So, a low-pass spatial filter is applied on the Navier-Stokes equations to provide the governing equations where the small scales are removed. However, the unclosed terms corresponds to the interaction between the small structures with the turbulent flows and are modelled with different approaches. LES are consequently able to capture the unsteadiness within the turbulent flows in complex geometries with high-fidelity and allows a dynamic representation of a single representation of the flow. Note that this technique tends to DNS when the spatial filter size tends to zero. Using compressible and reactive Navier-Stokes equations, the LES predictions are close
to the physics such as turbulent combustion and acoustic waves [Poinsot and Veynante, 2005] highlighting their potential in predicting turbulent flows in industrial applications.

### 3.1.1 The governing equations

The set of conservation equations for reacting flows can be written:

$$\frac{\partial \mathbf{w}}{\partial t} + \nabla \cdot \mathbf{F} = \mathbf{s} \tag{3.1}$$

where \( \mathbf{w} \) is the vector of the conservative variables:

$$\mathbf{w} = (\rho u, \rho v, \rho w, \rho E, \rho_k) \tag{3.2}$$

with respectively \( \rho, u, v, w, E, \rho_k \) the density, the three cartesian components of the velocity vector \( \vec{V} = (u, v, w)^T \), the energy per unit mass and \( \rho_k = \rho Y_k \) for \( k = 1 \) to \( N \) (\( N \) is the total number of species). The source term \( \mathbf{s} \) is decomposed for convenience into a chemical source term and a radiative source term such that:

$$\mathbf{s} = \mathbf{s}^C + \mathbf{s}^R.$$ 

It is usual to decompose the flux tensor \( \mathbf{F} = \mathbf{F}(\mathbf{w})I + \mathbf{F}(\mathbf{w}, \nabla \mathbf{w})^V \) into an inviscid and a viscous component. The three spatial components of the inviscid flux tensor \( \mathbf{F}(\mathbf{w}) \) are:

$$\begin{pmatrix}
\rho u^2 + P & \rho uv & \rho uw \\
\rho uv & \rho v^2 + P & \rho vw \\
(\rho E + P) u & (\rho E + P) v & (\rho E + P) w \\
\rho_k u & \rho_k v & \rho_k w
\end{pmatrix} \tag{3.3}$$

for the inviscid terms where the hydrostatic pressure \( P \) is given by the equation of state for a perfect gas, namely

$$P = \rho r T. \tag{3.4}$$

The components of the viscous flux tensor \( \mathbf{F}(\mathbf{w}, \nabla \mathbf{w})^V \) are

$$\begin{pmatrix}
-\tau_{xx} & -\tau_{xy} & -\tau_{xz} \\
-\tau_{xy} & -\tau_{yy} & -\tau_{yz} \\
-\tau_{xz} & -\tau_{yz} & -\tau_{zz}
\end{pmatrix}$$

\( \begin{pmatrix}
-(u \tau_{xx} + v \tau_{xy} + w \tau_{xz}) + q_x \\
-(u \tau_{xy} + v \tau_{yy} + w \tau_{yz}) + q_y \\
-(u \tau_{xz} + v \tau_{yz} + w \tau_{zz}) + q_z
\end{pmatrix} \)

\( J_{x,k} \)

\( J_{y,k} \)

\( J_{z,k} \).
Species diffusion flux \( J_{i,k} \): The species diffusion flux is based on the Hirschfelder Curtis approximation [Hirschfelder et al., 1969] where the diffusion velocity of species are expressed in terms of species gradients and a correction diffusion velocity \( V^c \) to take into account the convection velocity [Poinsot and Veynante, 2011] to satisfy the total mass conservation, namely

\[
J_{i,k} = -\rho \left( D_k \frac{W_k}{W} \frac{\partial X_k}{\partial x_i} - Y_k V^c \right), \quad i = 1, 2, 3
\]  

(3.6)

where \( D_k \) are the diffusion coefficients for each species \( k \) in the mixture, \( W \) and \( W_k \) the molar mass of the mixture and the specie \( k \) respectively, and \( X_k \) and \( Y_k \) the molar and mass fraction of specie \( k \) respectively.

Viscous stress tensor \( \tau_{ij} \): The stress tensor \( \tau \) is given by the following relations:

\[
\tau_{ij} = 2\mu \left( S_{ij} - \frac{1}{3} \delta_{ij} S_{ii} \right), \quad i, j = 1, 3
\]

(3.7)

and

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right), \quad i, j = 1, 3
\]

(3.8)

where \( \mu \) is the shear viscosity modeled with a Sutherland’s law.

Heat flux vector \( q_i \): For multi-species flows, an additional heat flux term appears in the diffusive heat flux. This term is due to heat transport by species diffusion. The total heat flux vector then writes:

\[
q_i = -\lambda \frac{\partial T}{\partial x_i} + \sum_{k=1}^{N} J_{i,k} h_{s,k}
\]

(3.9)

Heat conduction

Heat flux through species diffusion

where \( \lambda \) is the heat conduction coefficient of the mixture and \( h_{s,k} \) the sensible mass enthalpies of the specie \( k \).

The source term on the right-hand side of equation (3.1) is equal to

\[
\begin{pmatrix}
S_u \\
S_v \\
S_w \\
\dot{\omega}_T \\
\dot{\omega}_k
\end{pmatrix}
\]

(3.10)

where \( \dot{\omega}_T \) is the rate of heat release and \( \dot{\omega}_k \) the reaction rate of specie \( k \). The source terms \( S_u, S_v, S_w \) are used to impose source terms on momentum components. For
reacting flows, an Arrhenius law is often used to model the reaction rates $\dot{\omega}_k$ [Poinset and Veynante, 2011]. Furthermore, thermodynamic variables as sensible mass enthalpy or entropy are computed with JANAF tables [Stull and Prophet, 1971] and interpolated between 0K and 5000K.

### 3.1.2 Equations for compressible Large Eddy Simulations

Applying the spatial filtering, the quantity $\bar{f}$ is resolved whereas $f' = f - \bar{f}$ is the subgrid scale part due to the unresolved flow motion. The mass-weighted Favre filtering is well-adapted to the conservative equations, namely

$$\bar{\rho} \bar{f} = \rho f.$$  

(3.11)

So, the LES governing equations are obtained by filtering the equation (3.1):

$$\frac{\partial \bar{w}}{\partial t} + \nabla \cdot \bar{F} = \bar{s}$$  

(3.12)

where $\bar{\rho}$ is the mass density, $\bar{F}$ is the flux tensor. The spatial filtering where the cut-off corresponds to the mesh size divides the flux tensor $F$ and the source term $s$ into a resolved part and a subgrid scale component. The decomposition of $F$ presented in the previous section is also valid but leads to unclosed quantities, which need to be modelled. So, the inviscid terms of the flux tensor $F$ are equal to

$$\begin{pmatrix}
-\bar{\tau}_{xx} & -\bar{\tau}_{xy} & -\bar{\tau}_{xz} \\
-\bar{\tau}_{yx} & -\bar{\tau}_{yy} & -\bar{\tau}_{yz} \\
-\bar{\tau}_{zx} & -\bar{\tau}_{zy} & -\bar{\tau}_{zz}
\end{pmatrix} + \begin{pmatrix}
\bar{q}_x & \bar{q}_y & \bar{q}_z
\end{pmatrix}$$  

(3.13)

and the viscous terms take the form:

$$\begin{pmatrix}
-\bar{\tau}_{xx}^t & -\bar{\tau}_{xy}^t & -\bar{\tau}_{xz}^t \\
-\bar{\tau}_{yx}^t & -\bar{\tau}_{yy}^t & -\bar{\tau}_{yz}^t \\
-\bar{\tau}_{zx}^t & -\bar{\tau}_{zy}^t & -\bar{\tau}_{zz}^t
\end{pmatrix}$$  

(3.14)

Noting with the subscript $\bar{t}$ the subgrid scale turbulent terms of the flux tensor $F$, the subgrid scale turbulent tensor is

$$\begin{pmatrix}
-\bar{\tau}_{xx}^t & -\bar{\tau}_{xy}^t & -\bar{\tau}_{xz}^t \\
-\bar{\tau}_{yx}^t & -\bar{\tau}_{yy}^t & -\bar{\tau}_{yz}^t \\
-\bar{\tau}_{zx}^t & -\bar{\tau}_{zy}^t & -\bar{\tau}_{zz}^t
\end{pmatrix}$$  

(3.15)
Modelling of these unclosed terms is required to resolve the set of LES equations. The subgrid scale turbulent fluxes are modelled to be representative of the interaction between the large structures of the flows and the small ones, especially in terms of turbulent energy dissipation. Based on homogeneous isotropic turbulent flows, a historical model was proposed by Smagorinsky [1963] where the dissipative effect of the small structures was modelled with a turbulent viscosity $\nu_t$ proportional to the size of the spatial LES filter $\Delta$, the resolved strain rate tensor $\tilde{S}_{ij}$ and a constant value $C_s$, namely

$$\nu_t = (C_s \Delta)^2 \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}.$$  \hspace{1cm} (3.16)

Different approaches were proposed in the literature to reduce the dissipation induced by this model, as a filtering version of the Smagorinsky’s model where the resolved strain rate tensor is obtained by filtering with a high-pass filter the resolved velocity field [Ducros et al., 1998], or a better evaluation of the constant $C_s$ computing with a Germano’s inequality [Lilly, 1992]. However, some models are dedicated to deal with the flow dynamics close to walls [Nicoud and Poinset, 1999] or for transient flows where the assumption of a homogeneous isotropic turbulence is false. In the present work, the classical Smagorinsky’s model is used to perform the numerical productive simulations of the chapters 5 and 6. Preliminary simulations of the single sector are performed with the dynamic Smagorinsky’s model and the filtering Smagorinsky’s model to validate this choice, presented in the appendix E. The subgrid diffusive species flux vector is modelled using a diffusion coefficient based on a turbulent Schmidt number and a turbulent Prandtl number gives the subgrid heat diffusion coefficient in the subgrid heat flux vector.

Similarly to the flux tensor $\mathbf{F}$, the source terms are divided into a resolved component and a subgrid scale component. For combustion process, the reaction zone is generally thinner than the spatial filter size, i.e. the mesh size. As the interaction between the flame and turbulent structure are known to be primordial to solve flame dynamics, combustion models have been built to take into account these effects and overcome this issue. Different approaches have been developed in the literature:

- **Algebraic models based on an infinite thin flame and infinite reaction rate** [Fureby and Lofstrom, 1994].

- **Geometrical approaches** where the flame front is assumed to be thin compared to integral length scales [Pitsch, 2006]. The flame is then described as an interface between fuel and oxidizer (non-premixed) or between fresh and burned gases (premixed). These approaches use flame front tracking techniques (flame surface density assumption, G-equation or c-equation).
• Statistical analysis whereby scalar fields may be collected and analysed for any location within the flow. Filtered values and correlations are afterwards extracted via knowledge of filtered probability density functions (FPDF) to be determined either by a presumed assumption or by solving a FPDF-transport equation (presumed PDF, CMC, LEM) [Meneveau and Poinsot, 1991, Cook and Riley, 1994, Pierce and Moin, 1998].

• Artificially thickening the flame front. In this approach a flame thicker than the real one is considered, but that has the same laminar flame speed [Butler and O’Rourke, 1977, Angelberger et al., 1998, Colin et al., 2000].

The last model is used in the numerical simulations presented in this work. The main idea of the thickened flame model is the resolution of the flame front on a mesh in which the mesh size is larger than the reacting zone always encountered in LES. The thickening stiffens the flame and altered the interaction between the vortices and the flame front. The eddies smaller than the thickened reaction zone do not interact with the flame. Furthermore, the reaction rate is computed with an efficiency function to take into account the underestimation of the flame surface. So, the thickening is dynamically applied in the reaction zone [Légier et al., 2000].
Chapter 4

Operating points and numerical set-up

Abstract  The aim of this chapter is to introduce the numerical set-up and the operating points simulated in this work to apply the CONOCHAIN methodology. Two stabilized operating points are computed, representative to the lowest engine power performed in the TEENI experiment and the highest one which corresponds to the full power of the engine during take-off. Once the geometry is introduced, the numerical models and assumptions are presented.
4.1 Configuration and operating points

4.1.1 Description of the geometry

Gas generator of the TEENI engine is fitted with a reverse annular combustion chamber composed of $N_{\text{sectors}}$ sectors. A sketch of the LES domain of a single sector is shown in Fig. 4.1. The LES domain is composed of the casing where the air flow coming from the compressor is injected and the flame tube where the combustion takes place. In the real engine, the liquid kerosene is injected through two counter-rotating swirlers as shown in Fig. 4.2(a). In the simulations, no liquid phase is considered which induces a slight modification of the injection zone. To be representative of the fuel dispersion in the primary zone of the combustion chamber, the gaseous fuel is injected where the liquid spray impacts the swirler lips [Boudier et al., 2008] (Fig. 4.2(b)). Multi-perforated plates and dilution holes ensure the cooling of the burnt gases and the thermal protection of the flame tube in the primary zone. At the exit of the combustion chamber, the high-pressurized stator is removed. Thus, the wave extraction performed to compute the combustion noise with CHORUS is not impacted by potential effects generated by the blades. So, to conserve the sonic state of the stator, an equivalent nozzle is inserted at the chamber outlet to be representative of the mean curvilinear evolution of the Mach number through the blade vane. This methodology is described in

![LES domain of a 1/$N_{\text{sectors}}^{th}$ of TEENI annular combustion chamber](image)

Figure 4.1: LES domain of a 1/$N_{\text{sectors}}^{th}$ of TEENI annular combustion chamber
4.1.2 Operating points

The productive simulations correspond to stabilized experimental operating points where the first high-pressurized stator is choked. The single-sector LES are representative to an engine power equal to 344 kW and 904 kW while the full-scale simulation corresponds only to the higher engine power. As the outlet nozzle is choked, the mass flow rate injected through the air inlet is set to reach the experimental mean pressure within the combustion chamber. The fuel mass flow rate is computed with a given fuel-air-ratio. The complete main parameters of the operating points are provided in the Table. 4.1.

<table>
<thead>
<tr>
<th>Dimensionless c.c parameters</th>
<th>Operating points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Engine power</td>
<td>344 kW</td>
</tr>
<tr>
<td>Stagnation temperature</td>
<td>0.8</td>
</tr>
<tr>
<td>Stagnation pressure</td>
<td>7.5 bars</td>
</tr>
<tr>
<td>Fuel-air ratio</td>
<td>0.78</td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>2.20 kg/s</td>
</tr>
</tbody>
</table>

Table 4.1: Dimensionless combustion parameters at low and high powers.
4.2 Numerical parameters

The set of numerical simulations presented in this PhD is performed with the LES solver AVBP. This software is a fully unstructured compressible code including species transport and variable heat capacities. Based on a cell-vertex formulation, centred spatial schemes and explicit time-advancement are used to solve the reactive Navier-Stokes equations presented in the previous section. These numerical techniques allow controlling the numerical dissipations and capturing acoustic waves [Colin et al., 2000]. Productive simulations are performed using a TTG third order accurate in space and time scheme with a time step controlled by the speed of sound [Lamarque, 2007].

4.2.1 Boundary conditions

Since the objective of these simulations is to compute waves leaving the chamber to enter the turbine, boundary conditions require specific treatments to handle the unsteady flow features. In AVBP, the Navier-Stokes Characteristic Boundary Conditions (NSCBC) method is implemented decomposing each variation of the flow variables on the boundaries into ingoing and outgoing waves. In these computations, the NSCBC treatment is only applied at the inlet where the air mass flow rate and the flow temperature are known, while wall-law adiabatic conditions are imposed at all walls of the chamber except in the outlet nozzle where a slip condition is used. Since the throat of the nozzle is choked, no additional boundary condition is need at the chamber outlet. Contrary to the dilution holes which are resolved in the simulations, the multi-perforated plates are modelled with a homogeneous approach [Mendez and Nicoud, 2008] to take into account both the mass flow rate injected through the plates and the acoustic behaviour of these devices. This model is based on the computation of the instantaneous pressure gradients across the plates to compute the suction and injection sides and the associated flow velocities. Furthermore, the flow dynamic in the plate is described with the unsteady Bernoulli equation which can be seen as an acoustic impedance depending on the geometry. Preliminary simulations shown that the acoustic activity of the combustor are deeply impacted by the modelling of the multi-perforated plates which justified to use this model. These simulations are presented in the appendix E.

4.2.2 Chemistry and combustion model

Kerosene is a mixture of hydrocarbons mainly composed of alkanes ranging from $C_{10}H_{22}$ to $C_{14}H_{30}$ with a high specific energy and properties well-adapted to aviation requirements. However, this complex composition leads to a rather
complex chemical scheme of reactions involving a large amount of species to reproduce properly the combustion process. Therefore, the composition as well as the chemistry have to be modelled in the LES context.

