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Adaptative ionospheric electron content estimation method

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**BIOGRAPHIES**

*Sébastien Trilles* is PhD graduated in Mathematics (topology and real algebraic geometry) at Paul Sabatier University and post-master graduated in space engineering at the École Nationale Supérieure d’Aéronautique (ISAE-SUPAERO), France. He worked in space flight dynamics and precise orbitography determination. He was also involved in some current topics as three body problem, symplectic and variational integrators. He joined Thales Alenia Space in 2011 as an algorithm navigation engineer.

*Gonzague de la Hautière* is graduated from the École Polytechnique and was awarded a master in aeronautic and space engineering together with an intensive education in high level project management (DESIA) at the École Nationale Supérieure d’Aéronautique (ISAE-SUPAERO), France. He joined Thales Alenia Space in 2007 as a system engineer participating to the qualification of EGNOS release v2.2-ext. He is currently managing the EGNOS Sub-systems technical responsible for the development of new releases. In addition he is involved in many Research studies.

*Mathias van den Bossche* graduated with a Masters in Physics at the École Normale Supérieure (Paris), and got PhD in Theoretical Physics at the Paris University. He turned to Space industry in 2001, and joined the Navigation Unit at Alcatel Space in 2003 to work on Galileo. He is now is head of the Navigation System Performance department with Thales Alenia Space.

**ABSTRACT**

In 2011 and 2012, the increase of solar activity has severely disturbed the ionosphere, and caused strong performance degradations of both Safety-of-life Satellite Based Augmentation Systems, WAAS and EGNOS. As the dynamics of the ionosphere remains poorly understood, the algorithms at users level models the global ionosphere as a thin layer as defined by MOPS reference document.

The Triangular Interpolation model, also known as TRIN model, approximates this thin layer by a rigid polyhedron with triangular facets in a solar fixed frame. This model assumes that the ionosphere layer behaviour can be considered as spatially linear inside each facet. Since during a strong solar activity period, the variation may be significantly non-linear, the ionosphere dynamic may locally be outside the model validity domain.

Based on the analysis of the temporal and spatial distribution of ionosphere pierce points (IPP) - and associated L1 ionosphere delays, we have developed a set of invariants to discriminate different types of ionosphere dynamics around each vertex of TRIN model.

These invariants reflect the taxonomy of the IPP distribution around a mesh vertex. On this basis, we propose a method to develop a flexible and adaptive TRIN model by adding new mesh vertex where relevant in order to reduce the process model noise. In the case where a cluster distribution is detected, the process positions new mesh vertex near local optima. The method adapts to the ionosphere’s dynamic seeking periodically local optima. This gives a more realistic mapping of heterogeneities of the ionosphere and its developments.

A simple implementation of this method is being developed. In this paper we detail the methodology we are following in order to improve the total electric content computation for the Ionospheric Grid Point using a Flexible TRIN method.

**INTRODUCTION**

The accuracy and reliability of the position estimated using satellite-based navigation systems depends mainly on two factors. First the number of satellites to which measurements can be made and their geometrical distribution are impacting the navigation solution. Secondly position estimation process is fed by pseudo-distance measurements and is dependant of the accuracy of each of them.

Among the various errors that affect the pseudo-distance measurements, the main contributor is the ionospheric error. It corresponds to the delay that the navigation signal takes when crossing the high altitude, ionised layers of the atmosphere (ionosphere plasma). This delay depends on the level of ionisation of the ionosphere, and needs to be accounted for to obtain an accurate position solution. Satellite-based augmentation systems (SBAS) compute and broadcast corrections and integrity parameters so as to allow the use of the Global Positioning System or any other satellite navigation mean by civil aviation aircrafts in a way where safety is under control. Among the corrections the SBAS broadcast to aircrafts is an estimate of the ionosphere electron content, known as Total...
Electron Content (TEC). This correction allows the position estimation algorithm to account for the ionosphere delay, to improve position accuracy and reliability. The ionospheric error is of the order of several tenth of meters.

