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Logics for MAS: a critical overview

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Introduction
Introduction

Multi-Agent Systems (MAS):
agents with imperfect knowledge
perform actions
in order to achieve goals

- philosophical logic/KR view:
  - what are the main concepts?
  - what properties do they have?
  - how do they relate?

- formal, logical analysis
  ⇒ logics of action and knowledge
  ⇒ extensions of propositional logic by modal operators
Introduction: modal operators of knowledge

- **knowledge of individual** $i \in \text{Agt}$ :  
  
  $K_i \varphi = \text{“agent } i \text{ knows that } \varphi\text{”}$

- **knowledge of group** $J \subseteq \text{Agt}$ :  
  
  $E_{KJ} \varphi = \text{“it is shared knowledge in } J \text{ that } \varphi\text{”}$
  
  $= \text{“every agent in } J \text{ knows that } \varphi\text{”}$

  $C_{KJ} \varphi = \text{“it is common knowledge in } J \text{ that } \varphi\text{”}$
  
  $= E_{KJ} \varphi \land E_{KJ} E_{KJ} \varphi \land E_{KJ} E_{KJ} E_{KJ} \varphi \land \cdots$  

  $D_{KJ} \varphi = \text{“it is distributed knowledge in } J \text{ that } \varphi\text{”}$
  
  $= \text{“if each agent in } J \text{ tells all he knows to } J \text{ then } C_{KJ} \varphi\text{”}$
Introduction: modal operators of action and ability

- nonstrategic (ceteris paribus)

\[ \langle \pi \rangle \varphi = \text{“there is an execution of program } \pi \text{ after which } \varphi \text{”} \]
\[ \langle J \rangle \varphi = \text{“coalition } J \text{ can achieve } \varphi \text{ (while opponents don’t act)”} \]

- strategic (‘ceteris agentis’, ‘ceteris mutandis’)

\[ \langle\langle J \rangle \rangle \varphi = \text{“coalition } J \text{ can achieve } \varphi \text{ (whatever opponents do)”} \]
\[ \text{Stit}_J \varphi = \text{“coalition } J \text{ achieves } \varphi \text{ (whatever opponents do)”} \]
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Introduction: the grid of MAS logics

- aim of talk: overview the main MAS logics and highlight problematic points
  - KR point of view: which logical language?
  - semantic-free

- the grid of MAS logics:

<table>
<thead>
<tr>
<th>Knowledge / Action</th>
<th>S5^C</th>
<th>PAL^C</th>
<th>ATEL^C</th>
</tr>
</thead>
<tbody>
<tr>
<td>No uncertainty</td>
<td>S5</td>
<td>PAL</td>
<td>ATEL</td>
</tr>
<tr>
<td>Knowledge</td>
<td>PDL, CL-PC</td>
<td>ATL</td>
<td></td>
</tr>
<tr>
<td>Action</td>
<td>no actions</td>
<td>nonstrategic</td>
<td>strategic</td>
</tr>
</tbody>
</table>
1. No uncertainty, nonstrategic actions
2. No uncertainty, strategic actions
3. Individual knowledge, no actions
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5. Individual knowledge, strategic actions
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7. Group knowledge, nonstrategic actions
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No uncertainty, nonstrategic actions: PDL

- language of Propositional Dynamic Logic PDL:
  \[ \langle \pi \rangle \varphi = \text{“there exists a possible execution of } \pi \text{ after which } \varphi” \]
  \[ [\pi] \varphi = \text{“for every possible execution of } \pi \ldots” \]
  where \( \pi \) is a program (alias complex action):
  \[ \pi ::= a | \pi; \pi | \pi \cup \pi | \pi^* | \varphi? \]
  \[ \Rightarrow \text{“while } \varphi \text{ do } \pi” = (\varphi?; \pi)^*; \neg \varphi? \]

- in focus: reasoning about action/program effects
  \[ (\text{ActionTheory} \land \text{Init}) \rightarrow \langle a_1; \cdots; a_n \rangle \text{Goal} \]
PDL action theories must be augmented by frame axioms

\[ \text{BlockRed} \rightarrow [\text{moveBlock}_{L_1,L_2}] \text{BlockRed} \]

\[ \Rightarrow \text{PDL doesn’t solve the frame problem} \quad [\text{McCarthy & Hayes 1969}] \]

a lot of dedicated logical formalisms

SitCalc, EventCalc, FluentCalc, \( A, B, C, C+, BC \), separation logic, …

SitCalc basic action theories [Reiter 1991]:

\[ \forall x ([x] \text{BlockRed} \leftrightarrow (x = \text{paintRed} \lor (\text{BlockRed} \land x \neq \text{paintBlue}))) \]
No uncertainty, nonstrategic actions: PDL