First, Kerosene is replaced with a fuel surrogate equivalent to $C_{10}H_{20}$ which is quite similar in terms of thermodynamic properties [Franzelli et al., 2010]. The complex chemical scheme is replaced with a two-reaction scheme, namely

$$F + 10O_2 \rightarrow 10CO + 10H_2O$$  \hspace{1cm} (4.1)

$$CO + \frac{1}{2}O_2 \leftrightarrow CO_2,$$ \hspace{1cm} (4.2)

where $F$ is the surrogate fuel. This scheme is known to well-computed the laminar flame properties in lean conditions but can lead to an over estimation of the adiabatic temperature when the mixture composition is rich. Consequently, both reaction rates are fitted to retrieve a realistic adiabatic temperature. This reduced scheme was compared with a detailed chemical computation [Luche, 2003] for which very good agreements were found in our thermodynamic conditions of interest.

The Dynamic Thickening Flame LES model is based on the laminar flame characteristics. As two operating points are computed in this work, two laminar flame computations are performed to set the flame properties (Table 4.2) required in the DTFLES model at the stoichiometric and corresponding to the combustion chamber thermodynamic conditions.

<table>
<thead>
<tr>
<th>Operating points</th>
<th>344 kW</th>
<th>904 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flame speed (m/s)</td>
<td>0.652</td>
<td>0.745</td>
</tr>
<tr>
<td>Flame thickness (m)</td>
<td>5.92.10^{-5}</td>
<td>5.25.10^{-5}</td>
</tr>
</tbody>
</table>

Table 4.2: Laminar flame characteristics for the DTLES model

4.2.3 Modelling of multi-perforated plates in AVBP

Multi-perforated plates are cooling devices to ensure thermal wall protection in the primary zone of the flame tube by generation of local effusion films. The holes are characterized by the injection angle depending from the zone within the combustor and a radius equal to 0, 2 mm. A proper resolution of the flow within these devices requires mesh resolution and computational cost out of reach in the LES context. Based on these observations, Mendez and Nicoud [2008] proposed a modelling where the multi-perforated plates are considered equivalent to homogeneous inlet boundary conditions to ensure mass conservation and conserved the tangential...
momentum. For a LES of single sector with the casing, the mass flow rate injected through the plates is given by empirical laws and set in the computation. Consequently, the mass flow rate injected from the compressor is modified to take into account mass injection through the multi-perforated plates. This method is used in the design stage in industry because is able to reproduce the main aerodynamic characteristics of the plates whereas their acoustic behavior.

The impact of the perforated wall on acoustics is well-known: acoustic waves impacting the plate induce locally a flow through the hole which generates vortices and absorb partially the acoustic energy [Rienstra and Hirschberg, 2011]. Linear sound absorption through a perforated plate was addressed by Howe [1979] for which a model was proposed to evaluate the amount of energy transferred from the acoustic to vortical energy. Introducing the harmonic mass flow rate fluctuation $\hat{q}$ and the pressure fluctuations $\hat{p}^+$ and $\hat{p}^−$ for both sides of the apertures, the Rayleigh conductivity $K_R$ can be seen as an admittance of the plate, namely

$$K_R = \frac{i \rho \omega \hat{q}}{\hat{p}^+ - \hat{p}^-} \quad \text{(4.3)}$$

where $\rho$ is the density through the aperture and $\omega$ the pulsation. Howe [1979] proposed a complex expression for the Rayleigh conductivity in case of mean flow through the aperture giving

$$K_R = 2a(\gamma - i \delta), \quad \text{(4.4)}$$

where $a$ is the hole radius, $\gamma$ and $\delta$ depends on the Strouhal number and modified Bessel functions. The frequency dependence of this formulation is a problem for time-dependent solvers to reproduce accurately the acoustic response of the plates. Instead, an other approach was developed in AVBP based on the linearisation of the unsteady Bernoulli equation which provides the velocity fluctuation across the aperture in terms of the geometrical parameters and the pressure gradient $\Delta P$ [Mendez and Eldredge, 2009], namely

$$\Delta P = \rho \left( \frac{\pi a}{2} + \frac{h}{\sin(\alpha)} \right) \frac{\partial U_j}{\partial t} + \frac{(\rho U_j)^2}{2\rho C_D^2} \quad \text{(4.5)}$$

where $U_j$ is the bulk velocity in the aperture, $a$ and $\alpha$ the radius and the angle of the hole, $\rho$ the density and $C_D$ a discharge coefficient across the plate. This acoustic impedance was tested on a canonical test case where an incident acoustic wave was injected through a circular aperture to capture the conversion of acoustic energy into vortical energy and compared with the Howe model [Mendez and Eldredge, 2009]. Very good agreements were found by comparisons between the analytical approach and the numerical results in terms of acoustic impedances. Particularly,
the equation 4.5 is equivalent with the Howe model for a discharge coefficient equal to 0.64. In this work, this model is used to perform the CONOCHAIN LES. The impact of the multi-perforated plates on the acoustics within the combustion chamber is addressed in appendix E.
Chapter 5

Single-sector LES of the TEENI combustion chamber

Abstract  This chapter describes LES of a single sector of the TEENI annular combustion chamber with AVBP. The lower and higher operating points are simulated at 344kW and 904kW respectively. The mean flow predictions are found to be very similar between the two operating conditions. Entropy waves are created within the combustion chamber and caused by the interaction between the dilution jet flows and the burnt gases while the unsteady pressure fluctuations are related to the unsteady heat release. The acoustic field within the LES domain is investigated using the Helmholtz solver AVSP. The acoustic solver correctly predicts the acoustic longitudinal modes in terms of cut-off frequency and spatial structure contained in the LES. Moreover, using a modal decomposition at the exit of the combustion chamber, the pressure field is mainly carried by longitudinal components while the entropy field is disturbed and composed of higher azimuthal modes. Finally, LES is able to retrieve the experimental fluctuating pressure recorded during the TEENI project in terms of levels and patterns.
5.1 Mesh description

Both simulations of the TEENI sector are based on a full tetrahedral mesh composed of 12.8 millions of elements. As shown in Fig. 5.1, the primary zone of the combustor is described with tetrahedral cells of sizes close to 0.5 mm. The mesh dependency was investigated by Boudier et al. [2008] and Wolf [2011] where a single sector composed of 6.2 millions elements was sufficient to reach a statistical convergence. Our criteria to perform this mesh were a accurate description of the primary zone and the computation of the dilution jet interactions with the burnt gases as well as a proper description of the entropy waves convection through the flame tube. Considering that an entropy wave is well-described with 5 cells at least, the cut-off frequency above which the convective waves are altered is computed for a average velocity equal to 30 $m.s^{-1}$ corresponding to a Mach number close to 0.05, namely

\[ f_c = \frac{\hat{U}}{5.\Delta x_{cell}} . \tag{5.1} \]

In the scope of combustion noise, the frequency range can reach 2000 Hz and the mesh is built to ensure a convective wave convection up to 5000 Hz which requires a $\Delta x_{cell}$ of the order of 1 mm according to Eq. (5.1).
5.2 Mean flow description

The physical time simulated in these LES is of 82 ms with a time-step equal to $2 \times 10^{-8}$s corresponding to a maximum CFL number of 0.7. The time-step is strongly impacted by the smallest cells located to the swirler vanes. The air mass flow rate is coming from the compressor and impacting the flame tube in the casing. As the simulations are choked, there is a linear dependence between the mean pressure in the combustor and the exit mass flow rate. If $\dot{m}$ is the mass flow rate, using the stagnation pressure $P_0$ and temperature $T_0$ within the combustion chamber, we get

$$\dot{m} = AP_0 \sqrt{\frac{\gamma}{rT_0}} \left( \frac{\gamma + 1}{2} \right)^{-\frac{\gamma + 1}{2(\gamma - 1)}}$$

(5.2)

where $A$ is the nozzle section. Thus, the stagnation pressure within the combustor can be plotted according to the exit mass flow rate for non-reactive and reactive cases in Fig. 5.2. For a given operating point, the difference of stagnation pressure variation with the mean mass flow rate is caused by the difference of gas composition at the exit and thus by the mean density. The black squares corresponds to the stabilized operating points of the LES confirming the exit nozzle is choked in both cases.

![Graphs of stagnation pressure vs. mass flow rate for different powers](image)

(a) 344 kW  
(b) 904 kW

Figure 5.2: Stagnation pressure within the combustion chamber according to the exit mass flow rate for reactive (---) and non-reactive cases (- - -) and stabilized operating points (■).

As the multi-perforated plates are modelled with a dynamic coupled approach [Mendez and Nicoud, 2008] (see Eq. 4.5), the mass flow rates computed with the
pressure gradient across the plates in the LES can be compared with the empirical evaluation of flow splits using simple one-dimensional network approaches which compute pressure losses and flow split according to experimental correlations. Then, the COOLANT tool of Turbomeca was used to obtain these evaluations. The mean mass flow rates across the multi-perforated plates for both simulations are given in Table 5.1. For both operating points, a very good agreement is found between the empirical values and the computed ones. These results depend on a correct estimation of the pressure losses within the combustion chamber which are well-predicted in our simulations (close to 3%).

<table>
<thead>
<tr>
<th></th>
<th>344 kW</th>
<th></th>
<th>904 kW</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Prescribed</td>
<td>Computed</td>
<td>Prescribed</td>
</tr>
<tr>
<td>Plate 1</td>
<td>6.22%</td>
<td>6.24%</td>
<td>6.22%</td>
</tr>
<tr>
<td>Plate 2</td>
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<td>5.96%</td>
<td>5.84%</td>
</tr>
<tr>
<td>Plate 3</td>
<td>9.27%</td>
<td>9.69%</td>
<td>9.27%</td>
</tr>
<tr>
<td>Plate 4</td>
<td>11.06%</td>
<td>11.6%</td>
<td>11.06%</td>
</tr>
<tr>
<td>Pressure losses</td>
<td>2.92%</td>
<td>3.19%</td>
<td>2.92%</td>
</tr>
</tbody>
</table>

Table 5.1: Mean mass flow rate distribution within the flame tube (Fig. 5.3) and pressure losses for the operating points.
Figure 5.3: Sketch of the multiperforated plate locations.
Figure 5.4: Sketch of the different planes locations.
Figure 5.5: Sketch of the probes locations within the combustion chamber.
5.2.1 Mean velocity in the combustion chamber

The mean velocity fields presented in this section are scaled by the inlet mean velocity $U_{inlet}$. The $U_{inlet}$ values do not differ from 344 kW to 904 kW contrary to the mean mass flow rate because the density is higher for the 904 kW case. The mean dimensionless velocity amplitude slightly differs between the two operating points as shown in Figs. 5.9 and 5.8. The dilution holes generate high velocity jet flows within the flame tube where the upper flow is deviated by the interaction with the low flow within the primary zone. Thus, a large recirculation zone is generated around the swirler highlighted in Figs. 5.6 and 5.7 and leads to low-velocity levels except in front of the lips of the swirler. The upper recirculation zone is more important than the lower because of the deviation of the upper jet flow in the primary zone. In the second zone of the flame tube, the mass flow rate injected circumferentially by the multi-perforated plates induces the slight azimuthal component of the mean velocity field. The high degree of similarity between the two simulated powers is noticeable in terms of mean velocity amplitude as well as streamlines of the mean flow.

5.2.2 Mean Temperature field

The dimensionless mean temperature field is equal to $\frac{T-T_{inlet}}{T_{adiab}-T_{inlet}}$ where $T_{inlet}$ is the mean inlet temperature and $T_{adiab}$ is the mean adiabatic temperature reached in the simulations. The mean flame location is highlighted by the iso-lines of the mean heat-release as shown in Figs. 5.9 and 5.8. The most reactive zones are confined close the swirler as shown in Fig. 5.9 but the axial cut in Fig. 5.8 shows a reacting zone up to the dilution holes for both power settings. A larger and more spread-out heat release distribution as well as the stoichiometric mixture fraction
Figure 5.7: Streamlines of the mean projected velocity in plane $\Delta_2$ (Fig. 5.4).

Figure 5.8: Mean dimensionless velocity magnitude $\frac{w}{U_{inlet}}$ with white isolines of mean heat release ($W/m^3$) in plane $\Delta_1$ (Fig. 5.4).

Isolines (Figs. 5.10 and 5.11) are observed for the high power condition. The mean temperature fields are characterized by a primary zone in which combustion takes place to reach the adiabatic temperature for both simulations. The air flow coming from the dilution holes interacts with the burnt gases to reduce the mean
Figure 5.9: Mean dimensionless velocity magnitude $\frac{\nu}{U_{inlet}}$ with white isolines of mean heat release ($W/m^3$) in unwrapped plane $\Delta_2$ (Fig. 5.4).

Figure 5.9: Mean dimensionless velocity magnitude $\frac{\nu}{U_{inlet}}$ with white isolines of mean heat release ($W/m^3$) in unwrapped plane $\Delta_2$ (Fig. 5.4).

temperature in the center of the flame tube. As shown on the mean temperature fields in plane $\Delta_3$ (Fig. 5.12), mixing is more important in the swirler axis which leads to non-uniform mean temperature patterns.
Figure 5.10: Mean dimensionless temperature field $\frac{T - T_{inlet}}{T_{adiab} - T_{inlet}}$ with an isoline of stochiometric mixture fraction in plane $\Delta_1$ (Fig. 5.4).
Figure 5.11: Mean dimensionless temperature field $\frac{T - T_{inlet}}{T_{adiab} - T_{inlet}}$ with an isoline of stochiometric mixture fraction in unwrapped plane $\Delta_2$ (Fig. 5.4).
Figure 5.12: Mean dimensionless temperature field \( \frac{T-T_{inlet}}{T_{adab}-T_{inlet}} \) in unwrapped plane \( \Delta_3 \) (Fig. 5.4).
5.3 Unsteady activity in the combustion chamber

Tracking the combustion noise sources requires to investigate the acoustic activity within the chamber as well as the generation of hot spots and vorticity waves responsible for indirect combustion noise.

5.3.1 Unsteady pressure within the combustor

The root-mean-squared (RMS) pressure fields are plotted in Fig. 5.13. In both cases, the main pressure fluctuations are located to the swirler and the jet interaction zones. In terms of fluctuating pressure levels, the low power simulation is quieter than the higher operating point as expected. However, these pressure fields do not provide information about the acoustic activity within the combustor. To do so, averaged periodograms of fluctuating pressure are computed at different locations (Fig. 5.5). Fig. 5.14 shows that these spectra exhibit two main narrow-band components centered around 600 Hz and 3000 Hz in the primary zone and at the exit of the combustion chamber. The intermediate sensors located to the plane 4 in Fig. 5.5 fail to capture the second narrow-band hump which can correspond to a pressure node of a longitudinal acoustic mode. Using coherence spectra between
unsteady heat release integrated over the LES domain and pressure sensors located to the exit plotted in Fig. 5.15, the acoustic activity is strongly coupled with the turbulent combustion process up to 4000Hz in both cases. At very low frequency, the coherence spectra are altered by hydrodynamic fluctuating pressure which is not correlated with the unsteady heat release.

To investigate the acoustic field within the combustor in terms of modal decomposition, the Helmholtz solver AVSP is used on the computational domain of the LES. The smallest dilution holes are filled and treated as walls to keep only the swirler and the primary dilution holes. The mean sound velocity field comes from the average of the LES. Previous work [Boudier, 2007, Silva, 2010, Guillaud, 2010] showed that the inlet and outlet boundary conditions can be treated as walls (ie the normal acoustic velocity is set to zero). These computations are detailed in appendix F. In the following, the computed acoustic fields obtained with AVSP correspond to the high power case. Indeed, for the low power case, as only the thermodynamic conditions differ, the cut-off frequencies are shifted but the modal structures are not modified. A first acoustic longitudinal mode is found at 530 Hz for the high power case and 490 Hz for the low power case where the pressure node is located in the casing just below the inlet injection as shown in Fig. 5.16. In the flame tube, this frequency corresponds to the narrow-band humps found on PSD of LES signals (Fig. 5.14). The second hump close to 3000 Hz found in LES signals corresponds to a second acoustic longitudinal mode found at 2875 Hz and 3055 Hz by AVSP for the low and the high power cases respectively. This mode is not recorded by the pressure probes located to plane 4 because this plane is close
Figure 5.15: Coherence spectra between unsteady heat release integrated over the LES domain and exit pressure signals (plane 5 in Fig. 5.5) - 344kW (—) and 904kW (—).

to a pressure node as seen in Fig. F.5(a). In terms of phase, the phase shift of cross-spectra between pressure fluctuations in the primary zone (plane 1) and at the exit of the flame tube (plane 5) is plotted in Fig. 5.18. For the first mode, the phase shift is close to 0π at 600 Hz which corresponds to the phase of the first longitudinal mode found by AVSP (Fig. F.4(a)). The phase-shift found close to 3000 Hz is equal to π similarly to the phase of the second acoustic mode computed with AVSP (Fig. F.6(a)).

By considering only a single sector of the annular combustion chamber, the acoustic field cannot exhibit azimuthal or radial components at low-frequency which means that the longitudinal mode amplitudes captured by the LES should not be as strong as in the experiment. Using the modal decomposition provided by the extraction of the pressure field at the exit of the combustion chamber with CONOCHAIN (plane Δ4 in Fig.5.4), the normalized modal energy carried by each mode is computed in both cases and plotted in Fig. 5.19. These spectra prove that the pressure field is mainly carried by the longitudinal modes up to 4000 Hz.

