A MINI AERIAL VEHICLE AS A SUPPORT FOR MODELLING, CONTROL AND ESTIMATION TEACHING

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Abstract—At ENSICA several teams of students developed a mini aerial system that is now mature and is used for teaching. The system includes an original delta wing configuration able to bring a complete avionics system (sensors, actuators and controller) and a micro camera. We describe how a non linear model is obtained from wind tunnel tests and a linear state space model is then obtained. These models can be used for basic control theory. The board originally designed at ENSICA includes enough sensors to estimate the attitude and position of the plane. It is used at ENSICA as a basis for Kalman filtering theory teaching.

Keywords: Mini Aerial Vehicle, Modelling, Attitude control, Sensors, Strapdown Inertial System, Kalman Filtering

1. INTRODUCTION

PEGASE is a Mini aerial vehicle (MAV) system which was developed by ENSICA [5]. This project is close to many others like the Dragonfly project at Stanford University [2] and the miniature acrobatic helicopter of MIT [5]. Due to its small dimensions, the plane has a very fast dynamics and therefore is hard to pilot. The payload of PEGASE consists in a miniature video camera that we use to teleoperate the plane and a microcontroller surrounded by sensors and actuators. PEGASE can be operated at three levels of autonomy: fully manual, manually controlled with an inner stabilization loop and fully autonomous. Actually this system is the achievement of various students’ projects and is now used for teaching and research.

We will describe first the short story of this project and give a detailed description of the system. In a second part we will give some results about the model of the plane and its control. In the last part we describe how this system is used at ENSICA as a support for a Kalman Filtering course.

2. WHERE PEGASE COMES FROM

Founded in Paris in 1945, ENSICA is one of the French aerospace schools. ENSICA trains multidisciplinary engineers capable, at outcome, of controlling complex projects within an international context mainly in the aerospace industry. During their three year stage at ENSICA, students have the opportunity to lead a personal project of about 1500 hours. Teams of two to ten students benefit from the assistance of ENSICA’s professors and technician’s staff and facilities (wind tunnels, mechanical engineering, electronics, etc...)

The PEGASE project started in 1999 when a student’s project led to a demonstration of live video transmission from an RC plane. Since then a total of about “4 student-years” were dedicated to several projects related to mini and micro UAV. These efforts have now been capitalized into the PEGASE project.

3. WHAT PEGASE IS

PEGASE has a delta-wing configuration with two ailerons. The general characteristics of PEGASE are as follows:
- Wing span : \( b=0.5m \)
- Length : \( L = 0.34m \)
- Wing area : \( S_{ref} = 0.0925m^2 \)
- Aerodynamic mean chord : \( c = 0.185m \)
- Speed of cruising : \( V_0 = 20m.s^{-1} \)

The plane is able to bring a payload of approximately 300gr. It is equipped with a video camera and a board including sensors, actuators, communication and microcontroller.
Figure 1: PEGASE synoptic

Synoptic of the complete system is given in Figure 1. Sensors are: a set of three gyroimeters, a set of three accelerometers, a three axis magnetometer, a micro GPS and a temperature sensor. Gyroimeters and accelerometers are basically analogical devices and are sampled at 100 Hz. The magnetosensor is sampled at about 13Hz while the GPS is sampled at 1Hz. Future versions will include static and dynamic pressure sensors. Actuators are the two control surfaces (“aileron”) and the thrust. The plane is electrically powered. “RC” denotes the classical Radio Command system (emitter and receiver). “HF” denotes a 173MHz 19200 Bds downlink transmitter and receiver system. Next version will include a half duplex uplink and downlink system. Data is transferred and stored into a laptop via the serial link. Sensors, actuators and HF communication are connected to a single Motorola 68332 microcontroller. Total weight (included into the dashed line in Figure 1) is only 175gr.

The plane is either controlled like a classical RC plane or teleoperated via a video feedback from a miniaturised camera and video transmitter weighing only 10gr. The video feedback is displayed either on a portable video recorder or on video glasses.

