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Particle motion induced by a cavitation bubble

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Cavitation bubbles induce impulsive forces on surrounding substrates, particles, or surfaces. Even though cavitation is a traditional topic in fluid mechanics, current understanding and studies are limited to bubble collapse on fixed surfaces, which does not capture the effect of cavitation on suspended objects in fluids. In the present work, the dynamics of a spherical particle due to a cavitation bubble is experimentally characterized and compared with an analytical model. Three phases are observed: the growth of the bubble where the particle is pushed away, its collapse where the particle approaches the bubble, and a longer timescale post-collapse where the particle continues to move toward the collapsed bubble. The particle motion in the longer timescale presumably results from the asymmetric cavitation evolution at an earlier time. Our theory considering the asymmetric bubble dynamics shows that the particle velocity strongly depends on the distance from the bubble as an inverse-4th-power law, which is in good agreement with our experimentation. This study sheds light on how small free particles respond to cavitation bubbles in fluids.

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Cavitation is the physical process of bubble formation in a liquid medium by either decreasing pressure or increasing temperature. These cavitation bubbles are well known for causing undesirable damage in hydrodynamics [1] and offer advantages in many systems; e.g. in sonocatalytic reactors [2] and in drug delivery methods [3]. They are also present in the human body [4], in plants [5], or in everyday life [6]. Cavitation occurring near fixed solid boundaries has drawn a lot of attention, however cavitation in the vicinity of mobile objects has yet to be understood, even though it takes place in many situations such as non-invasive removal of kidney stones [7, 8], cavitation reactors for wastewater treatment [9], or hunting techniques of some crustaceans [10, 11].

During the cavitation mechanism, a vapor bubble is nucleated and is rapidly turned back to its equilibrium liquid phase [12]. The detailed dynamics of spherical bubbles far from any boundaries, described by the Rayleigh-Plesset equation, has been extensively studied [13–16]. Interactions between cavitation bubbles and solids have also been examined, primarily through experimental and numerical methods [17–19], but always in proximity to stationary surfaces. Specifically, the current understanding of cavitation near fixed solid surfaces does not capture the motion of freely moving objects near cavitation bubbles.

In this present work, we propose an experimental study of the interaction between a cavitation bubble and a freely moving particle whose radius is smaller than the maximum bubble radius. We identify the response of the particle to the bubble dynamics, and also develop an analytical model for the particle behavior after the disappearance of the bubble that is compared with our experimental data.

There are two ways to initiate cavitation in water according to its phase diagram [20]: by lowering the pressure or by raising the temperature. In practice, pressure-based mechanisms are widely used due to their relatively simple setups [21], however with these methods it is difficult to precisely control the cavitation bubble’s location. As an alternative method, superheated cavitation bubbles are generated by laser pulses [22] or by electric sparks [23, 24], which is more convenient to control and study a single bubble and its effects [25–27]. We have chosen the spark-induced approach in this study. The electric spark is generated by the discharge of capacitors (the equivalent capacitance of the circuit is 23.5 mF) that can be charged up to 50 V. Two tinned copper electrodes linked to the circuit, approximately 0.17 mm in diameter, are touched together at the desired location of the nucleation. A trigger initiates the discharge of the capacitors, creating a short-circuit and thereby a spark that nucleates a cavitation bubble. Experiments are performed in a plexiglas tank filled with filtered water at room temperature. The bubble is nucleated far enough from the tank walls and from the air-water surface to neglect the effects of these boundaries. Three different voltages are used to charge the capacitors: 40, 45 and 50 V. Below 40 V no significant movement of the particles has been observed. This nucleates bubbles with respective maximal radii of $R_{\text{b,max}} = 2.6 \pm 0.1, 3.3 \pm 0.2, 3.9 \pm 0.2$ mm, growing times $t_g = 0.45 \pm 0.03, 0.58 \pm 0.06, 0.68 \pm 0.08$ ms, and collapsing times $t_c = 0.31 \pm 0.04, 0.42 \pm 0.07, 0.51 \pm 0.08$ ms.