Finally, power spectral densities of the experimental pressure within the combustion chamber are compared with numerical pressure signals recorded close the upper dilution holes which corresponds to the experimental pressure probe location (Fig. 5.20): LES is able to capture the unsteady pressure in terms of levels
and patterns.

5.3.2 Generation of temperature fluctuations

The previous section suggests that LES performs a reasonable job in predicting pressure fluctuations in plane $\Delta_4$ (Fig. 5.4). These pressure fluctuations control direct noise. To capture indirect noise, the temperature fluctuations in plane 5 (Fig. 5.5) must also be predicted: these fluctuations at the exit of the combustion chamber will create the indirect combustion noise within the turbine stages. The mechanisms responsible for the generation of the “hot spots” within the combustion chamber are described in this section.

The root-mean-squared (RMS) temperature fields are plotted in Fig. 5.21 where two main zones of fluctuating temperature are visible. First, in the primary zone, temperature fluctuations correspond to the shape of the turbulent flame where turbulent structures interact with the flame front which convects fresh gases across the reacting zone. Second, the interaction of the dilution jet flows at the end of the primary zone generates the strongest temperature fluctuations in both cases as highlighted in Fig. 5.22. The dimensionless fluctuating temperature shows that the largest hot spots are generated in the second half of the flame tube.

To quantify these phenomena, averaged periodograms are computed using the
Figure 5.17: Dimensionless modulus (right) and normalised phase $\frac{\phi}{\pi}$ (left) of the second acoustic longitudinal mode computed with AVSP over the plane $\Delta_1$ in Fig. 5.4.

Figure 5.18: Phase-shift of cross-spectra between pressure probes in plane 1 and plane 5 (Fig. 5.5).

fluctuating temperature recorded on three planes described on the Fig. 5.5 with 90
Figure 5.19: Normalized modal power distribution of the pressure signal at the exit of the combustion chamber over the exit plane $\Delta_4$ (Fig. 5.4), (mode = 0 : ▼, mode = $-N_{\text{sectors}}$ : ▲, mode = $N_{\text{sectors}}$ : ●, higher modes : ♦).

Figure 5.20: Power spectral densities of pressure fluctuations (position A in Fig.5.5) - LES (- ● -) Experiment (—).
combustion chamber, a modal decomposition is performed on the temperature field to evaluate the energy carried by each mode in both cases. Contrary to the pressure field dominated by longitudinal component, the temperature field highlights a more complex structure. The planar mode carries only roughly 20% of the total signal energy and the azimuthal components are required to describe precisely the temperature fields. The energies carried by the first azimuthal modes are not equal because the azimuthal velocity of the mean flow at the exit of the combustion chamber.
Figure 5.23: Power spectral densities of unsteady temperature in flame tube - Plane 2 (■) Plane 3 (●) Plane 4 (+) in Fig. 5.5.

Figure 5.24: Normalized modal power distribution of the entropy fluctuations signal at the exit of the combustion chamber over the exit plane $\Delta_4$ (Fig. 5.4), (mode = 0 : ◀, mode = $-N_{\text{sectors}}$ : ▲, mode = $N_{\text{sectors}}$ : ●, higher modes : ♦).

combustor leads to a non-symmetric modal decomposition and highlight a slight peak close to 500 Hz in both cases.
5.4 Conclusions

LES of single-sector of the TEENI combustion chamber have been presented in this chapter for two experimental operating points, 344kW and 904kW. The mean flow predictions showed that the mean velocity field is only slightly modified when the engine power changes. The prediction of pressure losses and flow split within the combustion chamber was shown to be correct thanks to a dynamic modelling of the multi-perforated plates [Mendez and Eldredge, 2009] able to predict the injected mass flow rates as well as mimic the acoustic behaviour of these devices. The computed sound pressure levels in the flame tube are found to be similar to the experimental pressure fluctuations in both cases and exhibit a combination of acoustic longitudinal modes. Furthermore, the generated acoustic waves at low-frequency are highly-correlated with the unsteady heat release ie the turbulent combustion. The generation of entropy waves was also addressed and it was found that the mixing induced by the main dilution jet flows of the burnt gases is the first-order mechanism responsible for the largest temperature fluctuations convected at the exit of the combustion chamber. This is an interesting but not expected conclusion suggesting that indirect ”noise” combustion in this burner is actually not a ”combustion” noise but a ”mixing” noise between burnt gases (in the primary zone) and cold dilution jets. The role of dilution jets in these combustors is again highlighted as done already by Motheau [2013] who showed that these jets can create unstable combustion modes (in a different engine).
Chapter 6

Full-scale LES of the TEENI combustion chamber

Abstract  All combustion noise studies up to now had been limited to single sector LES either because the combustion itself was a single sector burner or because the computation of a full-engine was simply too expensive. This was a major limitation because the interaction between burners can (1) modify longitudinal modes because of cancellation effects between sectors and (2) create azimuthal modes. In this work the first full LES of a $N_{\text{sectors}}$ sectors TEENI engine was performed to make progress on these questions. This chapter presents this LES at the higher operating point performed in the TEENI project. The mean flow predictions are consistent with the single sector showing that the single-sector was sufficient to predict the mean flow features. Nevertheless, the unsteady activity becomes more complex. Azimuthal modes dominates the acoustic activity within the combustion chamber as well as the entropy field at the chamber exit. Finally, the fluctuating entropy generation is addressed where similar mechanisms identified in the chapter 5 are found. Contrary to the pressure field at the exit of the combustion chamber, the absence of spatial coherence of the entropy field impacts the modal decomposition used in the CONOCHAIN methodology (chapter 2). Particularly, the modal amplitudes depend on the number of simulated sectors and a filtering is proposed to remove this dependence for the planar mode.
6.1 Mesh description for full engine

The mesh used to build the full-annular LES domain is shown in Fig. 6.1 and is composed of 7.3 millions tetrahedral cells which leads to a final mesh composed of 110 millions of cells. The mesh properties are very similar in the swirler and close to the lips of injector to the single-sector mesh used in Chapter 5 to solve properly the flow and the flame in the primary zone. To improve the time-step used in this simulation, slight geometrical modifications are performed in the swirler to limit the smallest cell volume. To assess the validity of this mesh, preliminary simulations (not shown in this work) using this single-sector have been compared with the simulations presented in the chapter 5 in terms of mean flow features and unsteadiness: no difference has been found.

6.2 Mean flow features

The duration of the full-annular simulation is 43,2 ms with a time-step equal to 3,6 \times 10^{-8}s to get a maximum CFL number of the order of 0,7. The total CPU required for this LES was 800 000 hours on a IBM x3750 equipped with Intel Sandy Bridge 2,7GHz CPUs.

6.2.1 Mean flow description

The mean flow features within the full annular combustion chamber are found very similar to the single sector simulations presented in Chapter 5. Figure 6.2(b) shows
that the mean temperature field within the flame tube exhibits the same pattern than in the single sector (Fig. 5.10(b)). The distribution of the stoichiometric isoline of mixture fraction is also very close to the predicted one in the single sector simulation. Over the meridian plane $\Delta_2$ (Fig. 5.4), the footprint of the dilution holes is visible on the mean temperature field (Fig. 6.3) inducing lower temperature levels in respect of the injector axis and lateral lobes of burnt gases. The isoline of stoichiometric mixture fraction is found more compact than in the single sector simulation but this difference is caused by the mesh resolution. The mixture fraction computed in the burnt gases corresponds to low fresh gases mass fraction levels (close to 1%) and is thus very sensitive.

The mean velocity magnitude levels predicted for the full annular combustion chamber are also very close to the levels found in the single sector simulation (Figs. 5.9(b), 5.8(b)). The penetration depth of the dilution jet flows are well-predicted (Fig. 6.2(a)) which leads to similar reacting zones locations (Figs. 6.2(a), 6.5(b)). In the second half of the flame, the mean velocity magni-
Figure 6.3: Mean dimensionless temperature field $\frac{T - T_{inlet}}{T_{adiab} - T_{inlet}}$ over the meridian plane $\Delta_2$ (Fig. 5.4) with an isoline of the stochiometric mixture fraction.

tude pattern exhibits a slight azimuthal component cause by the multi-perforated plates 3 and 4 (Fig. 5.3). The slight differences between the single sector LES of section 5.2 and the full engine LES of the present section can be explained by the establishment of a mean azimuthal flow as observed by Wolf [2011]. This mean azimuthal flow is mainly driven by the lateral flow injected through multi-perforated plates in the flame tube. Fig. 6.4 shows the time evolution of the azimuthal velocity over the exit plane $\Delta_4$ in Fig. 5.4 for both single sector and full LES. A difference of 1.5 $m/s$ between the mean values appears which is similar to the azimuthal velocity value found by Wolf [2011] in his computations. Since all swirlers
Figure 6.4: Time-evolution of the azimuthal velocity (m/s) extracted from the single sector (•• and mean value - -) and the full-scale LES (■ and mean value —) over the exit plane \( \Delta_4 \) (Fig. 5.4).

turn in the convective time, performing a full LES allows a mean azimuthal flow to settle.
6.2.2 Outlet mean temperature

The mean temperature field at the exit of the combustion chamber is extracted over the planes used to apply the CONOCHAIN methodology. These fields are compared with the duplicated mean temperature field obtained in the single sector simulation (Fig. 6.6). The temperature lobes generated by the main dilution holes showed in Fig. 6.3 induce the temperature distribution at the exit. The central high-temperature zone found in the full annular combustion chamber corresponds to the predicted fields in the single sector computation. The last rows of dilution holes emphasizes the temperature stratification in both cases. By considering the
full-annular combustion chamber as $N_{\text{sectors}}$ sectors, the mean temperature profiles at the exit are plotted over the radial direction for different azimuthal positions and compared with the single sector predictions in Figs. 6.7 and 6.8. A weak variability of the mean temperature prediction sector-by-sector is found and these profiles are consistent with the single sector LES which means that a single sector is sufficient to predict the mean temperature at the exit of the combustion chamber.
Figure 6.7: Mean temperature profiles $T/T_{mean}$ at the exit of the combustion chamber over the exit plane $\Delta_4$ (Fig. 5.4) for different azimuthal positions for the single sector computation (■) and the full combustion chamber (—).
Figure 6.8: Mean temperature profiles $T/T_{mean}$ at the exit of the combustion chamber ver the exit plane $\Delta_4$ (Fig. 5.4) for different azimuthal positions for the single sector computation ($\blacksquare$) and the full combustion chamber ($\leftarrow$).
6.3 Unsteady field in the full annular chamber

Contrary to the single sector simulation, the full annular computation allows computing the complete unsteady fields especially in terms of pressure and temperature. Computation of combustion noise using the CONOCHAIN methodology is strongly dependent on the fluctuating primitive variable extraction at the outlet of the combustor.

6.3.1 Unsteady pressure within the chamber

The unsteady pressure activity within the combustion chamber can be firstly addressed by plotting the RMS pressure field over the meridian plane $\Delta_2$ (Fig. 5.4). In the flame tube, the structure of the pressure field corresponds to the first azimuthal mode which is not present in the casing as shown in Fig. 6.9. The full 360° LES is run at roughly 40 acoustic cycles for this first azimuthal mode. Using the Helmholtz solver AVSP (Appendix F), the first azimuthal acoustic mode is found at 655 Hz and exhibits identical structure compared with RMS pressure field (Fig. 6.9). This frequency shift was also found by Wolf [2011]. At the exit of the combustion chamber, this pattern is also visible as shown in Fig. 6.10 where pressure nodes are located at 0° and 180° and pressure anti-nodes are located at 75° and 270°. Using a row of 45 probes circumferentially located in the flame tube, the PSD of pressure fluctuations confirms the presence of a peak close to 780 Hz emphasized by the cross-spectrum amplitude between probes at 75° and 270° (Fig. 6.10). The phase-shift of this cross-spectrum is constant in the frequency range [400Hz, 1200Hz] which means that the pressure fluctuations do not exhibit propagating components. The modal distribution of the power contained in the pressure signal at the exit of the combustion chamber over the plane 5 (Fig. 5.4) at 780 Hz shows that the power levels carried by the two propagating clockwise and counter-clockwise first azimuthal modes are of the same order. Fig. 6.12(a) shows that two modes at 780 Hz contain most of the pressure fluctuation energy suggesting that this mode dominates the acoustic waves entering the turbine and are not dying during the simulation. This is a major problem for the single sector LES which totally ignores these azimuthal modes. Chapter 7 however will show that most of the direct noise is actually damped in the turbine so that the single sector LES may still be a valid approach. Finally, the experimental PSD of the pressure fluctuations at 904kW obtained in the chapter 1 is compared with the PSD of pressure fluctuations computed in the single sector (chapter 5) and the successive PSD of the pressure fluctuations over the $N_{\text{sector}}$ sectors (Fig. 6.12) in the full annular combustion chamber in the position A (Fig. 5.4). As the azimuthal acoustic mode dominates the numerical spectra in the full annular simulation, the numerical PSDs impacted by the peak at 780
Figure 6.9: Comparisons between the RMS pressure field extracted from the full-scale LES and the dimensionless acoustic pressure computed with AVSP corresponding to the first azimuthal mode (Appendix F) over the plane $\Delta_2$ (Fig. 5.4).

Hz are slightly different with the numerical PSD in the single sector and the experimental one (Fig. 6.13). Particularly, the broadband part of the experimental spectrum below 2000 Hz is well-predicted in the full annular simulation while the PSD levels dramatically drop above 2 kHz.
Figure 6.10: Radial integration of the RMS pressure field over the exit plane $\Delta_4$ (Fig. 5.4).
Figure 6.11: Power spectral densities of pressure fluctuations over the $N_{\text{sectors}}$ sectors (6.11(a)) and cross-spectrum between pressure probes within sectors 5 and 12.
Figure 6.12: Modal power distribution of the pressure signal at the exit of the combustion chamber over the exit plane $\Delta_4$ (Fig. 5.4), (mode = 0 : ▼, mode = -1 : ▲, mode = 1 : •, higher modes : ♦)

Figure 6.13: Power spectral densities of pressure fluctuations within the flame tube at the location A (Fig. 5.5) for single sector computation (■), full scale simulation (—) and experimental spectrum (—). The gray errorbars on the full LES spectra corresponds to the extrema of the $N_{sectors}$ PSDs circumferentially extracted at the location A.
6.3.2 Unsteady temperature in the combustion chamber

The RMS temperature field over the meridian plane $\Delta_2$ (Fig. 5.4) shows two main zones of fluctuating temperature generation close to the lips of the swirler where turbulent combustion takes place and just after the main dilution holes in the second half of the flame tube. As expected and shown in the single sector simulations (chapter 5), the mixing of the burnt gases induced by the dilution jet flows seems to be the main phenomenon responsible for hot spots generation (Fig. 6.14).

By dividing the full-annular domain into $N_{\text{sectors}}$ "single-sectors", the averaged PSD over planes 2, 3 and 4 are plotted in Figs. 6.16(a), 6.16(b) and 6.16(c) respectively. The gray errorbars correspond to the variability of the $N_{\text{sectors}}$ averaged PSDs for which the total averaged is the black curve. These PSDs are compared with the averaged PSDs obtained in the single sector simulation. In plane 2, the PSDs levels are dramatically lower than the periodograms in the single sector case while a good agreement is found between the averaged periodograms in planes 3 and 4.

The mixing process in the full annular combustion chamber leads to similar fluctuating temperature levels in the first part of the flame tube but is located more downstream than in the single sector case. The convection of the temperature spots from plane 4 to the exit (plane 5) of the combustion chamber is illustrated by the averaged periodograms in Fig. 6.16(d).

In the full annular simulation, the amplitudes of the fluctuating temperature sector-by-sector are close to the amplitudes computed in the single sector computation. Nevertheless, a significant variability between the sectors is found. Furthermore, these averaged periodograms allow characterizing the consistency between the simulations in terms of temperature fluctuations but do not correspond to the entropy planar mode at the exit of the chamber because of the loss of the phase in the averaged PSDs. The total power extracted from the entropy signal at the exit of the combustion chamber (Fig. 6.15(b)) exhibits the same broadband component centred around 600 Hz than in the averaged PSDs of the temperature fluctuation (Fig. 6.16(d)). However, the modal decomposition of the entropy field at the exit shows that the planar and the first azimuthal modes carried less than 5% of the total power contained in the entropy signal (Fig. 6.15). Similarly to the single sector case where the 8 first azimuthal modes were required to describe properly the entropy field, the modal decomposition is performed over the 100 first azimuthal modes which contained more than 95% of the entropy signal. Particularly, the entropy planar mode extracted from the full-annular simulation is lower than the one extracted from the single sector simulation.

To first order, this shows that:

- the temperature fluctuation in each sector are only weakly affected by the azimuthal acoustic mode identified in section 6.3.1. This differs from result
Figure 6.14: RMS temperature fields over the meridian plane $\Delta_2$ (Fig. 5.4).