Figure 2: example of GPS trajectory

Figure 2 shows an example of the plane trajectory during a short flight test. Position is obtained via the GPS, pre-processed into the onboard microcontroller and sent in real time to the ground station for display. All sensors output are sent to the ground station at a limited sampling rate of 13Hz due to the limited bandwidth of the transmission link.

This system is fully derived from students projects in which fundamentals of our aeronautic syllabus were involved:
- Flight dynamics,
- Sensors,
- Real time system.

4. MODELING AND CONTROL

In order to obtain a mathematic model of the MAV for the design and evaluation of guidance and control law, wind tunnel test has been carried out (Figure 3).

At a fixed airspeed of 20m/s, three force and moment coefficients where measured at various angle of attack (α from -30° to +30°), sideslip angle (β from 0° to 45°) and control surfaces (elevator δe from -5° to +5°, aileron δa from -5° to +5°). With these data and inertial moment of the MAV, by proper interpolation, the full degree nonlinear model of PEGASE can be obtained.

Figure 3: PEGASE during the wind tunnel test

To fulfill the mathematic model, force and moment coefficients are needed. They are multi-dimensional functions of angle of attack, side-slip angle and control surfaces, and can be obtained by interpolation of the discrete values from the wind tunnel test. From the six coefficients obtained above and the evaluation of inertial moment, a standard twelve-degree nonlinear equation can be derived.

A Matlab/Simulink full non linear model of PEGASE is made available for simulation.

It is not convenient to use the nonlinear equation to design guidance and control law; therefore, a linear state space model is computed from the full non linear model at a level straight flight condition:

\[ V_x = 20 \text{ m s}^{-1}, V_y = V_z = 0, \theta = \phi = \Psi = 0° \]
\[ \delta_e = -\delta^a, \delta_a = -\delta^e \]
where $V_s$, $V_z$, $V_e$ are velocities of MAV and $\theta$, $\psi$, $\varphi$ are pitch angle, heading angle and rolling angle. The linear equations of this flight state are:

$$
\begin{align*}
\dot{q} &= -0.02V_x - 2.44V_z - 1.81\delta_e \\
\dot{\theta} &= q \\
p &= -1.47\delta_o \\
\dot{\varphi} &= p
\end{align*}
$$

where $p$ is rolling angular rate, $q$ is pitch angular rate. The yaw angle can only be adjusted by controlling the rolling angle because only two control variables are available. Therefore a very simple attitude control law can be obtained:

$$
\begin{align*}
\delta_\omega &= -0.55 k_{11}[k_{12}(\theta_d - \theta) - q] \\
\delta_o &= -0.68 k_{22}[k_{22}(\varphi_d - \varphi) - p]
\end{align*}
$$

where $k_{11}$, $k_{12}$, $k_{21}$, $k_{22}$ are control parameters to guarantee enough bandwidth, $\theta_d$ and $\varphi_d$ are the desired pitch and roll of the MAV.

From the nonlinear simulation, conclusions can be drawn:
- The close loop attitude system is stable, the control law is efficient.
- Heading can be controlled by adjusting rolling angle.

For teaching purpose, the linear straight flight model can be used to design basic controllers. The controller can be then adapted to the Matlab/Simulink complete non linear model. At a basic level of control theory course, these models are a very good illustration of fundamental concepts:
- linearization,
- coupling and decoupling,
- state space control theory,
- non linearity due to actuator saturation and rate limiters,
- etc...

We plan to improve this tutorial approach with real flight tests and flight data analysis.

5. KALMAN FILTER DESIGN FOR ATTITUDE DETERMINATION

In order to control the attitude of the aircraft, three different angles (roll, pitch and yaw) must be determined. These angles are estimated from the fusion of sensors measurement into a very basic Kalman Filter as described for instance into [1]. This algorithm is used as a basis for Kalman Filtering teaching at the high level of control theory course at ENSICA.