The cavitation is initiated near a single solid spherical particle attached to a thin steel rod, whose mass is $m_{\text{rod}} = 0.07$ g, that hangs under gravity like a pendulum because the chosen particles are non-buoyant. The nucleation point and the center of the particle are po-
FIG. 1: Image sequences of a glass particle (left), 1.6 mm in radius, and a cavitation bubble (right). A 50 V spark (a) nucleates a cavitation bubble that begins to expand, (b) pushing the particle away. The bubble then reaches its maximum size (c) and begins to attenuate in volume. As it shrinks, it is no longer able to maintain its spherical shape (d) and will collapse in on itself to its nucleation point (e). The collapse of the cavitation bubble sucks the particle toward the nucleation site (f). In all of the images, the time represents the time since the electric spark. For details, see the supplementary video.

sitioned on a horizontal line, making the motion of the latter one-dimensional: considering the small horizontal displacements analyzed, less than 1 mm, and the length of the rod, \( l_{\text{rod}} \sim 65 \text{ mm} \), it is assumed that vertical displacements are negligible. The motion of the particle is hence only characterized by the distance from its center to the nucleation point, designated by \( X_p(t) \). Six different particles are used: three glass particles, whose radii \( R_p \) are 2.4, 2.0 and 1.6 mm; and three other 1.6 mm particles made of aluminium, brass and steel. Their masses \( m_p \) are respectively 0.14, 0.08, 0.04, 0.05, 0.14 and 0.13 g.

The motion of the particle and the evolution of the bubble are recorded with a Photron FASTCAM Mini high-speed camera, with 12,500 to 64,000 frames/s depending of the need of spatial and temporal resolution. The particle position is tracked using MATLAB codes. A sample image sequence is shown in Fig. 1. Dynamics of the bubble and the particle can be divided into the following three phases, illustrated by Fig. 2. (i) Just after the electric spark, the cavitation bubble grows and pushes the particle away (\( \dot{X}_p > 0 \)) until it reaches its maximal size. (ii) The bubble then starts to collapse in on itself, sucking the particle toward its center (\( \dot{X}_p < 0 \)). The bubble evolution is qualitatively similar to what has already been reported for a cavitation bubble near a fixed rigid convex surface [28]: it grows almost spherically, but takes a characteristic shape during the collapse as seen Fig. 1(d). (iii) Once the bubble has collapsed (see Fig. 1(e)), it rebounds and collapses again several times. The rebounds become progressively smaller, and then the bubble finally disappears. During this time, the particle continues to move toward the center of the bubble as illustrated in Fig. 1(f).

We studied the particle during the last phase of suction in which, as shown Fig. 2(b), it has a constant speed, denoted by \( \dot{X}_{p,f} \), for a long time compared to the timescale of the bubble dynamics. An analytical theory has been developed to evaluate the effect of the particle proper-

FIG. 2: (a) Evolution of the bubble radius generated by 40, 45 and 50 V sparks; measured without the presence of a particle. Due to the initial spark, the bubble radius is unmeasurable at the very beginning of the growth. (b) Displacement of a brass particle with a radius of 1.6 mm due to cavitation, showing the strong dependence on the initial distance and the bubble dynamics. The measured final velocity is given for each case.
ties, the initial distance $X_{p,i}$ from the nucleation point, and the bubble dynamics. The evolution of the bubble creates a flow in which the fluid velocity $u$ is found from the incompressibility of the fluid [29]: using spherical coordinates centered at the nucleation site, the velocity is radial: $u = u_{r}$ with

$$u(r, t) = \left(\frac{R_{b}(t)}{r}\right)^2 \ddot{R}_{b}(t),$$  \hspace{1cm} (1)$$

where $R_{b}$ is the bubble radius. An important geometric parameter for bubble dynamics near boundaries is the stand-off parameter $\gamma = (X_{p,i} - R_{b})/R_{b,max}$, the ratio of the initial distance between the solid boundary and the bubble center to the maximum bubble radius [30]. In our experiments, we only consider $\gamma > 1$. The drag force on the particle due to the flow is $F_{\text{drag}} = F_{\text{drag}}(t) \varepsilon_{r}$ with

$$F_{\text{drag}}(t) = \text{sign}(u) \frac{1}{2} C_{D} \rho_{w} A [u(X_{p}(t), t)]^2,$$  \hspace{1cm} (2)$$

where $A = \pi R_{p}^{2}$ is the projected area of the particle, $\rho_{w}$ is the density of water and $C_{D}$ is the drag coefficient taken as a constant.