Existing models of the ionosphere electron content fail to follow the very inhomogeneous ionosphere states observed during cyclic solar high activity periods. The goal of the method detailed in the following article is to improve estimation process of the ionosphere electron content in a way that is in the same time accurate and robust to inhomogeneous conditions. This method is based on a flexible TRIN model, that we called “adaptative ionospheric electron content estimation method”. It is worth to underling from now that this adaptative ionospheric TEC estimation method does not try to enhance the understanding of the atmospheric physical behaviour. Indeed the goal of the method is to have a model as close as possible to the effects of the ionospheric activity on the propagation of the electromagnetic waves.

**SBAS CURRENT ESTIMATION OF ELECTRON CONTENT**

In the 1990’s, California Institute of Technology [1] proposed a solution to estimate the electron content of the ionosphere as electron content maps using dual frequency pseudo-range measurement gathered by a set of sensor stations. Thales Research and Technology – UK (ex Racall), have further developed this method That is currently implemented in EGNOS, the European SBAS.

In this method, the ionosphere is considered as a thin layer, in which all of the ionosphere delay of the signal is accumulated. This thin layer allows considering that the amount of delay collected by a signal depends only on the TEC at the point where it pierces the thin ionosphere layer, that is called Ionosphere Pierce Point (IPP). The typical quantity to describe the delay at an IPP is the delay that a vertical signal would collect at this point. This is related to the **vertical TEC** (VTEC), i.e. the integral of the ionosphere electron density integrated along the vertical path of the signal through the ionosphere.

This method considers a model that uses a discrete VTEC map on this layer through a triangular mesh, at the vertex of which the VTEC is estimated. The model is built using a basic regular polyhedron that is refined by a subdivision of several deep as depicted in Fig. 1. It is also fixed with respect to a solar-magnetic system referential in such a way each vertex are at constant solar hour.

This estimate is done through the averaging by a Kalman filter, of the individual TEC values obtained for each measurement that lie in a defined area around each vertex of the mesh. The measures are built with a simple linear interpolation on each IPP with three vertex of the triangle containing the IPP. The benefit of a solar-magnetic triangular system is to smooth the TEC gradient of a vertex on the day and to allow the Kalman filter to follow easier the dynamic of the ionospheric layer along time.

This solution works however properly only when the geographical variation of the electron content is on one hand low and VTEC can be assumed as constant in the area around each vertex of the mesh and on the other one linear on each triangle of the model.

There are several ways in which this assumption cannot be met. When the solar activity is close to its 11-year cycle peak, the ionosphere gets significantly excited by the solar radiation, which resonates with Earth magnetic field to produce local high level of ionisation of the ionosphere. This results in either a strong variation (gradients) of the ionosphere over large distances or strong non-homogeneity of the ionosphere that are revealed through spots with very different electron content values. The functioning of the existing ionosphere estimation algorithms in this case gives either a very low accuracy, or if measurement coherence is controlled, impossibility to compute the Electron content.

**SBAS OBSERVED PERFORMANCES**

The SBAS current estimation content is not able to follow the ionospheric activity dynamic in a Solar cycle peak as the one we are facing.
The maps of the Fig. 2 and Fig. 3 present the behaviour of EGNOS facing ionospheric event raised on the 30/09/2011. The Fig. 2 shows the IGP monitoring availability for GEO 255. A strong performance degradation is observed on the South West of the service area. At this moment preliminary analysis have shown an high level of L1-L2 difference (greater than 25m) for RIMS A located over the Southwest ECAC area. The Fig. 3 highlights the GIVE continuity analysis for GEO 255. It is observed also a degradation of the continuity performances (in particular at South of Spain).

**ROBUST ESTIMATION OF THE TEC**

The following method proposes to estimate the ionosphere TEC in the frame of maps to be broadcast by a SBAS. This TEC is estimated by considering several estimation approaches that depend on the physical state of the ionosphere (smooth or rough). The choice of one of these approaches or another one is triggered in an adaptive manner based on automated criteria that use only the raw, non-estimated data. This proposed automated adaptive approach is however not running in open loop with, since various high-level control mechanisms are proposed. More details follow on our adaptative ionospheric electron content estimation method.