Figure 6.15: Modal power distribution of the entropy signal at the exit of the combustion chamber over the exit plane $\Delta_4$ (Fig. 5.4), (mode $= 0 : \triangledown$, mode $= -1 : \blacktriangle$, mode $= 1 : \bullet$, the higher modes $: \blacklozenge$)

obtained for pressure which were dominated by the azimuthal mode,

- the fact that the fluctuations in each sector are the same order does not imply that the global fluctuations for the whole combustor are similar to the single sector results because the phase of fluctuating temperature for each sector is not the same: interference effects between sectors lead to smaller
Figure 6.16: Averaged power spectral densities of temperature fluctuations for the single sector computation (■) and the full scale simulation (—).

Indeed, for the \( N_{\text{sectors}} \) sectors system to act like a single sector chamber, the entropy signals extracted from the \( N_{\text{sectors}} \) sectors would have to be in-phase. This phase can be examined in the LES by extracting these entropy signals from the \( N_{\text{sectors}} \) “single-sectors” at the chamber outlet (plane 5 in Fig. 5.5) and computing cross-spectra of these signals between adjacent pair of sectors. Consider the entropy time signal \((s'/c_p)_i\) of the planar mode related to the \( i^{th} \) sector split into \( N \) time samples and \( S_i \) the Fourier transform of each sample, the averaged cross-
Figure 6.17: Phase-shifts \( \phi_{i,i+1} \) of cross-spectra \( C_{i,i+1} \) (Eq. 6.1) between the entropy planar modes extracted from adjacent sectors of the full-annular combustion chamber (Sectors 1 to 3: ▲ - ● - ■, sectors 4 to 6: ▲ - ● - ■, sectors 7 to 9: ▲ - ● - ■, sectors 10 to 12: ▲ - ● - ■).

The spectrum \( C_{i,i+1} \) between the sector \( i \) and the adjacent one \( i + 1 \) is

\[
C_{i,i+1} = \sum_{k=1}^{N} S_i k S_{i+1}^* k
\]

where * represents the complex conjugate of a complex number. The phase-shift \( \phi_{i,i+1} \) is the argument of the averaged cross-spectrum \( C_{i,i+1} \). In case of entropy planar waves in-phase sector-by-sector, the phases \( \phi_{i,i+1} \) of the cross-spectra have to be identical between adjacent sectors. Yet, the phase-shifts are displayed in Fig. 6.17 and are randomly spread in the frequency range \([0,4000 \text{ Hz}]\) which means that sectors do not talk to each other in terms of entropy fluctuations: the entropy planar mode extracted from the full-annular simulation is not equal to the ”single-sector” entropy planar mode magnitude. The geometrical dependence of the entropy field will impact the prediction of indirect combustion noise generated by the entropy spots through the turbine stage in a single-sector simulation. However, an appropriate filter can be built to remove the impact of the number of simulated sectors on the entropy planar mode as proposed in the next section which uses the fact that all sectors have random phases to build a statistically meaningful average of the \( N_{\text{sectors}} \) sectors.
6.3.3 Entropy planar mode filtering

The previous section has shown that the entropy fluctuations produced by the full combustor were not the same as the fluctuations produced by a single sector because the burners activities interfere and decrease the overall signal level. Moreover, the LES of the full combustor showed that there was no correlation between sectors: this suggest a method to construct the entropy signal carried by the planar mode of the full combustor from the LES of a single sector as described in this section. The random phase distribution of the entropy planar modes extracted from the $N_{\text{sectors}}$ sectors implies that their magnitudes $w_i^s$ can be related to the amplitude of the planar mode $w_s$ of the full-annular LES by introducing a random phase $\phi$, namely

$$w_s = \frac{1}{N_{\text{sectors}}} \sum_{i=1}^{N_{\text{sectors}}} w_i^s \exp(i\phi_i).$$

(6.2)

Eq. 6.2 can be written in a more general form by considering an entropy planar mode extracted from $k$ sectors,

$$w_s = \frac{k}{N_{\text{sectors}}} \sum_{i=1}^{\lfloor k \cdot N_{\text{sectors}} \rfloor + 1} w_i^s \exp(i\phi_i) \kappa_i,$$

(6.3)

where $\kappa_i$ is unitary except for $i = \lfloor k \cdot N_{\text{sectors}} \rfloor + 1$ for which $\kappa_i$ is equal to $\frac{k}{N_{\text{sectors}}}$. Consequently, by assuming that the modal amplitudes $w_i^s$ are identical, the filter magnitude $F_k$ is

$$F_k = \frac{k}{N_{\text{sectors}}} \left| \sum_{i=1}^{\lfloor k \cdot N_{\text{sectors}} \rfloor + 1} \exp(i\phi_i) \kappa_i \right|.$$  

(6.4)

The accuracy of the filter depends on a sufficient number of realisations to get the random-phase distribution. Actually, the equation 6.4 is averaged over 100 000 realizations to converge, namely

$$\hat{F}_k = \frac{1}{N_{\text{realizations}}} \left( \sum_{1}^{N_{\text{realizations}}} \frac{k}{N_{\text{sectors}}} \left| \sum_{1}^{\lfloor k \cdot N_{\text{sectors}} \rfloor + 1} \exp(i\phi_i) \kappa_i \right| \right).$$

(6.5)

The filter magnitude tends to 1 when the number of considered sectors corresponds to the full-annular combustion chamber ($k = N_{\text{sectors}}$) and is equal to 0.23 for a single-sector simulation ($k = 1$).

To assess the validity of this technique, a first test case consists of splitting the exit plane in the full-annular simulation to extract $N_{\text{sectors}}$ "single-sector" entropy planar mode. Result are displayed in Fig. 6.18 where the "single-sectors" modes amplitudes are shifted by a constant value from the full-annular mode magnitude.
(a) No filtering  
(b) Filtering

Figure 6.18: Power spectral densities of the planar entropy modes extracted from the "single-sectors" (−) and the full-annular combustion chamber (■).

(Fig. 6.18(a)). By applying the filter defined in the equation 6.4, the "single-sector" modes levels are in agreement with the full-annular one. A second test case is based on the splitting of the full geometry into several planes composed of different number of sectors from 2 to $N_{\text{sectors}}$. Thus, the different filter amplitudes which are functions of the number of considered sectors are used. Similarly to the first test cases, the different entropy plane modes amplitudes are correctly predicted by the filtering operation to reach the target spectrum as shown in Fig. 6.19.

(a) No filtering  
(b) Filtering

Figure 6.19: Power spectral densities of the planar entropy modes extracted from the different sum of sectors (−) and the full-annular combustion chamber (■).
6.4 Conclusions

This chapter has described the first LES of a full 360 degree annular combustor corresponding to the TEENI geometry presented in the previous chapter. This 360 LES was compared to the single sector LES of Chapter 5 in terms of mean flow field, acoustic activity and entropy generation. Three types of conclusions have been reached:

- Regarding the mean flow, limited differences are observed between single sector and full 360 LES, probably due to the establishment of a weak global swirling motion in the 360 LES.

- The acoustic field, on the other hand, is very different in both LES: the 360 LES is dominated by an azimuthal mode which cannot be captured by the single sector LES. This suggests that the direct noise produced by the real engine differs from the single sector LES. We will see in the next chapters that this may be only a marginal problem because direct noise is found to be small in this configuration anyway.

- For indirect noise, the full 360 LES shows that all sectors create entropy spots which are comparable to the single sector LES results but are uncorrelated so that important cancellation effects appear when averaged over the whole machine. In other words, the overall entropy fluctuations of the 360 machine are smaller than the predictions of the single sector LES. However, if the entropy fluctuation phases between sectors are fully uncorrelated, as suggested by the 360 LES, a simple filtering model can be used to deduce the full machine fluctuating entropy from a single sector LES.

To conclude, except for direct noise, a single sector LES seems to be a reasonable way to predict the waves leaving the combustion chamber before entering the turbine. This last phase is described in the next sections.
Part III

Combustion noise computation in the turbine stages and in the far-field with CONOCHAIN tool
Chapter 7

Combustion noise computation within the turbine stages of TEENI engine

Abstract  The previous chapters have described LES of the combustion chamber in which the waves leaving the combustion chamber were estimated. The next point to compute combustion noise is to evaluate how these waves are transmitted (direct noise) or transformed in the turbine stages (indirect noise). The aim of this chapter is the computation of direct and indirect combustion noise levels from the high-pressurized turbine to the turbine exit. Experimental data is available for pressure signals from the TEENI test cases. A first part is dedicated to the acoustic role of each component of the turbine stages. The high-pressurized turbine is identified to be a strong generator of indirect combustion noise. Using the actuator disk theory CHORUS and the unsteady fields provided by the LES presented in chapters 5 and 6, the acoustic intensities are compared with the experimental wall-pressure fluctuations shown in chapter 1. Reasonable agreement is found between predicted acoustic levels through the turbine stages and experimental results for single-sector LES while full-scale LES improves significantly noise predictions.
Section 7.1 presents first only the transfer functions of the turbine: these results are independent of the LES waves imposed at the turbine inlet. In section 7.2, wave amplitudes at the combustion chamber exit are extracted from the LES of chapters 5 and 6. Both single sector LES and full 360° LES are compared. Finally, section 7.3 combines the transfer functions of section 7.1 with the inlet forcing data of section 7.2 to compute the noise at the turbine outlet (position D) as well as within the turbine stages (positions B and C in Fig. 7.1).

7.1 Acoustic characterisation of the turbine stages

Before computing combustion levels within the turbine stages, the actuator disk theory CHORUS presented in the chapter 2 is used to determine the acoustic transfer functions through the turbine stages at each experimental location (Fig. 7.1):

- the high-pressurized turbine (position B),
- the power turbine (position C),
- the exhaust (position D).

Using the wave notation displayed in Fig. 7.2, the wave vector \( \mathbf{W} = (w_s, w_v, w^+, w^-) \) is computed using the inlet excitation \( \mathbf{W}_{cc} = (1, 1, 1, w^-) \) and \( \mathbf{W}_{p2} = (w_s, w_v, w^+, 0) \). The matrices \([\mathbf{M}]^{(a)}, [\mathbf{M}]^{(d)}\) (Eq. 2.38), \([\mathbf{B}]^{(a)}, [\mathbf{B}]^{(d)}\) (detailed in section 2.1.3.4) are obtained by using information on the mean flow in the three turbine stages of Fig. 7.2. The input data required for this exercise is:

- the local upstream and downstream Mach number of each blade row,
- the local upstream and downstream flow angle,
- the local upstream and downstream sound velocity,
- the rotation speed of the rotor stages,
- the local upstream and downstream heat capacity ratio \( \gamma \).

These informations were obtained from TURBOMECA and are not given here for confidentiality reasons.
High pressure turbine (B)  

Power turbine (C)  

Exhaust (D)  

Combustion chamber (A)  

Figure 7.1: Sketch of TEENI experimental set-up and location of internal pressure and temperature sensors.

Combustion chamber  

Turbine stages  

Figure 7.2: Sketch of TEENI turbine stages corresponding to Fig. 7.1 and wave notation.
7.1.1 Acoustic-to-acoustic transfer functions

The acoustic-to-acoustic transfer functions are the amplitudes of the downstream-propagating acoustic waves $w^+$ at different locations in Fig. 7.2 normalized by the downstream-propagating acoustic wave $(w^+)_c$ at the combustor exit. These transfer functions are computed here at low and high power cases for the planar mode (i.e., the mode index is 0) with the matrix system defined in Eq. 2.79 and displayed in Fig. 7.3. The frequency dependence of the acoustic coefficients is due to the resonances between the blade rows. First, the blade rows act like an acoustic masking which reduces the maximum levels of the transfer functions from the high-pressurized turbine to the exhaust. Finally, no reflected acoustic wave is computed after the second power turbine because of the non-reflecting boundary condition (Figs. 7.3(e) and 7.3(f)).

7.1.2 Vorticity-to-acoustic transfer functions

The vorticity-to-acoustic transfer functions $(w^+) / (w_v)_c$ computed with Eq. 2.79 and defined in Fig. 7.2 of the turbine are displayed in Fig. 7.4. In both cases, their amplitudes are lower than 0.005. That means the vorticity waves extracted from the LES are strongly damped through the turbine stages. However, additional vorticity can be generated through the turbine blades by the convection of entropy or acoustic waves because of the mean flow deviation and acceleration but this generation does not take into account these transfer functions. Furthermore, the actuator disk theory used in this work is not able to compute the noise generation induced by the blade-vortex interaction which is known to be out of the frequency range of interest.

7.1.3 Entropy-to-acoustic transfer functions

The entropy-to-acoustic transfer functions $(w^+) / (w_s)_c$ defined in Fig. 7.2 and computed with the matrix product of Eq. 2.79 are shown in Fig. 7.5. The attenuation of the planar entropy wave is taken into account. In the high-pressurized turbine, the first stator is choked in both cases which leads to similar transfer functions amplitudes. (Figs. 7.5(a) and 7.5(b)). As displayed in Figs. 7.5(c), 7.5(d), 7.5(e) and 7.5(f), the entropy-noise generation within the power section differs significantly between the operating points because the mean flow acceleration through the turbine stages is higher in the high power case. Thus, the increase in the engine power leads to a potential increase in indirect entropy noise generation within the power section.
7.1.4 Entropy wave distortion through the turbine

In each stage, the entropy attenuation filter described in chapter 2 with the equation 2.59 is applied to build the entropy-to-acoustic transfer functions presented in the previous section. These filter magnitudes are displayed in Fig. 7.6 in both cases from the high-pressurized turbine to the exhaust (positions B, C and D in Fig. 7.2). As expected, the convection of the planar entropy wave through successive blade rows increases the attenuation of the wave amplitude. Furthermore, the entropy attenuation is more important in the low power case. Indeed, the mean temperature within the turbine stages drops from the high power case to the low power case. So, the lower mean axial velocity leads to the increase in the time-delay of the entropy particles through the blade rows and an increase in the attenuation effect.
Figure 7.3: Analytical acoustic-to-acoustic transfer functions at different locations within the turbine stages (Fig. 7.2).
Position B in Fig. 7.2
\((w^+)_{hp} / (w_v)_{cc}:\) and \((w^-)_{hp} / (w_v)_{cc}:\)

(a) Low Power

(b) High Power

Position C in Fig. 7.2
\((w^+)_{p1} / (w_v)_{cc}:\) and \((w^-)_{p1} / (w_v)_{cc}:\)

(c) Low Power

(d) High Power

Position D in Fig. 7.2
\((w^+)_{p2} / (w_v)_{cc}:\) and \((w^-)_{p2} / (w_v)_{cc}:\)

(e) Low Power

(f) High Power

Figure 7.4: Analytical vorticity-to-acoustic transfer functions at different locations within the turbine stages (Fig. 7.2).
Position B in Fig. 7.2

\[ \frac{(w^+)_\text{hp}}{(w_s)_\text{cc}}: - \quad \text{and} \quad \frac{(w^-)_\text{hp}}{(w_s)_\text{cc}}: \]

Position C in Fig. 7.2

\[ \frac{(w^+)_p1}{(w_s)_\text{cc}}: - \quad \text{and} \quad \frac{(w^-)_p1}{(w_s)_\text{cc}}: \]

Position D in Fig. 7.2

\[ \frac{(w^+)_p2}{(w_s)_\text{cc}}: - \quad \text{and} \quad \frac{(w^-)_p2}{(w_s)_\text{cc}}: \]

Figure 7.5: Analytical entropy-to-acoustic transfer functions at different locations within the turbine stages (Fig. 7.2).
Figure 7.6: Entropy-to-entropy transfer functions $(w_s)_{hp} / (w_s)_{cc}$: –, $(w_s)_{p1} / (w_s)_{cc}$: •, and $(w_s)_{p2} / (w_s)_{cc}$: ■ (Fig. 7.2).
7.2 Extracted waves from the LES entering the turbine stages

In this section, comparisons between the acoustic and convective wave amplitudes extracted from the LES presented in chapters 5 and 6 at the exit of the combustion chamber are performed. To do so, primitive variables \((p'/\gamma p, w'/c, s'/c_p, \theta')\) are used to compute the two acoustic and convective waves:

- upstream-propagating acoustic wave \((w^-)_{cc}\) (Fig. 7.2),
- downstream-propagating acoustic wave \((w^+)_{cc}\) (Fig. 7.2),
- entropy wave \((w_s)_{cc}\) (Fig. 7.2),
- vorticity wave \((w_v)_{cc}\) (Fig. 7.2),

with the matrix product defined in chapter 2 Eq. 2.37. In the single-sector LES, the "first" azimuthal modes extracted from the LES domain corresponds to the azimuthal modes for which the mode index is equal to the sector number \(N_{sectors}\) or \(-N_{sectors}\). So, this mode is the \(N_{sectors}^{th}\) azimuthal mode.

7.2.1 Downstream-propagating acoustic waves

The downstream-propagating acoustic waves amplitudes are shown in Fig. 7.7 for the single-sector LES of the chapter 5 for the planar mode and the \(N_{sectors}^{th}\) azimuthal modes. These amplitudes confirm that the longitudinal acoustic mode is dominant in both cases with similar patterns and levels between the single-sector simulations. The \(N_{sectors}^{th}\) mode carries very little energy. Moreover, the engine power variation does not seem to impact significantly the acoustic activity within the combustion chamber.

