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Multiple-Frequency Phase-Lagged Unsteady Simulations of Experimental Axial Compressor

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This paper presents an evaluation of the multiple-frequency phase-lagged approach, which enables the performance of unsteady Reynolds-averaged Navier–Stokes simulations on multistage turbomachinery configurations using a time-marching method. The major advantage of this approach is to reduce the computational domain to one single blade passage per row. The first part of the paper presents the method and discusses the associated assumptions and limitations. The method is then evaluated on the axial compressor configuration “Compresseur de Recherche pour l’Etude des effets Aérodynamiques et TEchnologiques” investigated experimentally at Laboratory of Fluid Mechanics and Acoustics. The computational fluid dynamics results are analyzed and compared both with experimental data and with a reference multipassage computation based on a sliding mesh approach. These comparisons enable the highlighting of the interests of this approach but also the underlining of its limits. The multiple-frequency phase-lagged approach enables the simulation of unsteady effects on a multistage turbomachinery and access to unsteady information that would not be available with a mixing-plane approach. However, if the method is capable of capturing unsteady effects linked to the adjacent upstream and downstream blade rows passing frequency, it fails modeling clocking effects, i.e., the relative influence between rows N and N + 2.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n, b_n, C_n$</td>
<td>Fourier coefficients</td>
</tr>
<tr>
<td>$f_m$</td>
<td>spinning-mode frequency, Hz</td>
</tr>
<tr>
<td>$h/H$</td>
<td>relative span height</td>
</tr>
<tr>
<td>$m$</td>
<td>circumferential wavelength</td>
</tr>
<tr>
<td>$N_p$</td>
<td>blade number</td>
</tr>
<tr>
<td>$N_{harm}$</td>
<td>Fourier series harmonic number</td>
</tr>
<tr>
<td>$N_p$</td>
<td>number of perturbations</td>
</tr>
<tr>
<td>$P_1$</td>
<td>total pressure, $Pa$</td>
</tr>
<tr>
<td>$P_1'$</td>
<td>total pressure fluctuation, $Pa$</td>
</tr>
<tr>
<td>$Q$</td>
<td>mass flow, $kg \cdot s^{-1}$</td>
</tr>
<tr>
<td>$R$</td>
<td>rotor</td>
</tr>
<tr>
<td>$S$</td>
<td>stator</td>
</tr>
<tr>
<td>$T$</td>
<td>$1/16$th of rotation period</td>
</tr>
<tr>
<td>$T_t$</td>
<td>total temperature, K</td>
</tr>
<tr>
<td>$x, r, \theta$</td>
<td>axial, radial and azimuthal coordinates, $m, m, \text{and rad, respectively}$</td>
</tr>
<tr>
<td>$\phi, \psi, \eta$</td>
<td>normalized mass flow rate, total pressure ratio, and isentropic efficiency</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>spinning-mode rotation speed, rad $\cdot s^{-1}$</td>
</tr>
</tbody>
</table>

### Subscripts

| Adim | nondimension value |

rot = rotor
stat = stator

### I. Introduction

The relative motion between adjacent rotor and stator blade rows encountered in turbomachinery configurations gives rise to a wide range of unsteady flow mechanisms such as wake interactions [1], potential effects [2], hot streak migrations [3], shock wave propagations [4], or unsteady transitional flows [5]. All these phenomena, which can have a crucial impact on the performance of gas turbines, cannot be captured accurately with a steady mixing-plane approach [6] since the averaging treatment at the rotor/stator interface filters all unsteady effects. It is therefore important for aeroengine designers to take into account these unsteady effects in the design process, at a reasonable cost. Yet the computational cost of a time-accurate full-annulus computation remains very high, despite the increase of computer resources and the availability of parallel computing. Indeed, a direct unsteady Reynolds-averaged Navier–Stokes (URANS) calculation in a three-dimensional multistage whole-annulus configuration requires by 3 orders of magnitude more computing time compared to a steady isolated blade row mixing-plane simulation [7]. It is therefore important to have access to numerical methods that reduce the computational domain (ideally one blade passage per row) and are at the same time efficient enough to simulate accurately the main unsteady effects.

To alleviate the problem, the phase-lagged method is commonly used for unsteady rotor/stator simulations to reduce computing resources without modifying the blade counts [8,9]. This method, limited to flow configurations including a single periodic perturbation (in the general case, one single-stage configuration), enables the computation of the periodic flow around one blade per row, using appropriate phase-shifted boundary conditions at the pitchwise boundaries. A detailed comparison of single-phase phase-lagged computations with a multiple-phase periodic domain computation was performed by Van Zante et al. [10] on a 2.5 axial compressor stage configuration. Based on the work of He [11], Li and He [12–14],
Neubauer [15], and Gerolymos [16], a generalization of the phase-lagged method to multiple stage configurations (with flows that, in the general case, include more than one periodic perturbation) has been implemented in a multiblock structured computational fluid dynamics (CFD) code [17].