The adaptative method implements five main steps, with a the background of a mesh representation of a thin ionosphere layer.

- \(\alpha\) – Identifying whether the ionosphere is approximately constant or significantly varying
- \(\beta\) – If varying, identify whether variation is smooth or rough
- \(\gamma\) – In the case where it is smooth, take gradients into account
- \(\delta\) – In the case where it is rough, neglect gradient, but create new vertex to follow non-homogeneities.
- \(\varepsilon\) – In either \(\gamma\) or \(\delta\) cases, check the assumption by computing estimated minus measured residuals

![Fig. 2: IGP monitoring availability during the storm of the 30/09/2011](image2)

![Fig. 3: GIVE continuity analysis during the storm of the 30/09/2011](image3)
DATA COLLECTION

Once collected with a set of sensor stations, pairs of pseudo-range measurements $\rho_1$ and $\rho_2$ in two frequencies $f_1$ and $f_2$ (e.g. GPS L1 and L2 or L5). The following simplified measurement errors is computed as per:

$$\rho_i = R + \Delta h + \frac{S_{TEC}}{f_i^2}$$

Where $\Delta h$ is the difference between the receiver and transmitter clocks, $R$ is the geometric propagation distance including the troposphere delay under consideration. It is related to the corresponding centre of phase of the antenna. $S_{TEC}$ is the ionosphere electron density integrated along the (slant) path of the line-of-sight through the ionosphere. It is assumed that the slant TEC ($S_{TEC}$) is related to a local, thin-layer vertical electron content (VTEC) at the IPP through a known multiplicative factor $s(EI)$ that depends only on the line of sight elevation $EI$, and which explicit formula may be found in the literature, such as [2].

$$S_{TEC} = V_{TEC} \cdot s(EI)$$

Where $V_{TEC}$ or VTEC is the quantity to be estimated and mapped. These two formulae allow the deduction for each pair of dual-frequency pseudo-range measurement a value of the VTEC at the ionosphere pierce point.

The next step consists in the definition of an initial ionosphere modelling mesh. The VTEC will be estimated at the vertex of this mesh. The choice made in EGNOS system is to use a Delaunay triangular mesh. Our prototype is based on the same Delaunay triangular mesh even if any other form could be used for a better modelling. This use enables a comparison of the monitoring reached by our prototype with the one of EGNOS.

MONO DIMENSIONAL EXAMPLE

To highlight the processing and current weakness of the current model let’s consider the following mono-dimensional relevant example by considering a sample of spatial VTEC distribution along one axis.

The model consists in performing a piecewise linear curve through the sample in Least Square sense by fixing points on x-axis. This is shown on the following Fig. 4 where points “1, 2 & 3” are fixed.

The adjustment of point “2” in the Fig. 4 is performed with the double constraint to fit a line on the right (toward point “1”) and on the left (toward point “3”) by passing at best through the cloud of points. The process of these constraints is propagating along the fixed point on the x-axis.

Once the model is fitted it is used to determine some particular VTEC associated with regular grid point as depicted in the Fig. 5. Several computed points have VTEC values quite far from the effect we want simulated (see blue samples). It is obvious that some linearity assumptions are mandatory to take benefit of the model.

Two different issues are revealed:
- First, to compute a precise VTEC on each vertex of the model, with a low difference between truth and simulated VTEC value,
- Secondly, to have a linear evolution of the ionospheric layer to provide a relevant interpolation VTEC evaluation.

If the ionospheric layer is smooth then both precision and linearity interpolation can be achieved. In other cases, the precise VTEC estimation can be made while the interpolation can reveal high no-linearity inside some facets.