It is interesting to compare the wave amplitudes at the chamber outlet for the single sector and 360 LES. For this regime (high power), a strong azimuthal acoustic mode appears in the 360 LES (Fig. 7.8) which is obviously impossible to capture in the single sector LES (Fig. 7.7(b)). This will modify the direct noise evaluations because this mode dominates the acoustic waves \((w^+)_{cc}\).

7.2.2 Vorticity waves

The vorticity extracted from the combustor engine corresponds to the a vorticity vector orthogonal to the meridian plane of the turbine stages. First, in both cases, the planar waves extracted from single-sector LES have the same order of magnitude of the \(N_{sectors}^{th}\) azimuthal modes as shown in Fig. 7.9 because the
Figure 7.7: Power spectral densities of downstream-propagating acoustic wave amplitude \((w^+)^{cc}\) in Fig. 7.2) extracted from single-sector LES of the chapter 5 (planar mode (-), \(\pm N_{\text{sectors}}\) azimuthal modes (\(\ast\))).

Figure 7.8: Power spectral densities of downstream-propagating acoustic wave amplitude \((w^+)^{cc}\) in Fig. 7.2) extracted from full-scale LES of the chapter 6 (planar mode (-), \(\pm 1^{\text{th}}\) azimuthal modes (\(\triangledown\)), \(\pm 2^{\text{th}}\) azimuthal modes (\(\ast\)) for the high power case to compare with the single-sector results of Fig. 7.7(b).
vorticity, similarly to the entropy, is a convective variable for which the planar mode is not sufficient to properly describe the signal. Moreover, the vorticity waves amplitudes are not strongly impacted by the regime variations. In the full-scale LES, the vorticity waves displayed in Fig. 7.10 are lower than in the single-sector LES of Fig. 7.9. Furthermore, the first mode observed on the vorticity spectra of Fig. 7.10 exhibits a peak at 780 Hz which corresponds to the cut-on frequency of the first acoustic azimuthal mode. Even if the velocity field related to the vorticity waves is supposed to be irrotational, the vorticity computation is affected by the acoustic velocity field: the azimuthal mode at 780 Hz induces vortices in the flow which are captured in the vorticity waves in Fig. 7.10.

7.2.2.1 Entropy Waves

Similarly to the vorticity waves, the entropy wave amplitudes are not significantly altered by the regime variations and the $N_{\text{sectors}}$ azimuthal modes carry significant signal power as displayed in Fig. 7.11 for the single sector LES for low and high power cases. In the full-scale LES of the high power case, the entropy waves magnitudes of the first 2 azimuthal modes (Fig. 7.12) are identical and do not exhibit a spatial dependence contrary to the vorticity waves (Fig. 7.10). Unlike vorticity, the first acoustic azimuthal mode does not modify the azimuthal distribution of the entropy field at the combustor engine.

This suggests that mixing in the azimuthal direction is strong and that the entropy waves retain only an axial component while most transverse fluctuations
Figure 7.10: Power spectral densities of vorticity wave amplitude ((w_v)_cc in Fig. 7.2) extracted from the full-scale LES of chapter 6 (planar mode (−), ±1^{th} azimuthal mode (▼), ±2^{th} azimuthal mode (●)).

Figure 7.11: Power spectral densities of entropy wave amplitude ((w_s)_cc in Fig. 7.2) extracted from single-sector LES of the chapter 5 (planar mode (−), ±N^{th}_{sectors} azimuthal mode (●)).

are dissipated by mixing.
Figure 7.12: Power spectral densities of entropy wave amplitude \((w_s)_c\) in Fig. 7.2) extracted from the full-scale LES of the high power case of the chapter 6 (planar mode (—), \(\pm 1^{st}\) azimuthal mode (▼), \(\pm 2^{nd}\) azimuthal mode (●)).

7.3 Noise predictions within the turbine stages

Experimental results are available in TEENI at different positions within the engine (Figs. 7.1 and 7.2) at the outlet of:

- the high-pressurized turbine (position B)
- the power turbine 1 (position C)
- the power turbine 2 (position D) corresponding to the turbine exit.

At these locations, pressure sensors provide wall-pressure fluctuations containing total noise signals. It is not possible to separate direct and indirect combustion noises in these experimental signals. Therefore, the following strategy is used to compare CHORUS results to the experiment:

- Section 7.3.1 evaluates the different components of combustion noise computed with unsteady fields of single-sector LES at two operating points (chapter 5) and compared with experimental results. Since CHORUS predicts only combustion noise, it is expected to give values lower than the experiments which measure all noise components. In these cases, only the longitudinal
convective and acoustic waves are considered (ie the mode index is 0). Indirect noise generation coming from entropy waves acceleration through the turbine stages is scaled to take into account entropy wave scattering through turbine stages described in section 2.1.3.4.

- The second section 7.3.2 shows predictions of combustion noise levels provided by the full-annular LES presented in chapter 6 by considering the first azimuthal modes. As a 360° LES does not correspond to industrial practices, the ability of a single sector to provide noise predictions in agreement with the full-scale results is discussed by taking into account the impact of dependence of geometry on the entropy wave amplitudes at the combustor exit (explained in section 6.3.3) in terms of combustion noise levels.

As a compact assumption is used in CONOCHAIN, the frequency limit of the compactness is computed with the ratio $\Omega$ between the axial blade chord of the first row of the turbine and the acoustic wavelength when $\Omega = 0, 1$. Results shown in the following sections are not valid above this frequency limit.

### 7.3.1 Noise predictions with a single-sector LES

#### 7.3.1.1 High-pressurized turbine

In the high-pressurized turbine (position B in Fig. 7.1), direct noise related to the downstream-propagating acoustic waves at the combustor exit $(w^+)_cc$ is lower than experimental pressure levels and the indirect noise predictions as shown in Fig 7.13. Note that the peaks observed on the experimental signals at the two pressure sensors of Fig. 7.13 (500, 1200, 1600, 2200 Hz) are assumed to be due to tonal noise created by the turbine blades and not by combustion noise. Our objective is not to match them but to capture the broadband activity below 1500 Hz.

Indirect noise computation uses both entropy and vorticity waves. Vorticity-generated noise induced by the longitudinal vorticity waves in the single-sector LES is negligible as shown in Fig. 7.14 where noise levels due to inlet vorticity waves $(w_v)_cc$ are much lower than experimental pressure fluctuations. Although significant wave amplitudes are found for the planar vorticity wave (Fig. 7.9), the vorticity-to-acoustic transfer functions (Fig. 7.4) are responsible for these low noise levels suggesting that the contribution of vorticity to total noise can neglected.

Consequently, the total combustion noise at position B in Fig. 7.1 contains both contributions of acoustic $(w^+)_cc$ and entropy waves $(w_s)_cc$ at the combustor exit. In practice, below 2000 Hz, the contribution of direct noise $(w^+)_cc$ is negligible so that combustion noise is dominated by indirect combustion noise in both cases (low
and high power). Moreover, the large experimental humps in PSD spectra at low-frequency (below 1200 Hz) are correctly predicted both in terms of magnitude and pattern and are related to indirect noise. The predicted indirect noise humps are centred around 500 Hz where the entropy plane mode induced by the combustion chamber topology contributes the most (Fig. 7.11) and the entropy-to-acoustic transfer functions (Fig. 7.5) also exhibit large values.

### 7.3.1.2 Power turbine 1

In the first power turbine (position C in Fig. 7.1), total noise spectra exhibit a broadband hump centred around 500 Hz which over-predict by 6 dB the experimental pressure fluctuations as displayed in Fig. 7.15 in both cases. Peaks visible on Fig. 7.15 at 400 and 700 Hz on experimental spectra are not created by combustion but by the rotating devices of the engine (shafts and rotating blade rows). We do not consider them in the present analysis. The broadband humps are related to high values of the transfer functions at this location (Figs. 7.5(c) and 7.5(d)). Similarly to the computed noise levels within the high-pressurized turbine, direct noise is lower than the entropy noise. Fig. 7.15 reveals another limitation of the single sector approach used in this section: from 100 to 700 Hz, indirect noise (■ is Fig. 7.15) is larger than the measured total noise (solid lines for the two pressure probes located at the position C in Fig. 7.1) suggesting that this indirect noise is over-estimated. We will come back to this issue when we will consider the full 360
Figure 7.14: Computed indirect noise corresponding to the planar vorticity mode (●) and experimental PSD of wall-pressure fluctuations (——) from the pair of probes in the high-pressurized turbine (position B in Fig. 7.1).

Figure 7.15: Computed combustion noise corresponding to the planar mode in the power turbine (position C in Fig. 7.1). Total predicted noise: ■, indirect noise: ●, direct noise: ▼ and experimental PSD of wall-pressure fluctuations (——) from the pair of probes.
7.3.1.3 Power turbine 2

In the turbine exhaust (position D in Fig. 7.1), direct noise levels are still lower than indirect noise predictions below 2000 Hz for the low power case (Fig. 7.16(a)) and 2500 Hz for the high power case (Fig. 7.16(b)). The total predicted combustion noise is very close to the experimental total noise for the low power case (Fig. 7.16(a)) but it is higher than experimental PSDs for the high power case as shown in Fig. 7.16(b). Longitudinal acoustic resonances also impact the total noise spectra in both cases.

![Figure 7.16: Computed combustion noise corresponding to the planar mode at the turbine exit (position D in Fig. 7.1). Total predicted noise: ■, indirect noise: •, direct noise: ▼ and experimental PSD of wall-pressure fluctuations (——) from a pair of probes.](image)

The CHORUS results using single-sector LES results as inlet waves show that indirect noise is the main acoustic source at low-frequency while direct noise is found to be low even if these simulations exhibit strong longitudinal acoustic modes within the combustion chamber. Furthermore, indirect noise is only composed of entropy noise because of low vorticity noise levels. However, as shown in chapter 6, longitudinal entropy waves at the combustor exit are still over-estimated in a single-sector suggesting that this may be the cause of the combustion noise overestimation obtained at all points for the high power case. Thus, the following section presents noise computation within the turbine stages in the high power case using waves extracted from the full-scale LES of chapter 6 including the azimuthal modes. Furthermore, a new formulation based on a filter applied to single-sector LES will be used and shown to be representative of a 360° LES in the scope of combustion noise.
7.3.2 Noise predictions using a full-scale LES and comparisons with a single-sector LES for the high power case

The noise computations of section 7.3.1 are repeated here but full 360 LES input waves \((w^+)_c\) and \((w_s)_c\) are used instead of single sector results. Our objective is to see if using the full-scale LES as inputs of CHORUS results avoid the over-prediction of combustion noise observed in Fig. 7.15 and Fig. 7.16.

In this section, azimuthal and longitudinal waves extracted at the combustor exit of the full 360° LES are used as inlet waves to perform noise computation with CHORUS. The higher azimuthal modes used to compute acoustic levels are selected with the azimuthal compact criterion described in section 2.1.5. Thus, only azimuthal modes for which mode index is higher than -2 and lower than 2 are considered in these noise computations (to respect \(L_y/\lambda_y < 0,1\) where \(L_y\) is the pitch length of the blade rows and \(\lambda_y\) the azimuthal wavelength). Furthermore, in this paragraph, indirect noise computations use azimuthal and longitudinal entropy modes corrected to take into account the entropy wave distortion through the blade rows with the filter built for the planar entropy wave (presented in chapter 2). Even if this filter is not dedicated to the distortion of the azimuthal convective waves through the blade vanes, the lack of numerical or experimental results available to properly model this phenomenon imposes to use this approach.

Similarly to single-sector case (section 7.3.1), predicted acoustic powers are compared with experimental pressure fluctuations from the high-pressurized turbine to the exhaust (positions B to D in Fig. 7.1) in Fig. 7.17. Total noise levels are lower than experimental spectra in the high-pressurized turbine (position B in Fig. 7.1) as displayed in Fig. 7.17(a). Peaks visible on total noise spectra correspond to the first and second azimuthal acoustic modes captured in the full 360° LES and dominate direct noise predictions. In the power turbine (position C in Fig. 7.1), predicted total noise matches the experimental spectra below 1000 Hz and drops above this frequency where direct and indirect noise contributions have the same order of magnitude (Fig. 7.17(b)). Noise predictions in the exhaust (position D in Fig. 7.1) are displayed in Fig. 7.17(c) and total noise levels are close to experimental spectra below 2000 Hz in which indirect noise contribution is dominant. As expected, direct noise amplitude reveals tonal peaks related to azimuthal acoustic modes at their cut-on frequencies.

Fig. 7.18 compares total noise predictions obtained with the full 360° LES, the single sector and the experimental spectra from the position B to D (Fig. 7.1) and shows that the 360° LES is a better setup to evaluate waves produced by the chamber than the single-sector setup waves used in Figs. 7.13 to 7.16.
(a) High-pressurized turbine (position B in Fig. 7.1)

(b) Power turbine (position C in Fig. 7.1)

(c) Exhaust (position D in Fig. 7.1)

Figure 7.17: Computed combustion noise using longitudinal and azimuthal modes extracted from a full 360° LES (up to mode index = 2) in different locations within the engine (Fig. 7.1). Total predicted noise: ■, indirect noise: •, direct noise: ▼ and experimental PSD of wall-pressure fluctuations (—) from the pair of probes in high power case.
(a) High-pressurized turbine (position B in Fig. 7.1)
(b) Power turbine (position C in Fig. 7.1)
(c) Exhaust (position D in Fig. 7.1)

Figure 7.18: Computed total combustion noise using full 360° LES (longitudinal and azimuthal modes (up to mode index = 2), ●) and single-sector LES (raw longitudinal mode, ■) in different locations within the engine (Fig. 7.1) in high-power case.
### 7.3.3 Comparisons between noise predictions using a full-scale LES and a filtered single-sector LES

Section 7.3.2 has shown that the full 360° LES gives much better results to feed the CHORUS chain than the single-sector LES. A 360° LES, however, is expensive. This section shows that an intermediate model can be considered where only a single-sector LES is used to measure waves but these waves are filtered to reconstruct the essential features captured by the 360° LES as proposed in section 6.3.3. The most essential property of the 360° LES is the phase scrambling of longitudinal entropy waves revealed by the LES in section 6.3.2, a characteristic which can be mimicked easily as shown here, using a single-sector LES and a entropy wave filtering.

In the single-sector computation, acoustic powers are computed with longitudinal waves in which entropy waves are corrected to take into account their over-estimation at the combustor exit as explained in section 6.3.3. The aim of this investigation is to evaluate the ability of a single-sector LES to provide noise levels in agreement with full 360° LES results presented in section 7.3.2 by using entropy wave filtering.

At different locations within the turbine stages (from position B to position D in Fig. 7.1), total noise levels are shown in Fig. 7.19 for the filtered single-sector and full 360° LES in high power case. Very good agreement is found between single-sector and full 360° LES noise predictions below 2000 Hz while some discrepancies appear above 2kHz related to the presence of first azimuthal modes in the full-scale simulation. Applying the entropy wave filtering on single-sector noise levels is crucial to remove over-estimation at low-frequency shown in total noise spectra (Fig. 7.16(b)) at the turbine exit (position D in Fig. 7.1). It also opens a simple path to predict the noise of the whole machine. Instead of measuring waves produced in the 360° machine, a single-sector LES can be used and then filtered to feed CHORUS.
(a) High-pressurized turbine (position B in Fig. 7.1)
(b) Power turbine (position C in Fig. 7.1)
(c) Exhaust (position D in Fig. 7.1)

Figure 7.19: Computed total combustion noise using full 360° LES (longitudinal and azimuthal modes (up to mode index = 2), ●) and filtered single-sector LES (longitudinal mode, ■) in different locations within the engine (Fig. 7.1) in high-power case.
7.4 Conclusions

This chapter was dedicated to comparisons of predicted combustion noise levels by CONOCHAIN with experimental TEENI results using three methods:

- CHORUS fed by waves \((w^+)_\infty\) measured in a single-sector LES,
- CHORUS fed by waves measured in a full 360° LES,
- CHORUS fed by a filtering operation (Eq. 6.5) on the waves measured in a single-sector LES.

Results confirm the dependence of the acoustic field everywhere in the turbine to the LES domain where the longitudinal downstream-propagating acoustic wave dominates in the single-sector while the full-scale LES simulation exhibits the first azimuthal acoustic mode.

Using the CHORUS approach presented in chapter 2, the single-sector LES of the chapter 5 were used to compute noise levels through the turbine stages in section 7.3.1. The first conclusion is that indirect noise is the main contributor to the overall radiated noise in which vorticity-generated noise is very-low. The predicted total acoustic powers (containing both direct and indirect noises) were compared with the experimental results shown in chapter 1 for which reasonable agreement was found from the high-pressurized turbine to the exhaust. At low-frequency, noise levels are however higher than the experimental spectra at the engine exit because of the over-estimation of the longitudinal entropy wave in a single-sector. The full-scale LES provides better results in CHORUS in terms of noise predictions in which the contribution of the azimuthal modes is found to be essential to correctly predict combustion noise.