To the authors’ knowledge, the multiple-frequency phase-lagged method developed by the previously mentioned authors has not been applied up to now to a practical three-dimensional (3D) URANS multistage turbomachinery environment of more than three rows, with arbitrary blade counts. Indeed, He [11] had initially tested this approach in an Euler solver on an oscillating flat plate cascade subject to inlet and outlet distortion, before applying it to a vibrating isolated fan with inlet distortion [12]. The method was then tested on a 1.5 transonic compressor configuration [13] on a quasi-3D section and successfully applied to blade vibration problems in a compressor stage configuration [14]. The multiple-frequency phase-lagged approach developed by Neubauer [15] and Gerolymos [16] was also tested on a 1.5 transonic compressor but in three dimensions. Moreover, few comparisons of the multiple-frequency phase-lagged method with a reference computation (for example, a full-annulus or multiple-passage computation) can be found in the literature. The present work attempts to contribute to the previously mentioned work by evaluating this multiple-frequency approach (MFA) on a 3.5 stage (seven rows) 3D compressor configuration.

Finally, it worth mentioning that frequency-domain approaches, developed and evaluated in the last few years [18–21], are very interesting alternatives to time-integration methods for unsteady blade row simulation. By frequency-domain approaches, one designates methods in which the problem of solving a set of unsteady flow equations is transformed to a problem in which one solves a set of time-independent steady flow equations. Following a Fourier modelling principle, these methods consist of solving steady flow equations for time mean flow and time harmonics. Nevertheless, if promising results can be obtained in terms of a CPU, recent validations [22,23] also tend to highlight the limitations of such approaches in terms of numerical stability, memory cost, or harmonic number limitation. Since these limitations have not been observed with the MFA (which is based on a time-integrations solver and therefore more expensive in terms of CPU time than frequency-domain methods), it deserves attention.

The first part of the paper describes the multiple-frequency phase lagged method, which has been implemented in the CFD code elsA. In particular, the limitations of this approach are discussed. The evaluation of this approach performed on the experimental compressor Compresseur de Recherche pour l’Etude des effets Aérodynamiques et Technologiques (CREATE) is then presented. The paper analyzes the results obtained by the MFA calculations by comparison with both experimental data from LMFA and with a reference multipassage computation.

II. Description of Multiple-Frequency Phase-Lagged Approach

A. Classical Phase-Lagged Approach

To introduce the multiple-frequency phase-lagged approach, this subsection very briefly reviews the principle of the classical phase-lagged approach used for single-stage configuration. The blade-to-blade phase shift periodicity, also called chorochronic periodicity, has been described by many authors, and more details can be found in [8,9].

Figure 1 represents a blade-to-blade view of a rotor/stator configuration. Consider two points A and B having the same axial and radial position, separated by one stator pitch and located at the upper and lower boundaries of the stator domain. With the hypothesis that the sources of unsteadiness are only due to the rotation of the wheel, the flowfield in each blade row can be assumed to be time periodic (with a different period in each row). For example, at point A, any aerodynamic variable represented in the cylindrical frame of reference is supposed to be a periodic function,

\[ g(x, r, \theta_A, t) = F_{x,r,\theta}(t) = F_{x,r,\theta}(t + f_{\text{stat}}) \]

with \( f_{\text{stat}} = N_{\text{stat}} \Omega / 2\pi \) its associated frequency. Moreover, the previous hypothesis enables the linking of the flowfield at points A and B, which experience the same values but with a phase lag in time,

\[ g(x, r, \theta_B, t) = g(x, r, \theta_B, t + \phi) \]

with \( \phi = (\theta_B - \theta_A) / \Omega \) being the phase lag. Analogous relationships can be written in the rotor frame of reference. The previous formulas enable the reduction of the computational domain to a single blade passage for each row, using specific boundary conditions at the azimuthal boundaries and at the interface, using Eqs. (1) and (2). During the computation, the flowfield in A and B, time periodic according to Eq. (1), is approximated by Fourier series for which the coefficients are updated at each time step using a sliding average technique. Equation (2) is used to perform the join condition between the flowfield in A and B. The boundary treatment at the rotor/stator interface relies on the same principle. At each time step, taking into account the rotation, the two sliding interfaces are positioned one with respect to the other. The flowfield for each computing cell of the donor interface, approximated by a Fourier series, is then reconstructed using time and the phase-lag information before being used for the join treatment.