Therefore our investigations have lead to the following relevant and convenient check, designed to verify linearity behaviours of the ionospheric layer. And in case of no-linearity detection the model is adapted to regain accuracy with respect to the physicals effects of the layer.
ADAPTATIVE IONOSPHERIC ELECTRON CONTENT ESTIMATION

IONOSPHERIC SPATIAL VARIABILITY CHECK (Step α). With all the IPP in the area surrounding each ionosphere modelling mesh vertex, a histogram is built with the computed VTEC values. Two estimation processes may be required.
- In the case where the histogram displays a single group of VTEC values (broad or narrow), ionosphere is conjectured as smooth, and a gradient approach is selected in the next step (step β).
- In the case where this histogram displays well separated sub-sets of VTEC values, the ionosphere is identified as ‘rough’ or ‘piecewise constant’, and an adapted mesh by a piecewise approach is selected in the next step (step δ).

GRADIENT APPROACH (step β). In the case where ionosphere has been identified as smooth, the TEC is estimated on the vertex in a way that takes into account gradients. This requires a multi-linear modelling to manages non-local coherence of the variations of the TEC. This allows removing the assumption that the VTEC is constant around each ionosphere modelling mesh vertex. Three unknowns are estimated at each vertex, namely the value of VTEC at the vertex, and two variation slopes (e.g. along geomagnetic NS and EW directions). These three values yield the equation of the plane that models the VTEC values in the vicinity of the vertex. These three values are estimated through a least square process or any other relevant state-of-the-art estimation process.

CONTROL OF GRADIENT MODELING (step ε). With the above criterion, this approach may be triggered in two different cases.
- If ionosphere is really smooth. In this case the slopes may be either very small (single narrow group in the histogram, in which case low variances could be set on the slopes) or significant (one broad group in the histogram), but really coherent.
- If ionosphere is extremely rough (there is one broad group in the histogram, but no geographical coherence). In this case, gradient modelling is likely to be very poor.

These two cases can be distinguished after the estimation process. The control method should be the following.
- The estimation residuals are computed as the difference between the measured VTEC and the modelled ones with the plane. In a first case where the residuals are small, the computation is validated and the vertex may be used to compute ionosphere corrections to be broadcast. In another case where the residuals are large, the computation should be invalidated, and a warning on the ionosphere estimation possibility at the corresponding vertex shall be raised.
- Finally, the gradient values should connect reasonably among neighbouring vertex. The differences correspond to higher order variation terms that should not exceed a certain fraction of the TEC itself. If this is not the case, computation should be invalidated too.

PIECEWISE APPROACH (step δ). Back to the check of the ionospheric state (see step α), in the case where it is rough, it is possible to identify local families of coherent measurements to adapt the mesh by introducing new vertex to describe individually each family in the modelling of the ionosphere. This enables a reduction of the model noise introduction in the process by approaching the model to linear situation, with a very positive effect in the ionospheric modelling.

Assumption is made here that spatial gradient is high inside an open ball of fixed radius around a vertex of the model (black central vertex in Fig. 6).

Fig. 6: IPP distribution around a vertex in an open ball

The taxonomy behaviour is computed around the vertex depending on the dynamic of the ionospheric layer. The taxonomy analysis can lead to two opposite situations. In the case where the sample of measurements can be gathered around the vertex in individual class reflecting homogeneous VTEC, we consider the case δ1 described hereafter where the VTEC are aggregated. In case no homogeneous VTEC groups appear, a stochastic distribution of the measurements will be considered and the case δ2 will be applied in order to further refine the mesh.

Case δ1: aggregate VTEC distribution. For high ionospheric conditions the VTEC is function of latitude and longitude of the piercing point IPP. Some experiments show that there exist, inside the open ball, some groups of measurements with comparable VTEC value as depicted in Fig. 7. We will then consider as new vertex the centre of inertia of each class. Each IPP can possibly be weighted using the mapping function to favour the most vertical line of sight. Obviously the number of class shall be limited so as to keep a good
representative of the geometric repartition. The new model illustrated on the Fig. 8 is locally built by adding edges between old and new vertex.