Nevertheless, a single-sector LES can be sufficient to correctly predict combustion noise levels generated by a turboshaft engine by filtering the entropy waves produced by the single-sector LES, a simple solution which is attractive in terms of computational cost.
Chapter 8

Propagation of combustion-generated noise within the far-field

Abstract The previous chapters have presented noise computation within the chamber (chapters 5 and 6) and through the turbine stages (chapter 7). The final missing element is a pure aeroacoustic tool for far-field noise evaluation. The objective is to compute how noise at the turbine outlet \((w^+)^p_2\) propagates to the far-field since the far-field noise is actually the only relevant quantity for noise in practice. In this chapter, the acoustic far-field propagation is computed with AVSP-f, a Helmholtz solver in which the prediction of acoustic wave at the turbine outlet \((w^+)^p_2\) (Fig. 7.2) computed in the previous chapter is used. Using a modelled jet flow to take into account the mean temperature field, the main characteristics of the experimental acoustic far-field shown in chapter 1 are correctly predicted even if important assumptions are made. These promising results confirm the importance of combustion noise in the acoustic far-field of a turboshaft engine and demonstrate the capacity of the present tool (CONOCHAIN) to offer a simulation method based on first principles and not on correlations: all the tools used here (LES, CHORUS, AVSP-f) operate only by solving conservative equations. No experimental or ad hoc correlation is used at any point, contrarily to most previous methods developed for combustion noise.
In the previous chapter, combustion noise computations have provided acoustic waves at the trailing edge of the last turbine row of the engine (position D in Fig. 7.1). To propagate these acoustic waves from the exhaust to the far-field at 19.2 meters where experimental far-field microphones are located, the acoustic solver AVSP-f is used. This Helmholtz solver described in chapter 2 can compute acoustic propagation of the turbine exhaust through the burnt gas jet to the far-field. In this chapter, acoustic far-field of the two simulated operating points (low and high power cases) are computed by considering total noise levels of single-sector LES (chapter 7).

8.1 Computational methodology

8.1.1 Numerical domain

The TEENI engine is placed on a platform at three meters height as shown in Fig. 8.1. The far-field microphones are placed circumferentially at 19.2 meters away from the engine every 10 degrees. To limit the computational cost, the numerical domain is composed of an eighth of the exhaust nozzle and a sphere of

![Figure 8.1: Schematic view of the TEENI test-bench with far-field microphones location (●).](image-url)
radius 5 meters in which the ground is not taken into account as shown in Figs. 8.2 and 8.4. Thus, predicted acoustic pressures are extracted from the AVSP-f domain on shifted microphone locations (dotted lines in Fig. 8.3) and corrected with a simple scaling law presented in section 8.1.4 to be comparable with experimental results. Furthermore, acoustic reflections induced by the ground are taken into account by removing 6 dB to experimental far-field acoustic spectra. Note however a specificity of this setup. The point $\theta_{\text{TEENI}} = 180^\circ$ for example does not correspond to the jet axis but is shifted by an angle $\Delta \theta$ such that $\Delta \theta = 8.88^\circ$ (and $\theta_{\text{AVSPf}} = 171, 11^\circ$ consequently) into the AVSP-f domain over the plane $(\vec{xOy})$ in Fig. 8.3. For clarity, we will refer to probe position using these two angles $\theta_{\text{TEENI}}$ and $\theta_{\text{AVSPf}}$.

![Figure 8.2: Schematic view of the TEENI test bench and the computational domain AVSP-f with the location of experimental microphones (Fig. 8.1) projected over sphere around the engine (dotted lines).](image)

The AVSP-f mesh contains 6.2 millions of tetrahedral cells where the mesh cell size varies linearly with radius from 1 cm in the nozzle to 5 cm as displayed in Fig. 8.5. This mesh criterion is used to ensure wave propagation through the far-field up to 1500 Hz (corresponding to wavelength of 35 cm resolved on 6 points at
least). The quality of the AVSP-f setup for this far-field computation will be tested in section 8.2.1. Non-reflecting boundary conditions are applied on the atmosphere surfaces (surfaces 3 and 4 in Fig. 8.5) and the normal acoustic velocity is set to 0 on the periodic boundaries (surface 2 in Fig. 8.5) similarly to the nozzle walls (surface 1 in Fig. 8.5).

8.1.2 Wave injection

The acoustic forcing is performed through the turbine exit in the nozzle as shown in Fig. 8.4. Wave definition differs from CHORUS to AVSP-f. Indeed, for scaling reasons in AVSP-f, the CHORUS downstream-propagating acoustic wave at the turbine exit \((w^+)_p\) in Fig. 7.2 is not identical to the forcing downstream-propagating acoustic wave \(A^+\) used in AVSP-f but is equal to

\[
A^+ = 2 \left( w^+ \right)_p \gamma p
\]  (8.1)
Figure 8.4: Sketch of the AVSP-f domain and the boundaries.

- 1: Exit nozzle
- 2: Periodic walls
- 3: Atmosphere
- 4: Atmosphere

Figure 8.5: Mesh of the AVSP-f domain with a zoom of the mesh refinement in the nozzle.

Forcing boundary

5 meters
where $\gamma$ is the heat capacity ratio (equal to 1.33 in the burnt gases) and $p$ is the mean pressure at the turbine exit. Thus, the acoustic waves computed in the previous chapter 7 with single-sector LES in the two operating points are considered. The propagation is computed for a set of 20 discrete frequencies between 100 Hz and 2000 Hz. For each forcing frequency, the amplitude of the wave $(w^+)_{p2}$ is given by CHORUS and used as forcing signal for AVSP-f. Each frequency computation is independent. The truncation of the numerical domain limits the acoustic computation to the far-field propagation of the planar acoustic waves. Indeed, injecting azimuthal modes would require the full-scale computation of the exhaust region.

### 8.1.3 Mean temperature field of the exhaust jet

The mean temperature field through the jet and atmosphere is built with the model proposed in chapter 2 in which the jet flow is divided into three main zones: the potential core, a transient zone and the fully-developed zone as displayed in Fig. 8.6 in both cases. The temperature variations are supposed to be proportional to the mean velocity field computed with scaling in each region even if the mean velocity is set to zero in the AVSP-f computations. Figure 8.7 shows the mean sound

![Figure 8.6: Mean temperature field through the jet flow scaled with the exit temperature in the high power case.](image_url)
velocity field for the two power cases. As expected, the length of the potential core is lower in the low power case than in the high power case.

Figure 8.7: Mean sound velocity field through the jet flow scaled with the exit sound velocity in the high power case.

8.1.4 Scaling of the acoustic pressure in the far-field

As mentioned in chapter 1, the acoustic far-field is measured over a sphere of radius 19.2 meters around the engine. However, we chose to limit the computational domain to 5 meters in order to save CPU time but also because the propagation between 5 and 19.2 m can be supposed to be fully radial. Comparisons of the numerical noise levels and the experimental results requires to extrapolate the predicted acoustic pressure from 5 to 19.2 meters. A important assumption is thus made to perform these comparisons: the predicted pressure levels are extracted from the computational domain at 5 meters in the azimuth of the far-field probe location. Assuming a monopolar radiation of the acoustic waves up to 19.2 meters and the conservation of the acoustic power, the predicted pressure levels are scaled as follows:

\[ p'_{r_2} = p'_{r_1} \frac{r_1}{r_2}, \text{ where } r_1 = 5m \text{ and } r_2 = 19.2m \]  

(8.2)

where \( p'_{r_2} \) is the quantity compared to the experimental results in the rest of the chapter.
8.2 Predictions of the acoustic far-field

The longitudinal acoustic waves injected at the engine exit correspond to the sum of the direct and indirect (vorticity and entropy) combustion noises computed with the single-sector LES and the filtered entropy waves (section 6.3.3).

8.2.1 Numerical verification of far-field noise tool

In a first step, the high power case was forced acoustically with arbitrary amplitudes \((w^+)_{p2}\) to verify the numerical behavior of the AVSP-f solver used for acoustic propagation. The acoustic pressure fields are plotted in Fig. 8.8 at different frequencies (100 Hz, 500 Hz, 1000 Hz and 2000 Hz). As displayed in Fig. 8.8(a), for the low-frequency acoustic forcing at 100 Hz, the temperature gradient through the jet flow has a negligible impact. However, the directivity patterns of the acoustic pressure fields at 500 Hz and 1000 Hz (Figs. 8.8(b) and 8.8(c) respectively) show that the acoustic waves are mainly deflected by the hot jet close to 130°-140° with respect to the engine axis (as described in chapter 1, Fig. 1.8). The acoustic forcing at 2000 Hz corresponds to the upper limit which can be reached on this mesh as shown in Fig. 8.8(d). Indeed, the acoustic wavelength corresponding to this frequency forcing is close to 17 cm and is solved over 4 points in the AVSP mesh which it probably not sufficient to properly propagate the acoustic waves.

8.2.2 Comparison with experimental far-field spectra

After the numerical verification of the AVSP-f set-up performed in section 8.2.1, the configuration was forced using the downstream-propagating longitudinal acoustic waves \((w^+)_{p2}\) predicted by CHORUS for the two operating points (low and high powers). As described in chapter 7, the single-sector LES are used to evaluate \((w^+)_{p2}\) for both operating points. Raw and filtered results (to take into account over-prediction of indirect noise in a single-sector, see Eq. 6.5) are presented. Of course, using a full 360 LES would probably yield better results but this full LES was available only for the high power case. Moreover, the present set-up of AVSP-f is axi-symmetric (Fig. 8.5) and does not allow including azimuthal modes which are the first asset of 360° LES. Finally, the industrial practice today for LES is based on single-sector runs so that is logical to focus on this type of tools for the moment.

The comparisons between the predicted acoustic pressures related to combustion in the far-field with the experimental results are displayed in Figs. 8.11 and 8.12 for the low power case and Figs. 8.9 and 8.10 for the high power case.

On the one hand, patterns of the experimental spectra in both cases from 100° to 160° are correctly predicted by CONOCHAIN which corresponds to the main
Figure 8.8: Acoustic pressure fields at different frequencies in the high power case induced by an unitary acoustic forcing through the turbine exit.
lobe of maximum acoustic pressure visible in Fig. 8.8. The wide hump up to 500 Hz is related to combustion noise as suggested in the experimental analysis of the chapter 1 where the three-sensors technique revealed a significant broadband noise component centred around 500 Hz generated in the high-pressurized turbine (Fig. 1.21). Furthermore, region in the azimuth 90° can be considered as silent zones where the accuracy of both simulations is low as noise levels are quite reduced. From 170° and 180° highlight important discrepancies above 1000 Hz where temperature gradients induced by the hot jet strongly impact the acoustic refraction in these azimuthal directions. Consequently, these discrepancies are probably caused by the temperature field modelling proposed in section 8.1.3.

In the high power case, predicted acoustic spectra match with experimental acoustic levels at 130° and 140° (Figs 8.9(f) and 8.10(b) respectively) when the entropy filtering is used (section 6.3.3). Below 1000 Hz, a good agreement between filtered numerical and experimental results is also found from 100° to 160° (Figs. 8.9(c) to 8.10(d)) while experimental spectra are bounded by noise predictions computed with and without entropy filtering in the low power case (Figs. 8.11(c) to 8.12(d)).
Figure 8.9: Acoustic pressure from 90°/90° to 130°/129.4° (θ_{TEENI}/θ_{AVSPF} in Fig. 8.3) in the far-field for the high power case computed with CONOCHAIN (combustion noise with the entropy filtering (section 6.3.3): ■, combustion noise without the entropy filtering: ●) and experimental acoustic pressure (—).
(a) Angles $\theta_{TEEN}$ of microphones (Fig. 8.2).

(b) 140°/139.2°

(c) 150°/148.8°

(d) 160°/158.9°

(e) 170°/166.6°

(f) 180°/171.1°

Figure 8.10: Acoustic pressure from 140°/148.8° to 180°/171.1° ($\theta_{TEEN}/\theta_{AVSPF}$ in Fig. 8.3) in the far-field for the high power case computed with CONOCHAIN (combustion noise with the entropy filtering (section 6.3.3): ■, combustion noise without the entropy filtering: ●) and experimental acoustic pressure (—).
Figure 8.11: Acoustic pressure from $90^\circ/90^\circ$ to $130^\circ/129,4^\circ$ ($\theta_{TEEN1}/\theta_{AVSPf}$ in Fig. 8.3) in the far-field for the low power case computed with CONOCHAIN (combustion noise with the entropy filtering (section 6.3.3): ■, combustion noise without the entropy filtering: ●) and experimental acoustic pressure (—).
Figure 8.12: Acoustic pressure from 140°/148.8° to 180°/171.1° (θ_{TEENI}/θ_{AVSPF} in Fig. 8.3) in the far-field for the low power case computed with CONOCHAIN (combustion noise with the entropy filtering (section 6.3.3): ■, combustion noise without the entropy filtering: ●) and experimental acoustic pressure (—).
8.3 Conclusions

This chapter presented the computation of far-field combustion noise levels through a modelled jet flow with AVSP-f and the comparisons of the predicted spectra with experimental ones for the TEENI engine. The results for two regimes (low and high powers) are promising but multiple discrepancies remain. They can be explained by various approximations used up to now:

- the absence of mean velocity field,
- the modelling of mean temperature stratification induced by the jet flow,
- the absence of azimuthal modes,
- the scaling of acoustic pressure from a radius equal to 5 meters to reach the far-field microphones (19.2 meters).

Despite these assumptions, the spectral characteristics of the experimental PSDs are correctly computed with CONOCHAIN at low-frequency as well as broadband levels below 1500 Hz. It is important to note that the present exercise is the first attempt to build a predictive tool starting from high-fidelity LES in the chamber and finishes in the far-field. This is obviously a complicated task, especially since the experimental pressure measurements are not able to separate combustion noise from other sources, making precise comparisons difficult. A second difficulty is the addition of multiple models (LES, analytical tools for turbine propagation, for far-field propagation). Despite these limitations, the present results show that the whole tool works and capture noise correctly in the turbine as well as in the far-field. Further improvement will of course be need but the methodology is clearly the right path to follow.
General conclusion

Evaluating combustion noise within a modern aero-engine faces challenges among which the capture of the unsteadiness responsible for direct and indirect combustion noise, the identification of acoustic role of each engine component from the combustion chamber to the last turbine stage and an appropriate numerical strategy able to provide noise levels from the combustor to the far-field. These questions are addressed in this thesis.

This work is based on experimental results obtained during TEENI project in which a turbo-shaft engine is instrumented from the combustor to the far-field to identify low-frequency broadband noise sources. Using original signal processing techniques on experimental database, it was shown that direct noise is characterized by a narrow-band component centred around 200 Hz while a broadband noise generation is identified just after the high-pressurized turbine which should be attributed to indirect noise. In absence of relevant fluctuating temperature measurements, the fact that entropy noise corresponds to this noise generation is not established.

The TEENI database is also used to validate a numerical strategy called CONOCHAIN dedicated to the computation of combustion noise levels from the combustion chamber to the far-field. Tracking combustion noise generation requires to get a proper description of the flow at the combustor exit, to model the low-frequency acoustic behavior of the turbine and the far-field propagation. So, CONOCHAIN uses LES of combustion chamber able to provide high-fidelity predictions of the unsteadiness of the flow to extract convective and acoustic waves at the combustor exit. LES are then coupled with an actuator disk theory named CHORUS to mimic combustion noise generation and propagation through turbine stages and the acoustic solver AVSP-f used to propagate acoustic waves from the turbine exit to the far-field.

Using LES of single-sector of TEENI annular combustion chamber corresponding to two stabilized operating points, mechanisms responsible for combustion noise generation are highlighted and noise levels through turbine stages up to the engine exit are compared with experimental results. As expected, acoustic activity within the combustor is mainly driven by the turbulent combustion in
both cases which corresponds to direct noise generation. However, it was found that entropy spots are not the direct result of flame-turbulence interactions but rather the consequence of mixing between burnt and fresh gases through the flame tube. These observations are confirmed with a full-scale LES representative of the higher operating point. Contrary to single-sector LES in which acoustic activity is mainly carried by longitudinal acoustic mode, the full-scale LES exhibits a strong azimuthal acoustic mode. Furthermore, considering the full geometry does not only impact the acoustics but also the entropy waves. Indeed, it was found that evaluating entropy waves used to compute combustion noise at the combustor exit depends on the number of simulated sectors which leads to a over-estimation of entropy wave magnitude in single-sector by neglecting phase cancellation effects sector-by-sector between longitudinal entropy waves. Nevertheless, a filter is built to correct this effect and allow considering a single-sector to compute combustion noise.

By injecting unsteady fields provided by LES into the actuator disk theory CHORUS, direct and indirect combustion noise levels are evaluated through turbine stages and compared with experimental pressure fluctuations. Entropy noise is found to be the major contributor of combustion noise in both simulations and is responsible for a broadband hump up to 1500 Hz while vorticity noise can be neglected. Taking into account entropy waves scattering caused by their convection through turbine stages is a necessary assumption to compute indirect noise. Furthermore, the direct noise narrowband component found in the experiment is not captured by CONOCHAIN. Thanks to 360° LES, contributions of azimuthal modes are also investigated which add significantly acoustic powers and lead to a better estimation of combustion noise levels. Finally, far-field propagation is also addressed in a prospective way with the acoustic solver AVSP-f in which combustion-generated acoustic waves at the turbine exit are injected to be propagated through the exhaust hot jet to the far-field for single-sector computations where the main characteristics of acoustic far-field at low-frequency are reasonably predicted.