Consider now the general case of a configuration composed of three or more blade rows, with independent blade counts and different rotating speeds such as represented in Fig. 2. On such a configuration, the classical phase-lagged approach is not valid anymore since each blade row experiences at least two blade passing frequencies corresponding to the upstream and downstream rows, which are different in the general case. Therefore, each blade row experiences multiple disturbances with unrelated frequencies. Now the question is the following: to reduce the computational domain to one blade passage for each row, which numerical treatment should be done at the upper and lower azimuthal boundaries; in other words, which link can be assumed between the flowfield in \( A(x, r, \theta_A) \) and \( B(x, r, \theta_B) \)?

B. Assumptions

The development of the multiple-frequency phase-lagged approach in the CFD code considered for this study (elsA) was described in [17]. The main hypothesis of this method is to assume that the flowfield can be decomposed in a linear combination of spinning modes, each mode being characterized by a spatial wavelength \( m \) and a rotation speed \( \Omega_{m} \) as proposed by Tyler and Soffrin [24]. Each aerodynamic variable written in the relative frame of reference can be decomposed in a sum of disturbances.
corresponding to circumferential traveling waves, for which a phase-shift periodicity can be defined:

\[ g(x, r, \theta, t) = \sum_{m=0}^{N} g_m^r(\theta, t) \]  

(3)

\[ g_m^{x,r}(\theta, t) = a_m^{x,r} \cos(m(\theta - \Omega_m t)) + b_m^{x,r} \sin(m(\theta - \Omega_m t)) \]  

(4)

In the following formulas, the superscript \((x, r)\) is dropped, and two gauges \(A\) and \(B\) are considered, located at the same axial and radial position but at different azimuth \(\theta_A\) and \(\theta_B\). Figure 3 represents the evolution of a spinning mode \(g_m\) in a time-azimuth diagram.

One can observe from Fig. 3 that gauges \(A\) and \(B\) experience the same signal, characterized by the frequency \(f_m\) and pulsation \(\omega_m\), such that

\[ f_m = \frac{m\Omega_m}{2\pi} = \frac{\omega_m}{2\pi} \]  

(5)

but with a phase lag in time:

\[ g_m(\theta_A, t_1) = g_m(\theta_B, t_2), \quad t_2 = t_1 + \frac{\theta_B - \theta_A}{\Omega_m} \]  

(6)

Assume now that the aerodynamic field at point \(A(x, r, \theta_A)\) is a linear combination of spinning modes, each disturbance being time periodic and having its own phase shift periodicity,

\[ g(\theta_A, t) = \sum_{m=0}^{N} g_m(\theta_A, t) \]  

(7)

with \(g_m(\theta_A, t) = a_m \cos(2\pi f_m t) + b_m \sin(2\pi f_m t)\); then, one can obtain the value of the aerodynamic field in \(\theta_B\) by phase shifting each mode one by one:

\[ g(\theta_B, t) = \sum_{m=0}^{N} g_m(\theta_B, t) \]  

(8)

Fundamentally, this approach is the same as for the classical phase-lagged approach. It is just more general, as it enables taking into account more than one fundamental frequency. Indeed, for a classical phase-lagged configuration (rotor/stator), the values of \(m\) correspond to the multiple integers of the blade count of the adjacent row, and \(\Omega_m\) is the adjacent blade row rotation speed. The multiple-frequency phase-lagged approach is more general in the sense that one allows the use of any values of \(m\) and \(\Omega_m\). For example, \(m\) can take the value of the upstream and downstream blade row numbers (which can be different), associated to a value of \(\Omega_m\) corresponding to the opposite blade row rotation speeds (which can be different). Also, \(m\) and \(\Omega_m\) can be a linear combination of the upstream and downstream blade numbers (blade rotation speed). Any kind of deterministic modes corresponding to periodic unsteadiness induced by relative blade motion can be selected for the spinning-mode decomposition.

C. Boundary Condition Treatment

A multiple-frequency phase-lagged calculation consists of using a classical time-marching solver (as, for example, Runge–Kutta, dual-time stepping, or Newton methods) with specified boundary conditions at the periodic boundaries and at the interfaces. The previous assumption enables reducing the computational domain to a single blade passage for each row, using specific multiple-frequency phase-lagged conditions both at azimuthal boundaries and at the rotor/stator interfaces. The boundary condition treatment is performed in two steps:

1) The flow variables are first approximated by a sum of Fourier series, for which the coefficients are updated at each time step.

2) A time shift is performed on the Fourier series, in order to determine time-shifted variables that are used in ghost cells at the opposite adjacent boundaries.