Fig. 7: aggregate VTEC distribution around a vertex

Fig. 8: new modified mesh

The process includes some geometric constraints in order to avoid degenerated new facet by checking geometrical characteristic of each facet. The prototype used for the experimentations includes a check of the height of each triangular facet and doesn’t process to the mesh refinement if the height is too small. The initial value of VTEC with respect to the new vertex is computed with a classic least square process. It is the average of VTEC of the associated class.

The process aims at putting a new vertex close to each local optimum of VTEC. The benefit of this refined process is to localize local optimum following the dynamic of the ionospheric layer using the variability of each sample of measurements. The benefit is to approach to the realist ionospheric behaviour.

This behaviour has been observed in the case of 22/10/2011. Fig. 9 shows the location and vertical TEC (TECU) of the 70 IPP in the vicinity of a node that participate on the spatial gradient estimation. Three group of same magnitude are identified: one group of about 30 TECU, another (in yellow) around 60 TECU and the last (in red) around 100 TECU. In this case the process would remove the node and replace it by three new nodes located at inertia centre of the three identified group.

Fig. 9: aggregated IPP distribution

As the dynamic of the ionosphere evolve in time the previous identified groups change in location and in magnitude. The following Fig. 10 shows the situation in the neighbourhood of the same node roughly one hour after. The number of IPP to evaluate the dynamic is now 55. We have now 30 TECU of magnitude for the first group, 80 TECU for the second and 60 TECU for the last one. In case the spatial dynamic becomes smooth the rational to add new node disappear and the algorithm will run on the initial mesh.

Fig. 10: smooth ionospheric dynamic

Case 82: stochastic VTEC distribution. In case of stochastic distribution there is no geometrical privileged position to add some vertex. Thus it can be proceed to a geometric sub-division of the network locally around the mean vertex. Several methods can be applied in function of the complexity to be refined mesh targeted. The
purpose of such a sub-division is to have a refined mesh with smaller cells that verify the assumption of a constant TEC for each vertex. Two configurations are proposed here: a level-4 facet subdivision and a barycentre facet subdivision.

This first configuration consists in applying a Level-4 facet subdivision. As illustrated in the Fig. 1, the first three levels of subdivision lead to the current mesh over the whole planet. The Level-4 facet sub-division consists in performing a new subdivision of polyhedron model locally around the mean vertex as depicted in Fig. 11. Note that this refinement could be performed to any Level n+1 facet sub-division. If n is the number of different facets with the common mean vertex, the process adds 2\*n new vertex (the total number of vertex is 2\*n+n+1=3\*n+1) and 4\*n triangles.

Another possible implementation of a stochastic VTEC distribution consists in the use as new vertex of the barycentre of each triangle containing the vertex. This is depicted in Fig. 12. The number of new triangles is also reduced involving a lower complexity than with the Level-4 facet sub-division. If n is the number of different facets with the common mean vertex, the process adds n new vertex (leading to 2\*n+1 vertex) and 3\*n new triangles.

Obviously various geometric mesh refinements are possible. Among the mesh refinement possibilities that are envisaged, our prototype implements the Barycentre sub-division, since it matches the most the samples of measurements distribution.

**BOUNDARIES OF THE REFINEMENT (step ε).**

Once the need of a refinement is detected after performing the step α, it is of value to verify whether new vertex and facet improve the electron content computation. Indeed each refined mesh vertex may contain fewer samples than the original mesh.

![Fig. 11: Level 4 facets sub-division](image1)

![Fig. 12: barycentre sub-division](image2)

The insight is to recalculate the spatial variability on each new vertex (or new facet following the proposed relevant check) taking into account that less of points are available to estimate the figure. In term of variance a criteria can be formulated as follow. The refinement improves the computation if the following condition is met:

\[ \sigma_k \chi^2_{n_k}(k_i) < \sigma_{k_i} \chi^2_{n_{k_i}}(k) \]

Where \( \sigma_k \) the spatial variability of a vertex k computed

Where \( \sigma_{k_i} \) the spatial variability of new vertex \( k_i \)

\( n_k \) and \( n_{k_i} \) are the number of points used for the computation of the respective vertex.

Where the \( \chi^2 \) parameter is intended to weight the variances with respect to number of point used for evaluating.