This work shows the first full numerical evaluation of far-field combustion noise based on blind LES coupled with CONOCHAIN tools for propagation through both turbine stages and far-field. The results for the two regimes (low and high powers) are promising but multiple discrepancies remain. They can be explained by various approximations up to now:

- In the LES, both simulations are performed without the first stator row. The presence of this device in the LES domain can modify the mean flow at the combustor exit by potential effects where the extraction of the unsteady fields to build waves entering the turbines is performed. Furthermore, considering only a single-sector has an impact on noise computation as previously
discussed.

- The actuator disk theory is also based on important assumptions. First, the turbine stages (and the flow consequently) are considered two-dimensional. The compact assumption used to model blade rows as thin interfaces is valid to propagate acoustic waves but can be discussed for convective waves. Particularly, entropy wave scattering through blade vanes which is a non-compact phenomenon has to be taken into account in the actuator disk theory with a simple filter proposed in this work. This filter should be extended and validated to be representative of different operating conditions encountered within the turbines. Moreover, the non-reflecting boundary condition at the turbine exit also impacts the acoustic transfer functions of the turbine.

- In the acoustic solver AVSP-f, the absence of mean velocity field is the major assumption which can alter acoustic propagation even if the turboshaft exit jet flow is low-Mach number. Moreover, the mean temperature field should be extracted from numerical simulations to provide accurate mean temperature gradients. Furthermore, scaling the acoustic pressure from the acoustic domain boundaries to the far-field by considering monopolar acoustic sources impacts far-field noise levels. Finally, the absence of azimuthal acoustic modes in the acoustic forcing at the turbine exit is also a significant bias introduced in these noise computations.

Considering the complexity of the overall task, the levels of agreement for such a blind test are reasonable. Moreover, the improvements required now on the CONOCHAIN tool will need further work. The complete tool feasibility is demonstrated even if certain elements should be developed.
Appendices
Appendix A

Analytical transfer function of experimental pressure probe in harsh conditions

An analytical model to mimic the acoustic response of the experimental pressure probe used in the TEENI experiment is presented. It is based on one-dimensional propagation in junction tubes [Rienstra and Hirschberg, 2011] and is similar to what has been used for remote microphone probes by Perennes and Roger [1998], Moreau and Roger [2005] and Chauvin et al. [2014].

A.1 Sketch of the sensor

The remote microphone is cooled by a controlled air flow injected from 3. The mass flow rate is set to ensure an efficient cooling and avoid noise generation. Along each pipe a x-coordinate with a positive direction outwards from 2 is defined.

A.2 Acoustic modeling

No mean flow is considered in the sensor because the outlet 3 is closed. Mean temperature and mean pressure are assumed to be equal to the inlet combustion chamber values. Acoustic perturbations are supposed one-dimensional and harmonic. Thus, acoustic pressure fluctuation $p'$ is written as

$$p'(x,t) = p^+ e^{i(\omega t-k^+ x)} + p^- e^{i(\omega t+k^- x)} \text{ with } k = \frac{\omega}{c_0} \quad (A.1)$$
and the associated acoustic velocity fluctuation $u'$ is

$$ u'(x, t) = \frac{1}{\rho_0 c} \left( p^+ e^{i(\omega t - k^+ x)} - p^- e^{i(\omega t + k^- x)} \right) $$  \hspace{1cm} (A.2) 

The analytical transfer function is computed between the acoustic pressure in 4 and the acoustic pressure in 1. Closed ends are considered in 3 and 4 to set infinite acoustic impedances. In 2, a compact T-junction gives pressure and velocity jump conditions using mass and pressure conservation. These conditions provide a set of six equations

$$ p'_I(0, t) = p'_{II} \hspace{1cm} (A.3) $$
$$ p'_I(0, t) = p'_{III} \hspace{1cm} (A.4) $$
$$ u'_I(0, t).S_I + u'_{II}(0, t).S_{II} + u'_{III}(0, t).S_{III} = 0 \hspace{1cm} (A.5) $$
$$ p'_I(l_I, t) = 1 \hspace{1cm} (A.6) $$
$$ p'_{II}(l_{II}, t) = Z_3 u'_{II}(l_{II}, t) \hspace{1cm} (A.7) $$
$$ p'_{III}(l_{III}, t) = Z_4 u'_{III}(l_{III}, t). \hspace{1cm} (A.8) $$

Using the plane wave formulation in Eqs. A.1 and A.2, the set of Eqs. (A.3-A.8)
is expressed only with downstream and upstream acoustic waves. So, the system can be written in matrix form:

\[ MX = B \]  

(A.9)

where

\[
X = \begin{pmatrix}
p_I^+ \\
p_I^- \\
p_{II}^+ \\
p_{II}^- \\
p_{III}^+ \\
p_{III}^-
\end{pmatrix}
\quad \text{and } B = \begin{pmatrix}
1 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\]  

(A.10)

and the matrix \( M \) is

\[
\begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & e^{i(-kL_{III})} (1 - \frac{Z_4}{\rho_0 c}) & e^{i(kL_{III})} (1 + \frac{Z_4}{\rho_0 c}) & 0 & 0 \\
1 & 1 & -1 & -1 & 0 & 0 \\
1 & 1 & 0 & 0 & -1 & -1 \\
S_I & -S_I & S_{II} & -S_{II} & S_{III} & -S_{III}
\end{pmatrix}
\]  

(A.11)

Solving the system of equations gives directly the acoustic pressure fluctuation recorded by the microphone. However, this modeling is not sufficient to explain the acoustic resonance visible in the PSD of pressure fluctuations in Figs. A.2(a) and A.2(b) because the effect of kinematic viscosity, responsible for the ability of the probe to absorb standing waves within the sensor, is not taken into account. To do this, a modified wave number \( k_m \)

\[
k_m = \frac{1}{4} (1 - i) \frac{L_p}{A} \sqrt{\frac{2\rho_0}{\omega}} k \left( 1 + (\gamma - 1) \frac{1}{\sqrt{Pr}} \right) + k
\]  

(A.12)

is used to introduce an imaginary part able to mimic damping effect of the kinematic viscosity, as proposed by [Rienstra and Hirschberg, 2011] to consider acoustic damping within boundary layers. Note that the modified wave number defined in (A.12) tends to \( k \) when the kinematic viscosity tends to zero.

Finally, several acoustic transfer functions are plotted in Fig. A.2 for pressure probe mounted in the combustion chamber (position \( A \) in Fig. 1.7) and in the power turbine (position \( C \) in Fig. 1.7). Note that the mean pressure is higher in the combustion chamber for both power settings. Thus, damping effect in the sensors decreases with increasing pressure. Finally, the multiple peaks correspond to the maxima observed in the PSD of pressure in the combustion chamber (Fig. A.2(a)), suggesting that the modulation observed on all experimental PSD is due to the transfer function of the pressure probes.
Figure A.2: PSD of pressure fluctuations in the combustion chamber and the power turbine (positions A and C in Fig. 1.7 respectively) (——) and analytical transfer functions (---).
Appendix B

Acoustic propagation and generation in a low-Mach number jet

This appendix deals with the contribution of jet in the overall radiated noise and presents a simple model showing that the directivity pattern can be qualitatively explained by the difference between the jet and the ambient air temperature. Directivity pattern due to the exit mean flow is characterized by a zone of silence in the jet axis. It was found that this effect is negligible for low-Mach number jet [Goldstein, 1976]. Moreover, the hot jet exiting the engine is composed of several successive zones where temperature decreases to mimic sound velocity gradients induced by shear-layer mixing as shown in Fig. B.1. To deal with acoustic refraction due to temperature gradients, a dimensionless factor can be introduced, equal to $\frac{\nabla c_0}{\omega}$ where $\nabla c_0$ is the mean sound velocity gradient across the shear layers and $\omega$ the pulsation. When this factor is smaller than 1, a geometrical approximation can be made to consider acoustic waves as acoustic rays. Consequently, acoustic refraction across the jet can be computed using a Snell’s law [Rienstra and Hirschberg, 2011]. This straightforward modelling is not a proper description of the aerodynamic field but its aim is to understand the impact of temperature gradients on acoustic directivity.

Introducing subscripts $u$ and $d$ to describe the upstream and the downstream of an interface $i$ respectively between two regions gives the following angle relations.

1. $\tan \theta_i = \frac{x_i}{R_0}$, \hspace{1cm} (B.1)

2. $\alpha_{iu} = \theta_i - \gamma_i$, \hspace{1cm} (B.2)

3. $\gamma_{i+1} = \theta_i - \alpha_{id}$. \hspace{1cm} (B.3)
Snell’s law is
\[
\sin \alpha_{id} = \sin \alpha_{iu} \frac{c_{id}}{c_{iu}}. 
\] (B.4)

Using iteratively this set of equations, acoustic ray deviation angle across the hot jet is computed as well as propagation paths. It is also possible to consider only one interface and use these equations to get the deviation. Considering the interface \(i\), the upstream incident ray path is equal to
\[
y = \tan \gamma_i x + y_0. 
\] (B.5)

The equation of the interface segment between the upstream zone \(i - 1\) and the zone \(i\) is
\[
y = -\frac{R_0}{x_i} + R_0 
\] (B.6)

So, the intersection point coordinates of this interface and an acoustic ray are \((x_i, y_i)\) where
\[
x_i = \frac{R_0 - y_0}{\tan \gamma_{i-1} + \frac{R_0}{x_i}}. 
\] (B.7)
\[ y_i = \frac{R_0}{x_i} \frac{R_0 - y_0}{\tan \gamma_{i-1} + \frac{R_0}{x_i}} + R_0. \] (B.8)

Considering incident acoustic rays collinear with respect to the engine axis, ray paths layers are computed as shown in Fig. B.2. Sound velocity varies from 560 m/s in the jet to 340 m/s in the far-field. By means of Snell’s law, the main computed deviation angles \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \) are close to 130° degrees which correspond to the experimental deviation angle visible in Fig. 1.11. However, this modelling is not frequency dependent while diffraction effect of the hot jet flow occurs above 500 Hz. Indeed, at low-frequency, temperature stratification region induced by the shear layers enclosing the hot jet is compact with regards to the acoustic wavelengths which weakly impacts the acoustic ray transmission and directivity pattern.

Figure B.2: Modelling of the exhaust jet and acoustic ray path diagram (- - -).
Appendix C

Acoustic wave propagation within annular and cylindrical ducts

This appendix presents the analytical computation of the cut-off frequencies of azimuthal and radial modes within cylindrical and annular ducts with an axial mean flow. Once the geometry presented, the analytical solution of the cylindrical D'Alembertian equation for the pressure fluctuation is written.

C.1 Geometry

A infinite annular duct is considered in this work which can be extended to a cylindrical case if \( R_1 \) tends to zero (Fig. C.1). A steady homogeneous axial mean flow passes through the pipe.

C.2 Acoustic pressure field within the duct

The D'Alembertian operator for the fluctuating acoustic pressure in the presence of mean flow but without source term is

\[
\left[ \frac{D^2}{Dt^2} - c_0 \nabla^2 \right] (p),
\]

where

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}
\]

and

\[
\frac{D^2}{Dt^2} = \frac{\partial^2}{\partial t^2} + 2u_0 \frac{\partial^2}{\partial x \partial t} + u_0^2 \frac{\partial^2}{\partial x^2}.
\]
As the problem is not bounded in x-direction, the acoustic pressure \( p \) is expressed in terms of time \( t \) and the three spatial coordinates \((x, r, \theta)\). In this problem, we consider harmonic fluctuations and the acoustic pressure becomes

\[
p(t, \theta, r) = \hat{p} \exp(i\omega t)R(r)\Theta(\theta)X(x).
\]

Separation of variables leads to a first equation for the function \( X \) which gives

\[
X(x) = \hat{X} e^{i(k_x x + \phi_x)},
\]

where \( k_x \) is the complex axial number and \( \phi_x \) is an unknown phase-shift. Combining the pressure function (C.4) and the \( X \)-function (C.5) into the propagation equation (C.1) gives

\[
-\omega^2 R\Theta - 2k_x \omega u_0 R\Theta(u_0 k_x)^2 R\Theta - c_0^2 \left( \Theta \frac{\partial^2 R}{\partial r^2} + \Theta \frac{\partial R}{r \frac{\partial r}{\partial r}} + \frac{R \partial^2 \Theta}{r^2 \frac{\partial \theta}{\partial \theta}} \right) = 0
\]

and, after some algebras, we have

\[
k^2 + 2k_x M_0 + (M_0 k_x)^2 + \left( -k_x^2 \frac{1}{R} \frac{\partial^2 R}{\partial r^2} \ + \frac{1}{Rr} \frac{\partial R}{\partial r} \ + \frac{1}{\Theta r^2} \frac{\partial^2 \Theta}{\partial \theta^2} \right) = 0.
\]
The last term of the left-hand side of the equation (C.7) is the single term depending on \( \theta \), and can be solved easily. First, the \( \Theta \) function is \( 2\pi \) periodic and \( \Theta(\theta) = \Theta(\theta + 2\pi) \). Thus, the \( \Theta \) is equal to

\[
\Theta(\theta) = \hat{\Theta}e^{i(m\theta+\phi_\theta)}
\]  

(C.8)

where \( m \) corresponds to the azimuthal mode number and \( \phi_\theta \) is a phase-shift. Collect terms into the equation (C.7):

\[
\frac{1}{R} \frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} - \frac{m^2}{r^2} + k^2 + 2k_z k M_0 + (M_0 k_z)^2 - k_x^2.
\]  

(C.9)

Noting

\[
K^2 = k^2 + 2k_z k M_0 + (M_0 k_z)^2 - k_x^2
\]  

(C.10)

we have

\[
\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} + R \left( K^2 - \frac{m^2}{r^2} \right) = 0
\]  

(C.11)

or

\[
\frac{\partial^2 R}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial R}{\partial \eta} + R \left( 1 - \frac{m^2}{\eta^2} \right) = 0
\]  

where \( \eta = Kr \).  

(C.12)

The generalized equation (C.12) is well-know and the solution \( R(\eta) \) is a linear combination of first and second order Bessel’s functions [Abramowitz and Stegun, 1964, Watson, 1995]:

\[
R(\eta) = \alpha J_n(\eta) + \beta Y_n(\eta), \alpha, \beta \in \mathbb{R}.
\]  

(C.13)

The integer \( n \) is related to the ”mode index” of the radial mode. To describe the acoustic field within the duct, inner and outer wall boundary conditions have to be specified. The normal velocity fluctuations are equal to zero. In terms of pressure, this condition yields:

\[
\left. \frac{\partial p}{\partial r} \right|_{r_2} = 0 \text{ and } \left. \frac{\partial p}{\partial r} \right|_{r_2} = 0.
\]  

(C.14)

For a cylindrical pipe, the second boundary condition is replaced with a symmetric condition. In other words, the second term of the equation (C.13) is removed because of the singularity of the Bessel function of the second kind \( Y_n \) at \( r = 0 \). Hereafter, only the annular duct is considered. Using the boundary conditions defined in (C.14), the system is:

\[
\begin{cases}
0 = \alpha J_n'(\eta_1) + \beta Y_n'(\eta_1) \\
0 = \alpha J_n'(\eta_2) + \beta Y_n'(\eta_2)
\end{cases}
\]  

where \( \eta_1 = Kr_1 \) and \( \eta_2 = Kr_2 \).  