Consider a pair of mesh points \(A\) and \(B\) located at upper and lower azimuthal boundaries as represented on Fig. 2. At the periodic boundaries of a single-passage computational domain, one approximates the flowfield as a sum of periodic functions corresponding to multiple periodic disturbances. By gathering all of the modes associated to a common perturbation (a fundamental frequency and
The flowfield in $B$ can then be deduced by phase shifting each periodic perturbation,

$$g(\theta_B, t) = \sum_{i=1}^{N_{\text{sim}}} F_i(t) - \sigma_i$$

with $\sigma_i = \frac{\theta_{m,i} \pi}{N_{\text{stat}}}$ being the phase lag corresponding to the $i$th perturbation. During the time-marching integration, the boundary conditions are applied at both upper and lower boundaries, a similar treatment being done at both upper and lower boundaries, a similar perturbation. During the time-marching integration, the boundary

A key point to mention concerning the stability of the method is to apply an underrelaxation of the Fourier coefficients when they are being updated as proposed by Li and He [12]:

$$a_{ni(\text{stored})} = (1 - \alpha) a_{ni(\text{stored})} + \alpha a_{ni(\text{new})}$$

with a similar treatment for $b_{ni}$. Experience shows that this relaxation forbids Fourier coefficients to evolve too quickly and ensures numerical stability. In the presented calculation, the parameter $\alpha$ is set to 0.1, ensuring a good stability of the method [17].

E. Theoretical Limits of Multiple-Frequency Phase-Lagged Approach

If the advantages of the multiple-frequency phase-lagged approach are easy to understand (unsteady effects are modeled with a reduction of the computational domain to one blade passage for each row), it is important to keep in mind the limits of this method. The first limit is that the spinning-mode hypothesis [Eqs. (3–4)] assumes that the sources of unsteadiness are linked to the wheel rotation and that the unsteady phenomena can be modeled by spinning modes (supposing that information travels in the azimuthal direction). The method assumes that the flowfield is governed by deterministic frequencies linked to the periodicity of the adjacent rows; therefore, it is not valid for flow configurations including a priori unknown deterministic frequencies such as a rotating stall or vortex shedding. A second limitation of the MFA approach is that the user needs to select which spinning modes will be used for the mode decomposition [Eqs. (3–4)], in other words, which modes exist in the flowfield. If spinning modes corresponding to adjacent blade row motion are easy to determine ($\omega$ is a multiple integer of the adjacent blade count, and $\Omega_n$ is the adjacent blade row rotation speed), there can also exist modes for which the wavelength and rotation speed are a linear combination of the previous modes. It is possible to take into account perturbations due to interaction between the upstream and downstream frequency ($f_1 + f_2$, $f_1 - f_2$, $2f_1 + f_2$, etc.), but it is difficult to guess a priori which modes will be dominant. In fact, they are difficult to guess in advance unless one performs a full-anulus calculation (which is precisely what this paper is trying to avoid). Therefore, a simple rule to apply to the computations is to take into account only the modes associated to the disturbances from the neighbouring blade rows ($N_{\text{col}} = 2$); the modes wavelength will correspond to multiple numbers of both the upstream and downstream numbers of blades and their associated rotation speed. This choice is done in the calculations presented hereafter.

Last but not least, the most important limitation of the method is that it relies on formulas that are not valid anymore for nonspinning modes, i.e., if $\Omega_m = 0$. The behavior of such a mode in a time-azimuth diagram is presented in Fig. 5 (left). One sees that two gauges $A$ and $B$ located at two different azimuthal positions $\theta_A$ and $\theta_B$ experience a different constant value. There is no way knowing the flowfield at $A$ will help evaluate it at $B$ and vice versa. Such a particular mode can be encountered, for example, on a stator/rotor/stator/2 configuration with independent blade count for the two stator rows as represented in Fig. 5 (right). In the second stator, two adjacent stators (S2a and S2b) experience a time-periodic flow (corresponding to the rotor passing frequency) but are not related by any phase-lag relationship. Indeed, wakes of the upstream stator, after being convected in the rotor, impact the downstream stator, but with a different position with respect to the second stator (in the figure, the upstream stator wakes impact S2a but not S2b). There is no easy way to take into account this effect with accuracy and to link the upper and lower boundaries, as it was explained by Van Zante et al. [10].

Therefore, the MFA approach is not able to take into account nonrotating modes, i.e., rotor/rotor or stator/stator clocking effects. It represents a limitation of this numerical approach, meaning that the impact of blade rows $N - 2$ and $N + 2$ on blade row $N$ are not correctly modeled. This modeling error can lead to errors of continuity and conservation losses, which need to be checked. In the

During the time-marching integration, the Fourier shapes continuously

$$\Lambda' (x,r,\theta_A + \Omega \times t) = B'(x,r,\theta_B), \quad \theta_B = \theta_A + k \times 2\pi / N_{\text{stat}}$$

Fig. 4 Treatment of the rotor/stator interface.
following presented calculation, the relative error between the upstream and downstream mass flows did not exceed 0.1% (order of magnitude equivalent to mixing-plane calculations).