The particle and the rod form a pendulum system that follows the angular momentum equation $I \ddot{\theta} = -l_{\text{rod}} D$, where $I$ is the moment of inertia of the system around its axis of rotation. With small angular displacements $\theta$ ($\theta = (X_{p,i} - X_{p})/l_{\text{rod}}$) and particles smaller than the rod length ($R_{p}/l_{\text{rod}} \ll 1$), the angular momentum equation becomes $m_{\text{eff}} \ddot{X}_{p} = D$, with $m_{\text{eff}} = m_{\text{rod}}/3 + m_{p}$. The velocity of the particle can be evaluated by integrating this equation of motion over the first two phases. For simplicity, the particle displacement during the growth and the collapse of the bubble is neglected: in the drag expression (2), we assume $X_{p} = X_{p,i}$. This approximation is experimentally justified: the particle position was measured in our experiments during these phases, giving a maximum departure from the initial position $\Delta X_{p,i}/X_{p,i} \sim 0.3 – 5\%$. Therefore, we obtain

$$\dot{X}_{p,f} = \frac{1}{2} C_{D} \rho_{w} \pi R_{p}^{2} X_{p,i}^{4} m_{\text{eff}} \left( t_{c} \int_{0}^{t_{g}} R_{b}^{5} \dot{R}_{b}^{2} dt - \int_{t_{g}}^{t_{g}+t_{c}} R_{b}^{5} \dot{R}_{b}^{2} dt \right),$$  \hspace{1cm} (3)$$

where $t_{g}$ and $t_{c}$ are respectively the duration of the growing and collapsing phase of the bubble. We assume that the bubble follows the same form of evolution during the growth and the collapse, i.e. a self-similar solution $f$ such
as:

\[
\frac{R_b(t)}{R_{b,\text{max}}} = \begin{cases} 
    f \left( \frac{t}{t_g} \right) & \text{if } t \in [0; t_g] \\
    f \left( \frac{t + t_c - t_g}{t_c} \right) & \text{if } t \in [t_g; t_g + t_c] 
\end{cases} \quad (4)
\]

This is motivated by the fact that the collapse of an empty spherical bubble follows a similar scaling [31]. The assumption is verified in Fig. 3, showing a small deviation between the growing and collapsing phases of the bubble. Using Eq. (4), the integral term in Eq. (3) becomes \( \int_0^1 f^4 f^2 \times R_{b,\text{max}}^5 / \tau \), where \( \tau = t_g t_c / (t_g - t_c) \) is a characteristic time of the bubble. This characteristic time is always positive because the collapse is faster than the growth, and \( \tau \) can be seen as a measure of the asymmetry of the bubble evolution. From these results, dimensionless velocity and position are defined respectively as \( X_{p,f} = -\dot{X}_{p,f} \times m_{\text{eff}} \tau / \left( \rho_u R_b^2 R_{b,\text{max}}^2 \right) \) and \( X_{p,i} = X_{p,i} / R_{b,\text{max}} \), leading to the following power-law relation:

\[
\dot{X}_{p,f} \propto \left( X_{p,i} \right)^{-4} \quad (5)
\]

Figure 4(a) shows the particle velocity as a function of the distance with 3 different particle sizes, 4 different materials, and 3 different voltages. By normalizing the velocity and the distance based on the above analytical theory, we find that experimental data collapse well into Eq. (5) as shown in Fig. 4(b).

In conclusion, we observed and characterized the effect of a spark-induced cavitation bubble on a distant and small tethered spherical particle. This present study focuses on the final phase of the coupled dynamics, where the particle continues to move toward the center of the bubble because of the asymmetric bubble dynamics. The particle velocity inversely depends on the density and radius of the particle, and approximately on the 4th power of the initial distance from the bubble.

Our analytical theory is under the assumption of spherical bubbles and is then only valid for large \( \gamma \) values. We then observed that when the spark occurs very close to the particle, the particle can go away from the bubble because of the strong influence of the rebound. Nevertheless, the inverse-4th-power law is close to the experimental results with \( \gamma > 1 \), even though it underestimates the effect of the distance (the best-fit slope on our data is \(-4.8 \pm 0.3\)) which is believed to be a consequence of the deformations of the bubble. In future work, we plan on investigating the detailed consequence of the precise bubble dynamics on the particle response for \( \gamma \) values close to and less than 1.

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