For this course we focus in a generic attitude measurement system. Equations are derived from [5] and [3]. We only consider two frames: the earth frame, supposed to be Galilean, and the body frame. As we make no assumption on the aircraft dynamics, the system dynamic model is, in the discrete case:

$$
\begin{align*}
Q_{k+1} &= Q_k + \frac{1}{2}T_e \Omega_k Q_k + T_e w_Q \\
\omega_{k+1} &= \omega_k + T_e \omega_\alpha \\
\alpha_{k+1} &= \alpha_k + T_e \omega_\alpha \\
e_k &= e_k + T_e w_e \\
with
\Omega_k &= \begin{bmatrix}
0 & -p_k & -q_k & -r_k \\
p_k & 0 & r_k & -q_k \\
q_k & -r_k & 0 & p_k \\
r_k & q_k & -p_k & 0
\end{bmatrix}
\end{align*}
$$

If we consider $X_k$ the complete state $X_k = [Q_k, \omega_\alpha, \alpha_\alpha, \omega_\alpha]^T$ the equation above becomes:

$$
X_{k+1} = f(X_k) + T_e w_k
$$

Figure 4: Simulation results for attitude control

Nonlinear simulation has been done to verify the efficiency of attitude controller. The Figure 4 shows the step input response of pitch angle and rolling angle. At the same time, the variation of angle of attack, side-slip angle, control surfaces, heading angle and height are also given.
$T_e$ is the sample time. $Q_k$ is the attitude (attitude of body frame relative to fixed frame, expressed in fixed frame) of the aircraft as a quaternion. $Q_k = [q_0, q_1, q_2, q_3]^T$ is therefore a $4$-dimension vector. Transformation from quaternion to Euler angles is straight-forward and is not described here. $\alpha_k = [f_1, f_2, f_3]^T$ is the $3$-dimension vector of angular rates of turn (rate of turn of body frame relative to fixed frame, expressed in body frame). The state vector is augmented with two additional states: $\alpha$ is the drift of the accelerometer and $\varepsilon$ is the drift of the gyroimeters. For the design of the Kalman filter we must take into account a set of disturbance w (with subscript corresponding to each state) suppose to be Gaussian white noise with covariance $Q$.

The measurement model is as following:

$$
\begin{align*}
A_k &= C_k^T (\Omega_k + g) + \alpha_k + v_A \\
G_k &= \Omega_k + \alpha_k + v_G \\
M_k &= C_k M_k + v_M
\end{align*}
$$

where:

$$
C_k^T = 
\begin{bmatrix}
a^2 + b^2 - c^2 - d^2 & 2(bc + ad) & 2(bd - ac) \\
2(bc - ad) & a^2 - b^2 + c^2 - d^2 & 2(cd + ab) \\
2(bd + ac) & 2(cd - ab) & a^2 - b^2 - c^2 + d^2
\end{bmatrix}
$$

$C_k^T$ is the transformation matrix (from fixed frame to body frame) and is related to the quaternion as the equation above states. $A_k$ is the vector of three accelerometer outputs. As seen in the equation above, this measurement is related to the earth gravity $g = [0, 0, 9.81]^T$ and to the actual acceleration $\ddot{x}$ of the airplane's center of gravity. We make here the assumption that this acceleration is mean null$^1$ and all derivatives fall into $v_A$. $\alpha_k$ is the drift of the accelerometer. $G_k$ is the vector of three angular rates measurement. The technology of this gyroimeters (cheap miniaturized piezoelectric vibrating gyroimeters from Murata) makes them very sensitive to temperature: the drift $\varepsilon$ is quite important and must be estimated online. The last measurement vector is $M_k$ the three components of earth magnetic field. $v_M$ is the vector of earth magnetic field in earth frame. $v$ (with appropriate subscript) is the measurement noise supposed to be Gaussian white noise.