III. Description of CREATE Configuration

In this study, the MFA approach is evaluated on the 3.5 experimental compressor stage CREATE investigated at École Centrale de Lyon in the LMFA laboratory [25]. This axial compressor from Société Nationale d’Etudes et de Construction de Moteurs d’Aviation (SNECMA) is representative of the median stages of modern high-pressure compressors. A meridian view of the compressor with the investigated experimental planes is presented in Fig. 6. Measurements are carried out both with pneumatic and unsteady pressure probes, detailed by Mersinligil et al. [26]. Two-dimensional velocity measurements are also available through laser-doppler anemometry. On account of the large amount of available experimental data, the comparisons between the calculations and experiments shown hereafter mainly focus on pressure probe measurements. Tables 1 and 2 indicate the blade counts (multiples of 16) and the compressor characteristics at the design point.

IV. Computational Grid

A view of the computational grid is presented in Fig. 7. The numerical domain is discretized with a multiblock approach, using an O–H meshing strategy for each passage of the compressor. O blocks are used around the blades, and H blocks fill the rest of the passage. The rotor tip clearance is meshed with an additional O–H block. To minimize the computational cost, the wall cell size is set to ensure a normalized wall distance $y^+$ below 20 everywhere in the domain. Wall functions are then applied to improve the quality of numerical results [27]. This approach gives very satisfying results for non-separated boundary layers but results for massively separated flows, which are not the main goal of this work, should be considered with caution. A blade passage is meshed with about 1.08 million points (289 points around the blade and 109 in the radial direction, including 25 points in the tip clearance). The total number of nodes to represent the three compressor stages is 6.5 million points (32 blocks) when only one passage per blade row is meshed and 36 million points (178 blocks) to reach a $2\pi/16$ sector to perform the reference computation, taking into account the natural compressor periodicity.

The blade of the inlet guide vane (IGV) is not meshed, yet its influence is modeled in the calculation by applying at the inlet boundary a map corresponding to flow conditions downstream the IGV (such as total pressure deficit and flow angles), based on experimental data.

V. Computational Procedure

As indicated in Table 1, the blade numbers allow a reduction of the computational domain to $1/16$th of the circumference (22.5 deg), which is a reasonable hypothesis for stabilized operating points. Therefore, in this study, three types of calculations are performed:

1) A URANS multiple-frequency phase-lagged calculation (MFA) is performed, taking into account one single blade passage for each row. The MFA boundary conditions previously described are imposed both at the periodic boundaries and at the rotor/stator interfaces.

2) A URANS multiple-passage reference calculation (RC) [28] is performed on the same grid as the multiple-frequency phase-lagged...
approach, but with each blade passage being duplicated in order to reach a (22.5 deg) sector. Spatial periodic join conditions are applied at the periodic boundaries, while sliding mesh join conditions are used at the rotor/stator interfaces.

3) A RANS mixing-plane calculation (MPC) is performed, taking into account one single blade passage for each row.

The flow solver used for this study is the elsA software [29,30] that considers a cell-centered approach on structured multiblock meshes. Numerical aspects and boundary conditions have been detailed in [25]. The key point to mention is that the three calculations are performed on the same grid with identical numerical parameters (the third-order scheme of Roe [31]) and turbulence model (∂t of Wilcox [32]) in order to quantify the errors induced by the multiple-frequency phase-lagged boundary conditions, both at periodic boundaries and at rotor/stator interfaces.

For the MFA calculations, in each blade row, the flowfield is approximated as a sum of two periodic functions associated to the upstream and downstream blade passing frequencies (Np = 2). Each periodic function is approximated by a Fourier series composed of Nharmonic = 16 harmonics. Here, it deserves to be mentioned that in the MFA method, contrary to the frequency-domain methods cited in the introduction, increasing the number of harmonics used in the Fourier decomposition does not lead to significant penalties of memory or CPU cost. Preliminary calculations had been performed in quasi three dimensions before running the 3D calculations in order to investigate how many harmonics are required [17], and it was observed that 16 harmonics were more than enough to have a flow solution that is harmonic independent. One can also mention that some attempts have been performed to expand and enrich the frequency content of the flow decomposition, to decompose differently and include frequencies different than the blade passing frequencies (Np > 2). Up to now, these tests have not been fruitful (and are therefore not presented); either one can encounter robustness problems or no significant effect is observed. Therefore, it is good advice to use the simple rule of a basic two periodic function decomposition (Np = 2), which is a practical approach for designing a new machine for which no a priori information is available.