This number of point defines the degree of freedom of the law. In the case where several iterated refinement are on going the process will be stopped as soon as distance between on new vertices and others around are inferior to the new one and pierce point around.

The adaptative ionospheric electron content estimation method has been described. It consists in an adaptation of the well-known TRIN model. This adaptation is not a densification of the vertex that has been already analysed without success (Ref [3]). This Adaptative Ionospheric electron content estimation method is focused on a track to the local extrema, in order to stick better to the variations. This is well illustrated on the Fig. 13 and Fig. 14 compared to Fig. 4 and Fig. 5.

**PROTOTYPING ACTIVITIES**

The method here described is being implemented it in a prototype of ionospheric computation for SBAS. The purpose is to perform test is with recorded data including high ionospheric activity. The results are promising even if not consolidated enough to be detailed hereafter.
Therefore the prototyping activities are details here together with deepen considerations.

The benefit of the method is to approach to the realist ionospheric behaviour. The refined mesh with an aggregate VTEC distribution (case $\delta_1$) targets the points (blue points on the Fig. 5) raised above to have a good estimation of VTEC on each new vertex and to have a linear VTEC evolution on each new facet (see Fig. 13).

![Fig. 13: aggregate mesh around local optima](image)

The sub-divisions according to a stochastic VTEC distribution (Case $\delta_2$) intend to obtain a sub-set of facet where the evolution of VTEC is near to linear. The situation is illustrated in 1-dimension by the refinement of the x-axe to approach a curve (see Fig. 13).

![Fig. 14: stochastic mesh improvement of the TEC](image)

Moreover it appears necessary to control the evolution of mesh along time. Indeed the flexible TRIN model shall be refined in case of need (e.g. high spatial gradient) but also it shall come back in initial state when the ionospheric event disappears.

Once the model is refined a trade-off is conducted between two options: the first option is to allow the engagement of a new refinement whether the spatial variability is again considered as too high but inferior to the previous one. The system is intended to control the number of new vertex in order to limit the time of computation. A second check of gradient variability can be performed on new vertex to evaluate the level of linearity as already discussed above. If the variability remains considered as too high but inferior to the previous one, then a new subdivision can be performed. The second option consists in not allowing any deeper refinement. This is illustrated on the following Fig. 15, by one possible implementation.

![Fig. 15: control of the flexible TRIN model](image)

In addition, the process is to be limited by a trade-off between the complexity of the network and the number of IPP closed each vertex to feed the Kalman process. The benefit of refinement is to decrease the process model noise of the Kalman filter. But the measure noise has also to be decreased because difference between truth and simulated VTEC is intended to become lower. The value for measurement process noise can be chosen in function of the temporal variability of VTEC on the vertex, especially after a diurnal variation (after a large period without IPP).

It could remain after the piecewise modelling approach that the level of VTEC variability in the different groups is too high to allow a safe estimate of the VTEC in some group. To manage this aspect, the control method of the piecewise modelling (see idea $\epsilon$ above in the case of piecewise constant VTEC) may be the following.

- For each group resulting in a new vertex, the estimation residuals are computed as the difference between measured VTEC and the piecewise modelled ones.
- If at new vertex, the residuals are small (this may be seen through a variance), the computation is validated and the corresponding vertex may be used to compute ionosphere corrections to be broadcast.
- If at new vertex, the residuals are large (this may be seen through a variance), the computation should be invalidated, and a warning on the ionosphere estimation possibility at the corresponding vertex shall be raised.
CONCLUSION
The Adaptative ionospheric electron content estimation method briefly presented in the above article seems very promising for ionospheric delays estimation. Indeed it enables to track to the local extrema, in order to follow the variations of ionospheric activity where the current SBAS models lose the tracking.
This method is based on five steps that enable to adapt the computation of the ionosphere TEC when it is significantly varying or not; when the variation is smooth or rough.
At the time of the publishing this article, the method is under prototyping and testing. Since it seems promising, the method is presented from now the method whereas a next article will present the results obtained.

REFERENCES