If we consider $Y_k$ the complete measurement vector $Y_k = [A_k, G_k, M_k]^T$ the equation above becomes:

$$
Y_{k-1} = h(X_k) + v_k
$$

Let us just give the very well known Kalman Filter equations that makes the best estimate of $X$. (The subscripts $k$ and $k+1$ are now omitted as we describe here the algorithm, the equation $X_{k+1} = f(X_k)$ is written $X = f(X)$)

**Algorithm 1**

**Step 1**: compute the predicted state estimate:

$$
\hat{x} = f(\hat{x})
$$

**Step 2**: compute the covariance matrix extrapolation:

$$
P = P \cdot P^T + Q
$$

**Step 3**: compute the Kalman gain:

$$
K = P \cdot C^T \cdot (C \cdot P \cdot C^T + R)^{-1}
$$

**Step 4**: compute the covariance matrix update:

$$
P = P - K \cdot C \cdot P
$$

**Step 5**: compute the state estimate update with measurement $Y$:

$$
\hat{x} = \hat{x} + K \cdot (Y - h(\hat{x}))
$$

$F$ and $C$ are respectively the Jacobian matrixes of $f$ and $h$.

The algorithm that we actually use is described in Figure 5 and derived from [1].

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$^1$ This assumption leads to a good estimation of the attitude as long as the hypothesis is true. It is not true for circling flight.
Algorithm 2

Step 1: compute the predicted state estimate:

\[ \hat{X} = f(\hat{X}) \]

Step 2: compute the covariance matrix extrapolation:

\[ P = F.P.F^T + Q \]

Step 3i: compute the Kalman gain, corresponding to the arrival of measurement \( n \)

\[ K = P.C_i^T (C_i P C_i^T + R) \]

\( C_i \) is now the \( i \)-th line of matrix \( C \). \( R_i \) is the \( i \)-th element of matrix \( R \). The strong assumption is that \( R \) is diagonal i.e. that measurement noises are uncorrelated.

Step 4i: compute the covariance matrix update:

\[ P = P - K C_i \]

Step 5i: compute the state estimate update with measurement \( Y_i \):

\[ \hat{X} = \hat{X} + K(Y_i - h(\hat{X})) \]

\( Y_i \) is the \( i \)-th measurement.

Algorithm 2 is proven to be equivalent to algorithm 1 as long as the matrix \( R \) of noise measurement covariance is diagonal.

Compared to algorithm 1, algorithm 2 as two advantages:
- Step 3i is not a matrix inversion but a scalar inversion,
- Measurement can be asynchronous.

Actually algorithm 1 cannot be used because of two reasons: huge matrix inversion cannot be processed in real time by the microcontroller and measurements are basically asynchronous.

For teaching purpose, the algorithm is done offline with Matlab routines.

Figure 7 shows the convergence of the algorithm. Only the gyrometer output (scale \(^\circ/\mathrm{sec}\)) is drawn but accelerometers and magnetosensors are included into the algorithm. One can see that the gyro output is not centered at 0.

About 30s of convergence is necessary to estimate the gyro drift. One can see the estimated angular rates that converge to 0 \(^\circ/\mathrm{s}\) after 30s. At 38s a few +/-90\(^\circ\) rotations are made along the three axes and the attitude quaternion is estimated. After computation, the attitude is visualized in an animation (Figure 6).

For teaching purpose, students have the opportunity to work with a real attitude system. They can manage everything from the sensor output to the algorithm and drive their tests. Important notions of Kalman Filtering Theory and INS theory are illustrated in this course:
- Basis of Kalman Filtering. (For instance: filter is tuned by \( Q \) and \( R \)).
6. CONCLUSION

Mini UAV projects provide efficient yet inexpensive capabilities of conducting air vehicle experimental research and teaching. Energy, sensors and actuators miniaturization make now possible to reach very small dimensions and prices. In this paper we focused in the teaching aspects at an aeronautical engineer’s level. The PEGASE project can be a starting base for improvements by student’s projects. It is also used for academic teaching. Of course this system is rich and versatile enough to serve research experiments. Our current research interests are teleoperation, telesupervision and telepresence in a 3D and high dynamics environment, and advanced control of very small aircrafts.

REFERENCES