Table 3 summarizes the CPU performances obtained for the three types of computations on the vectorized computer Nec-SX8+. For the MFA and RC computation, the CPU time corresponds to the time to perform one revolution, while for the steady mixing-plane calculation, it corresponds to the time to reach steady convergence. The CPU cost of the unsteady MFA calculation is one order of magnitude more expensive than the steady mixing-plane calculation. Compared to the unsteady RC computation, the MFA method enables a CPU gain proportional to the number of simulated blade passages. Yet one must keep in mind that the reference calculation is performed on 1/16th of the machine. Therefore, compared to a (360 deg) calculation, the CPU gain would be 16 times larger.

It is important to mention that Table 3 just aims at giving rough orders of magnitude. Since the actual time of the computation includes a transient phase, a more precise estimation of the cost should include the number of revolutions needed to get a converged solution. Focusing on the nominal operating point, one rotation is enough for the RC computation, while for the MFA calculation, it takes approximately two rotations. For an operating point near the surge, it is approximately two rotations for the RC computations against four rotations for the MFA calculations. However, the relaxation coefficient, discussed in Sec. II.D, used in the MFA computations was set to a rather small value (0.1), and by optimizing it, one could accelerate the convergence. Moreover, the objective of the present paper is not to quantify with high precision the CPU gain but to analyze the difference in the flow solutions obtained between the two approaches.

### VI. Results

The following paragraphs analyze and compare results obtained with the three numerical simulations and the experiments such as compressor maps, flowfield analysis, circumferentially and time-averaged radial distributions, and, finally, time-fluctuation analysis. The following comparisons aim at providing answers to the following questions:

1) Compared to the reference computation, is the MFA method capable of reproducing the main unsteady effects encountered on the CREATE compressor configuration?

2) Compared to the reference computation, which effects are not captured by the MFA method?

3) How important are the discrepancies between the computations compared to the ones existing between experiments and the reference simulation?

4) Compared to a standard mixing-plane calculation, is there a significant benefit in using the MFA approach apart from the fact that unsteady effects are modeled?

#### A. Global Performance Results

The comparison of the compressor maps obtained for the three types of computations, and the experiments are presented in Fig. 8. If the three types of computations overestimate the total pressure ratio

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**Table 3** Mesh characteristics and calculation time

<table>
<thead>
<tr>
<th></th>
<th>Blocks</th>
<th>Points</th>
<th>Central processing unit hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC</td>
<td>178</td>
<td>6.5 × 10⁶</td>
<td>760</td>
</tr>
<tr>
<td>MFA</td>
<td>32</td>
<td>6.5 × 10⁶</td>
<td>85</td>
</tr>
<tr>
<td>MPC</td>
<td>32</td>
<td>6.5 × 10⁶</td>
<td>7</td>
</tr>
</tbody>
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**Fig. 7** Computational grid.

**Fig. 8** Compressor maps.
and efficiency compared to experiments, all of the calculations give close results for all of the operating points from blockage ($Q \sim 1.08$) to nominal points ($Q \sim 1.02$). Discrepancies between the computations begin to appear for operating points close to the stall. The MFA underestimates the stall margin compared to the RC and MPC computations. Indeed, the RC and MPC computations exhibit a last stable operating point around $Q \sim 0.97$ and $Q \sim 0.96$ respectively, while the last stabilized computation for the MFA calculation is at $Q \sim 0.99$. Moreover, the compression ratio near the stall is underestimated by about 3% compared to the RC computation. As it will be analyzed afterward, this difficulty for the MFA approach to obtain accurate results for such operating points can be explained by the fact that the MFA model relies on hypotheses that are no longer valid near the stall, like the spinning-mode hypothesis [Eqs. (3–4)] or the assumption that the dominating flow frequencies are the blade passing frequencies and their harmonics. Concerning the efficiency prediction, it is overestimated by the three calculations compared to the experimental values. One can notice that the efficiency prediction obtained by the MFA approach is close to the RC calculation. In the following analysis, computations associated to the nominal point ($Q \sim 1.04$) are compared with the closest experimental point ($Q \sim 1.00$).

**B. Flowfield Qualitative Analysis**

The MFA calculations are performed with only one blade passage per row. Yet at the convergence of the computations, one can obtain a reconstruction of the flowfield on adjacent blade passages using the spinning-mode hypothesis. This is achieved by computing the Fourier decomposition [Eq. (3)] for the cells located inside the computational domain, using the iterative algorithm of He [11]. Once the Fourier decomposition is known, the flow values on adjacent passages are obtained from the current blade passage by applying a phase shift in the flow as indicated by Eqs. (7–9).