(C.15)
Solving the system (C.15) is equivalent to find the Wronskian’s roots:

\[ J'_n(Kr_1)Y'_n(Kr_2) + J'_n(Kr_2)Y'_n((Kr_1) = 0. \tag{C.16} \]

Note that the equation (C.16) is equal to \( J'_n(Kr_2) = 0 \) for the cylindrical case. Using the roots \( K_{mn} \) of the Wronskian (C.16) in the equation (C.10), we can write the axial wave number \( k_x \) of an azimuthal and radial mode \((m, n)\), namely

\[ k_x^\pm = \frac{k M_0 \pm \sqrt{k^2 - (1 - M_0^2) K_{mn}^2}}{1 - M_0^2} \tag{C.17} \]

The subscripts \( \pm \) correspond to the upstream and downstream propagating modes within the duct, respectively. The second term of the left-hand side in the equation (C.17) allows computing the cut-off frequency \( f_{mn} \) of the corresponding mode. Indeed, the mode \( (m, n) \) is propagated through the duct for real axial wave number \( k_x \). Finally, the cut-off frequency \( f_{mn} \) is

\[ \frac{c_0 K_{mn} \sqrt{1 - M_0^2}}{2\pi}. \tag{C.18} \]
Appendix D

Geometrical method to build a nozzle from a blade vane

To conserve the curvilinear section variation across the blade vane, a technique is proposed to build an equivalent nozzle from the blade vane geometry. This geometrical method does not allow computing a section perpendicular to the mean streamlines of the flow. Without informations about mean flow in the stator, the computed section does not match with the normal section of the streamlines. As shown in Fig. D.1, a mediator cone is built between the outer and inner turbine walls which can be considered as cones. The mean angle $\theta_m$ and the abcissae of the edge $x_m$ are

$$\theta_m = \arctan \left( \frac{\tan \theta_i x_i \tan \theta_o x_o}{x_i + x_o} \right) \quad \text{and} \quad (D.1)$$

$$x_m = \frac{x_i + x_o}{2}. \quad (D.2)$$

A radial projection of a three-dimensional closed blade profile provides the ”footprint” of the blade on the mediator cone. Noting $(x, y, z)$ the three coordinates of a blade profile point and $(x_r, y_r, z_r)$ the projected point, the geometrical transformation leads to

$$x_r = x,$$

$$y_r = y \frac{R}{R_m},$$

$$z_r = z \frac{R}{R_m}, \quad (D.3)$$

where

$$R = \sqrt{y^2 + z^2},$$

$$R_m = |x - x_m| \tan \theta_m. \quad (D.4)$$
To compute the osculating circles, the projected blade profile is unwrapped to get a two-dimensional footprint. The 2D-polar coordinates of the unwrapped blade profile \((r_{2D}, \alpha)\) are expressed in terms of the 3D polar coordinates \((x_r, r, \beta)\) and the mediator cone parameter, namely

\[
\begin{align*}
  r_{2D} &= \sqrt{x_r^2 + r^2}, \\
  \alpha &= \sin \theta_m \beta.
\end{align*}
\]

The next step consists of the computation of the curvilinear line using the center of the osculating circles which are tangential to the blade profiles as highlighted in Fig. D.3. Once the circles are computed, segments corresponding to their diameter perpendicular to the curvilinear locus of the circle centres are projected over the three-dimensional median cone. The last step is a radial projection of these segments over the inner and outer cones to compute a normal area giving the equivalent nozzle shape. The LES geometry is thus modified just before the leading edge of the high-pressurized stator to set the equivalent nozzle.
Figure D.2: Sketch of the radial projection of a three-dimensional blade profile over the mediator cone.
Figure D.3: Osculating circles according to the curvilinear abscissae along their center points
Appendix E

Preliminary results about LES of a single sector of the TEENI combustion chamber

The aim of this appendix is to present preliminary simulations of a single sector of the TEENI combustion chamber at 904kW to investigate the effect of the modelling of the multi-perforated plates as well as the subgrid-scale turbulent models. To do so, a first simulation is performed where the mass flow rate injected through the multi-perforated plates is set by semi-empirical laws. Contrary to the productive simulations presented in chapter 5, there is no a dynamic coupling between the both sides of the plates and thus the acoustic behavior of these devices is missed. In the second section, two subgrid scale models (the dynamic Smagorinsky model and the filtering Smagorinsky model) are compared with the classical Smagorinsky model. The duration of the set of simulations is equal to 50 ms.

E.1 Impact of the multi-perforated plates on the acoustic activity

The simulation is based on the same mesh used in the productive simulations presented in chapter 5 and composed of 12,8 millions of tetrahedral cells. The boundary conditions and the numerical parameters are identical except for the multi-perforated plates.

E.1.1 Mean flow predictions

The mean dimensionless temperature and velocity magnitudes are plotted in Fig E.1 over the plane $\Delta_1$ (Fig. 5.4) and compared with the results of LES presented in
the chapter 5. The mean temperature field does not differ significantly when the

![Mean dimensionless temperature field](image1)

Figure E.1: Mean dimensionless temperature field \( \frac{T - T_{inlet}}{T_{adiab} - T_{inlet}} \) with an isoline of stochiometric mixture fraction in plane \( \Delta_1 \) (Fig. 5.4).

![Mean dimensionless velocity magnitude](image2)

Figure E.2: Mean dimensionless velocity magnitude \( \frac{u}{U_{inlet}} \) in plane \( \Delta_1 \) (Fig. 5.4).

multi-perforated plates modelling changes. The stochiometric mixture fraction presents the same shape while the mean dilution jet flows are slightly moved into
the primary zone as visible in Fig. E.2. To compare the flow topology within the swirler, a modified swirl number is used in which the pressure gradient is not taken into account, namely

\[ S_{\text{modified}} = \frac{\int_{R_{\text{in}}}^{R_{\text{out}}} \rho u_x u_\theta 2\pi r^2 dr}{\int_{R_{\text{in}}}^{R_{\text{out}}} \rho u_x^2 2\pi rdr}, \]  

(E.1)

where \( \rho \) is the density, \( u_x \) the mean axial velocity and \( u_\theta \) the mean azimuthal velocity. The modified swirl number is computed in each vane of the swirler as shown in Fig. E.3 and the results are plotted in Fig. E.4 where identical values are found. The mean flow is not altered by the multi-perforated plates modelling as confirmed by the mean axial velocity profiles shown in Fig. E.5 taken in the primary zone (Fig. E.3).

In terms of mean flow features, the dynamic coupling of multi-perforated plates does not seem to be crucial. Contrarily, it was found that the unsteady predictions such as the acoustic waves are altered by taking into account the acoustic behavior of these cooling devices.

### E.1.2 Unsteady features within the combustion chamber

First, the RMS values are compared over the plane \( \Delta_1 \) (Fig. 5.4) between the simulations (Fig. E.6) where no significant difference was found. By computing the PSD of fluctuating pressure (Fig. E.7(a)) at the exit of the chamber (plane \( \Delta_4 \) in Fig. 5.4), the spectral pattern of the hard simulation exhibits a tonal acoustic
activity at 580Hz which corresponds to the first acoustic longitudinal acoustic mode. This mode dominates the acoustic activity within the flame tube and leads to the absence of significant component at higher frequencies and especially at 3000 Hz. The PSDs of fluctuating temperature at the exit of the combustor (Fig. E.7(b)) are impacted by the acoustic activity where a peak is also found at 580Hz even if the broadband pattern is similar.

### E.2 Impact of the sub-grid scale models

Similarly to the analysis performed in the previous section, the effects of subgrid scale models are analysed in terms of mean flow predictions and unsteady-flow analysis. The different simulations are denoted by A, B and C for the Smagorinsky model, the dynamic model and the filtered model respectively.

#### E.2.1 Mean flow predictions

The mean temperature predictions do not show significant difference induced by the SGS models (Fig. E.8). However, the isoline of stoichiometric mixture fraction is thinner according to the swirler axis for the for the case C (Fig.E.8(c)). Contrary to the simulations A and C, the mean velocity magnitude field in the simulation C shows that the upper dilution jet flow does not penetrate within
Figure E.5: Mean dimensionless axial velocity profiles $u_x/U_{inlet}$ over different lines just after the lips of the swirler (Fig. E.3) according to the dimensionless radius - Coupled (−) and Uncoupled multi-perforated plates (−−−).

The primary zone but impacts vertically the lower dilution jet flow as shown in Fig. E.11. Using the modified swirl number defined in equation E.1, the swirl number are computed in the inner and outer vanes of the swirler in Fig. E.10. The swirl numbers computed close to the lips of the injector are identical while slight differences appear at the back of the vanes. This zone corresponds to the last part of the swirler between the back wall and the nose of the fuel injector where compact recirculation zones exhibit complex flow features which can lead to different swirl number estimation. The axial velocity profiles in the first plane
(a) Pressure fluctuations  
(b) Temperature fluctuations

Figure E.7: Power spectral densities of pressure and temperature fluctuations at the exit of the combustion chamber (plane 5 in Fig. 5.4) - Coupled (---) and Uncoupled multi-perforated plates (- - -).

(Fig. E.11(a)) show very good agreement between the simulations for the central part while the classical Smagorinsky model predicts the high velocity magnitude. Contrarily, the azimuthal velocity profiles are not altered by a change of the SGS model (Fig. E.12). The over-prediction of the axial velocity magnitude by the classical Smagorinsky model is responsible for the outer swirl number differences.
E.2.2 Unsteady features within the combustion chamber

Contrary to the mean fields, the RMS pressure fields differ according to the SGS model (Fig. E.13). The high RMS pressure levels are found when the dilution jet flows interact after the primary zone. The footprint of RMS pressure fields are related to the mean velocity fields within the dilution jet flows presented in Fig. E.9. Furthermore, the simulation C presents the higher RMS pressure levels within the flame tube and is very similar to the acoustic pressure field found for the first longitudinal mode of the combustion chamber (Fig. 5.16). For the simulations A and B, identical RMS pressure fields are found in terms of pattern but stronger levels are computed in the simulation A. The PSDs of pressure
fluctuations (Fig. E.14(a)) confirm the presence of the first longitudinal acoustic mode within the simulation $B$. Furthermore, the longitudinal mode close to 3000 Hz in the simulation $A$ is not present in the simulations $B$ and $C$. At the exit of combustion chamber, the PSD of temperature fluctuations (Fig. E.14(b)) for the simulation $B$ exhibits two peaks below 1000 Hz while the simulation $C$ predicts the lowest PSD levels. Particularly, above 1000 Hz, the simulations $B$ and $C$ under-estimate the temperature fluctuations contrary to the simulation $A$. 

Figure E.9: Mean dimensionless velocity magnitude $\frac{u}{U_{inlet}}$ in plane $\Delta_1$ (Fig. 5.4).
Figure E.10: Mean swirl numbers computed with the equation E.1 in the outer and the inner vane of the swirler (Fig.E.3) - Classical Smagorinsky (×) - Dynamic Smagorinsky (■) - Fitering Smagorinsky (●).
Figure E.11: Mean dimensionless axial velocity profiles $u_x/U_{inlet}$ over different lines just after the lips of the swirler (Fig. E.3) according to the dimensionless radius - Classical Smagorinsky ($\times$) - Dynamic Smagorinsky (■) - Fitlering Smagorinsky (●).
Figure E.12: Mean dimensionless azimuthal velocity profiles $u_\theta/U_{inlet}$ over the first line just after the lips of the swirler (Fig. E.3) according to the dimensionless radius - Classical Smagorinsky ($\times$) - Dynamic Smagorinsky (■) - Filtered Smagorinsky ($\bullet$).
Figure E.13: RMS pressure within the flame tube in plane $\Delta_1$ (Fig. 5.4).
Figure E.14: Power spectral densities of pressure and temperature fluctuations at the exit of the combustion chamber (plane 5 in Fig. 5.4) - Classical Smagorinsky (×) - Dynamic Smagorinsky (■) - Filtering Smagorinsky (●).
E.3 Conclusion

These numerical simulations of the single sector of the TEENI combustion chamber show that taking into account the acoustic behaviour of the multi-perforated plates is required to exhibit more complex acoustic activity within the combustion chamber. The classical Smagorinsky sub-grid scale model is found to be sufficient to perform simulations used in this work in the scope of combustion noise.
Appendix F

Acoustic analysis of the TEENI combustion chamber with AVSP

Tracking of acoustic modes within the full-scale combustion chamber of the TEENI experiment is performed in this appendix. Using the Helmholtz solver AVSP, the sound velocity field is provided by the mean predictions of the single-sector LES presented in the chapter 5 at 904kW. As the acoustic solver uses a no-Mach number assumption, the nozzle is cut at the half length of the convergent part. A first section deals with the computation of a single sector to highlight the impact of the secondary dilution holes of the bend of flame tube. The second section presents the full-scale simulation to list the acoustic modes which can be present in the LES simulations.

F.1 Acoustic analysis of a single sector

F.1.1 Numerical parameters

F.1.1.1 Mesh

The meshes are composed of 345000 tetrahedral cells which is sufficient to compute the acoustic modes in the frequency range of interest. As shown in Fig F.1, the swirler is conserved as well as the main characteristics of the geometry. In the first case, the dilution holes located to the bend of the flame tube are taken into account while these holes are filled in the second mesh. Previous simulations not presented here revealed that the presence or the absence of the swirler do not significantly impact the computation of acoustic modes of the combustion chamber.
F.1.1.2 Boundary conditions

The boundary conditions applied on the walls and on the periodic boundaries correspond to a acoustic normal velocity set to zero. It was found that the inlet condition has a minor impact in the acoustic mode computation [Silva, 2010] and this condition is treated as a wall. The outlet boundary condition is also considered as a wall to highlight the impact of the dilution holes.
F.1.1.3 Sound velocity field

The acoustic solver AVSP takes into account sound velocity gradients within the combustor mainly induced by the combustion. To be consistent with the LES, the sound velocity field is extracted from the mean LES fields of the high power simulation (904kW). As shown in Fig F.2, the sound velocity exhibits consequently a pattern identical to the mean temperature field.

F.1.2 Impact of secondary dilution holes

The computation of acoustic modes within a single sector is performed on similar meshes where boundary conditions and mean sound velocity fields are identical. The cut-off frequency of the longitudinal acoustic modes are summarized in the Table F.1. The computed cut-off frequencies are strongly impacted by the presence of secondary dilution holes which can be illustrated by the magnitude and phase of the acoustic pressure of the first and seventh longitudinal mode in both cases (Figs. F.3, F.4, F.6 and F.5). In the casing, the dilution holes induce a shift of the pressure node to be in phase with the pressure node of the flame tube. Filling these dilution holes breaks the coupling between the cavities and allows the shifting of the pressure node and the cut-off frequency drop.

\[
\begin{array}{|c|c|c|}
\hline
\text{Dilution holes} & \text{Yes} & \text{No} \\
\hline
1-L & 897 & 523 \\
2-L & 1672 & 1119 \\
3-L & 1817 & 1542 \\
4-L & 2357 & 1983 \\
5-L & 2797 & 2284 \\
6-L & 2891 & 2683 \\
7-L & 3423 & 3247 \\
\hline
\end{array}
\]

Table F.1: Cut-on frequencies of the acoustic longitudinal modes (Hertz) according to the presence of secondary dilution holes.

F.2 Acoustic analysis of the full-scale combustor

The acoustic computation of the full-scale LES is based on a single-sector in which the secondary holes are considered and the injector is removed. The full-scale mesh is construct by duplicating the single-sector mesh. Using the mean sound velocity field used in the single-sector simulations, the cut-on frequencies of the acoustic
Figure F.2: Mean dimensionless sound velocity field $\frac{c-c_{\min}}{c_{\max}-c_{\min}}$ over plane $\Delta_1$ (Fig. 5.4).

modes are computed for which $A$ corresponds to the azimuthal modes, $L$ is related to longitudinal ones and $R$ to the radial modes. As shown in Table F.2, the full-
Figure F.3: Dimensionless modulus of acoustic pressure field for the first longitudinal mode 1-L (Table F.1) in the combustion chamber with and without secondary dilution holes.

Table F.2: Cut-on frequencies of the acoustic modes (Hertz) within the full-scale combustion chamber.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-L</td>
<td>523</td>
</tr>
<tr>
<td>1-A</td>
<td>655</td>
</tr>
<tr>
<td>1-L 1-A</td>
<td>1012</td>
</tr>
<tr>
<td>2-L</td>
<td>1119</td>
</tr>
<tr>
<td>1-L 2-A</td>
<td>1137</td>
</tr>
<tr>
<td>1-L 1-A 1-R</td>
<td>1349</td>
</tr>
<tr>
<td>3-A 1-R</td>
<td>1511</td>
</tr>
<tr>
<td>2-L 2-A 1-R</td>
<td>1541</td>
</tr>
<tr>
<td>3-L 7-A</td>
<td>1542</td>
</tr>
</tbody>
</table>

The scale combustor exhibits longitudinal modes as well as azimuthal and radial modes contrary to the single-sector in which only longitudinal modes are present at low-frequencies. The acoustic modes computation is limited to the first 10 azimuthal modes because the acoustic activity within the full-scale LES is dominated by the first azimuthal mode. However, the cut-on frequency computed with AVSP for
Figure F.4: Normalised phase $\frac{\Phi}{\pi}$ of acoustic pressure field for the first longitudinal mode 7-L (Table F.1) in the combustion chamber with and without secondary dilution holes.

This mode (655 Hz) is lower than the frequency found in the LES. This frequency shift was also found by Wolf [2011] for this burner at a different operating point.
Figure F.5: Dimensionless modulus of acoustic pressure field for the seventh longitudinal mode 1-L (Table F.1) in the combustion chamber with and without secondary dilution holes.
Figure F.6: Normalised phase $\frac{2\pi}{n}$ of acoustic pressure field for the seventh longitudinal mode 7-L (Table F.1) in the combustion chamber with and without secondary dilution holes.
Bibliography

M. Abramowitz and I. Stegun. *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, volume 55. Dover publications, 1964. URL http://books.google.com/books?hl=en&amp;lr=amp;id=MtU8uP7XMvoC&amp;oi=fnd&amp;pg=PR5&amp;dq=Handbook+of+Mathematical+Functions&amp;ots=-DQLSqO8Gf&amp;sig=C3rG1kPcL5pju_C9eN-vt0F83Yk.


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the propagation of waves through turbine blades. In 3rd Colloquium INCA, Toulouse (France), November 17-18 2011.


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Publications