An example of flowfield reconstruction for the MFA calculation is illustrated in Fig. 9, which represents entropy snapshots at two span heights (50 and 80%). They are compared to snapshots obtained with the RC computation and with the mixing-plane steady flowfield. One can observe that the main flow structures seen in the RC computation are quite well captured by the MFA simulation. The main convective effects (wake migration) and segregation effects between low- and high-entropy zones are well reproduced by the MFA model. In particular, at 80% of the span height, the high-entropy zone associated to the tip leakage flow observed on the pressure side at R3 for the reference computation is well reproduced by the MFA model.

![Fig. 9 Entropy snapshots at $h/H = 50\%$ and $h/H = 80\%$.](image)

![Fig. 10 Time- and circumferentially averaged total pressure radial distribution. Left: downstream rotor planes (26A, 27A, and 28A). Right: downstream stator planes (270, 280, and 290).](image)
These figures are also interesting since they enable the highlighting of the weaknesses of the MFA model. Indeed, one can observe that clocking effects are not accurately simulated by the MFA; for example, the wakes emanating from R1, which are convected through S1, are not transmitted continuously in R2 at the S1/R2 interface since the MFA approach does not take into account any phase-lag influence between R1 and R2 (clocking effects). One notices that most of the wakes are cropped, except the wakes crossing the actual passage of the single-passage computational domain, which are correctly transmitted. The cropped wakes that are reproduced using the phase-lagged assumptions are the ones that are not correctly transmitted. This observation is also discussed in Sec. VI.D.

Finally, it is also interesting to compare the MFA snapshots with the steady flowfield obtained by the mixing-plane calculation. In the MPC calculations, due to the averaging treatment at interfaces, the wakes are lost at each interface, and only the mean level is transmitted uniformly in the downstream row. Wake migration and segregation effects between low- and high-entropy zones can therefore not be captured by the MPC model. In particular, at 80% of the span height, the high-entropy zone associated to the tip leakage flow observed on the pressure side at R3 for the reference computation is reproduced, but it is much more diffused compared to the MFA and RC computations. Even though rigorously one should compare the steady flow of the MPC computation with the time-averaged flow of the RC and MFA calculations (rather than snapshots, and it is done hereafter in Sec. II.D), this comparison is interesting in order to highlight the advantages of the MFA approach compared to the MPC calculations in the sense that the MFA gives access to unsteady and local effects that cannot be obtained with the MPC approach.

C. Radial Distribution

The time- and circumferentially averaged radial distributions of total pressure and total temperature obtained in the absolute frame of reference at the different measurement planes are plotted in Figs 10 and 11. Compared to the experimental values, one can observe an

![Fig. 11 Time- and circumferentially averaged total temperature radial distribution. Left: downstream rotor planes (26A, 27A, and 28A). Right: downstream stator planes (270, 280, and 290).](image)

![Fig. 12 Time-averaged total pressure contours at planes P26A/P270 (stage 1).](image)
overestimation of the computed total pressure level (maximum 1%), as previously observed on the compressor maps. On the total temperature distribution, all of the calculations overestimate the casing wall temperature. The purpose of this work is not to attempt to explain discrepancies between CFD and experiments, which is always a difficult task since discrepancies can be explained by many factors (unrealistic boundary conditions, geometrical details not taken into account [33], turbulence modelling, grid density, numerical errors, etc.) The important messages to deliver here are that, on this compressor configuration for the analyzed operating point, the following statements are true:

1) A very close agreement is obtained between the MFA and RC computations, in particular concerning the shapes and the levels of the radial distributions, which are well reproduced.

2) Discrepancies existing between the RC calculation and experiments are much larger than the gap separating the three CFD approaches. As a consequence, the error induced by the MFA boundary conditions is quite small if one compares it to errors induced by other effects such as numerics, turbulence modeling, etc. This means that if one performs a MFA calculations and observes discrepancies with experiments predictions will probably not be significantly improved by running a full-annulus computation.

3) One observes that there is no significant improvement of these radial distribution predictions with the three CFD approaches (RC and MFA) compared to the steady (MPC) approach. Therefore, if in an industrial design context one is only interested in the global values prediction as radial distributions of time-averaged results, the mixing-plane model is well adapted. Results will probably not be improved with unsteady approaches.

D. Total Pressure Time Averaged Field

Time-averaged two-dimensional total pressure contours obtained in the absolute frame of reference at the different axial experimental planes are represented on a $2\pi/16$ sector in Figs 12–14. They...
highlight more precisely the differences between calculations and experiments, for which the radial extension is limited between 15 and 98% of the span height. Concerning the MFA results, it should be remembered that only one blade passage is computed, yet the total pressure field can be reconstructed on a $2\pi/16$ pitch using phase-lagged decomposition. Planes 270, 280, and 290 are located downstream of the stator rows. As a consequence, the time-averaged total pressure fields exhibit the wakes emanating from the upstream stator rows. A good agreement is observed between the MFA and RC computations, which overestimate total pressure and underestimate the wakes thickness compared to the experiments. Planes 26A, 27A, and 28A are located downstream of the rotor rows. As a consequence, the time-averaging process leads to a flowfield for which the spatial periodicity corresponds to the previous upstream stators blade counts. Therefore, at plane 26A, one can observe two low total pressure zones near the hub, corresponding to the IGV secondary flows, seen both by the experiments (a) and by the RC computation (b). However, the MFA approach does not reproduce this phenomenon with the correct spatial frequency (c), and it is also the case with the mixing-plane calculation (d). This error, due to the fact that MFA approach does not simulate clocking effects, is also observed on plane 27A: the experiments and RC computations capture six wake structures (e, f) corresponding to S1, while the MFA and MPC erroneously exhibit seven wakes (g, h). Finally, on plane 28A, experiments and RC simulations exhibit seven wakes (i, j) corresponding to S2, while eight wakes (k, l) are obtained with the MFA and MPC methods.

**E. Total Pressure Fluctuations**

Figure 15 represents a time-azimuth diagram of the total pressure fluctuations obtained at section P26A (R1/S1 interface) at 50% of the span height. The azimuth is represented at the abscissa of the diagram and corresponds to a $2\pi/16$ sector, while the time, corresponding to a period of $1/16$th of the rotor revolution, is plotted at the ordinate. The total pressure fluctuation is defined as the instantaneous value minus the time-averaged value:

$$P_t(x, r, \theta, t) = P_t(x, r, \theta, t) - \frac{1}{T} \int_0^T P_t(x, r, \theta, t) \, dt$$  \hspace{1cm} (12)$$

The experimental values are compared with the two unsteady calculations (the MPC calculation, which is a steady approach, cannot provide these unsteady fluctuations).

Both calculations, which underestimate the total pressure fluctuations compared to the experiments, give similar results and qualitatively exhibit the same flow physics. The MFA method accurately captures the convection of the four wakes emanating from R1, represented by the four diagonal streaks (a). Moreover, the MFA calculation captures the potential effects emanating from S1, corresponding to the 6 vertical streaks (b).

To quantify more precisely the differences between the three results, the total pressure fluctuations signals are compared in Fig. 16, for two azimuthal positions ($\theta = 0$ deg and $\theta = 5$ deg). They are represented in terms of time harmonics of the Fourier series associated to the time period $T$ corresponding to $1/16$th of the rotor revolution.
revolution. \( C_n \) corresponds to the magnitude of the Fourier coefficients associated to the \( n \)th harmonic. It must be mentioned here that one could also choose to plot the Fourier coefficients associated to the spatial harmonics, as done by Courtiade et al. [34], but plotting the temporal modes enables the clear highlighting of the difference between the two computations in terms of temporal frequency content. One notices for both azimuthal positions that the dominating mode is mode 4, corresponding to the blade counts of R1, followed by the subharmonics \((8,12)\). The two calculations exhibit close results despite the fact that the amplitude of harmonic 4 is significantly underestimated compared to experiments.

Figure 17 represents the time-azimuth diagram of total pressure fluctuations at section P28A (R3/S3 interface) at 80% of the span height. Once again, compared to the experiments, both calculations underestimate the total pressure fluctuations. Two kinds of streaks can be observed in this diagram: five diagonal streaks \((a)\) corresponding to the wake convection of R3 and eight vertical streaks \((b)\) corresponding to the potential effects of S3. Clocking effects \((\text{in the sense rotor/rotor or stator/stator interaction})\) are clearly visible for the experiments and the reference computation, since two azimuthal positions separated by one stator pitch experience the same flowfield, even with a phase lag. A different behavior is obtained in the experiments and the reference computation, since two azimuthal positions separated by one stator pitch do not experience the same flowfield, even with a phase lag. A different behavior is obtained in the experiments and the reference computation, since two azimuthal positions separated by one stator pitch do not experience the same flowfield, even with a phase lag. A different behavior is obtained in the experiments and the reference computation, since two azimuthal positions separated by one stator pitch do not experience the same flowfield, even with a phase lag.

Concerning future work, it is finally mentioned that the MFA method will be evaluated on open rotor configurations and that preliminary evaluations have been performed on aeroelastic configurations [35].

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**References**


**Fig. 18 Histogram of the temporal Fourier harmonics at P28A, \( \theta = 0 \) deg and \( \theta = 5 \) deg.**