- PDL action theories must be augmented by frame axioms
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  SitCalc, EventCalc, FluentCalc, \( \mathcal{A}, \mathcal{B}, C, C+, BC \), separation logic, …
- SitCalc basic action theories \([\text{Reiter 1991}]\):
  \[ \forall x ([x]\text{BlockRed} \leftrightarrow (x = \text{paintRed} \lor (\text{BlockRed} \land x \neq \text{paintBlue}))) \]
DL-PA: a dialect of PDL solving the frame problem

- Reiter’s basic action theories can be expressed in Dynamic Logic of *Propositional Assignments* DL-PA
  
  \[\text{BlockAt}_{L_1} := \bot\]
  
  - atomic programs: assign propositional variables to formulas
  
  - successor state axioms become DL-PA programs:
    
    \[\text{moveBlock}_{L_1,L_2} = (\text{Free?}; \text{BlockAt}_{L_1} := \bot; \text{BlockAt}_{L_2} := \top)\]
    
    hyp.: in \(\forall x ([x]p \leftrightarrow \gamma_p(x))\), if \(a \notin \gamma_p(x)\) then \(\gamma_p(a) \leftrightarrow p\)

- nice properties
  
  - complexity of satisfiability just as PDL
  
  - model checking as complex as satisfiability checking
  
  - Kleene star eliminable
  
  - every formula reducible to a boolean formula

- claim: DL-PA = Assembler language for logics of change...
No uncertainty, nonstrategic actions: CL-PC

- **language of Coalition Logic of Propositional Control CL-PC:**
  \[ \langle J \rangle \varphi = \text{“coalition } J \text{ can achieve } \varphi \text{ by modifying its variables (while opponents don’t act)”} \]

- each propositional variable *controlled* by some agent;
  action of \( i \) = change of some of \( i \)'s variables (cf. bool. games)

  [van der Hoek & Wooldridge, AIJ 2005; JAIR 2010]

- in focus: reasoning about nonstrategic (ceteris paribus) ability

  \[ (\text{AbilityTheory } \land \text{ Init}) \rightarrow \langle \{i_1, \ldots, i_n\} \rangle \text{Goal} \]
No uncertainty, nonstrategic actions: CL-PC

- captures strategic ability
  \[ \langle J \rangle \overline{J} \varphi = "J \text{ can achieve } \varphi \text{ whatever the opponents in } \overline{J} \text{ do}" \]

- can be embedded into DL-PA:
  \[ \langle i \rangle \varphi = \langle \pi_{i,\varphi} \rangle \varphi \]

  with \( \pi_{i,\varphi} \) polynomial in \( \varphi \)

[H et al., IJCAI 2011]
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No uncertainty, strategic actions: ATL

- **Language of Alternating-time Temporal Logic ATL:**
  - $\langle J \rangle X \varphi = "\text{the agents in } J \text{ have a strategy such that whatever the other agents do, next } \varphi"$
  - $\langle J \rangle G \varphi = "\ldots, henceforth } \varphi"$
  - $\langle J \rangle \varphi U \psi = "\ldots, } \varphi \text{ until } \psi"$

- **In focus:** reasoning about the existence of strategies

  $\text{(AbilityTheory } \land \text{ Init}) \rightarrow \langle \{i_1, \ldots, i_n\} \rangle \text{Goal}$
ATL: the problem of strategy revocability

- problem: strategies can be canceled
  \[⟨⟨i⟩⟩G(married ∧ ⟨⟨i⟩⟩X¬married)\] is satisfiable
  \[⇒\] reason: strategies are “unsung heroes” [van Benthem]

- solution: commit to a strategy
  - ATL with irrevocable strategies [Ågotnes et al., TARK 2007]
  - ATL with strategy contexts [Brihaye et al., LFCS 2009]
    - make adoption and canceling of strategies explicit
    - undecidable [Troquard & Walther, JELIA 2012]
  - Strategy Logic (SL) [Mogavero et al., FSTTCS 2010]
    - uses strategy variables; undecidable
  - ATL with explicit strategies [Walther et al., TARK 2007]
    \[⟨⟨{i}⟩⟩_i:σ G(married ∧ ⟨⟨{i}⟩⟩_i:σ X¬married) \rightarrow ⊥\]

- more principled: commit to an action
  - ATLEA = ATL + Explicit Actions [H, Lorini & Walther, LORI 2013]
    \[⟨⟨{i}⟩⟩_i:staymarried^∞ G(married ∧ ⟨⟨{i}⟩⟩_i:staymarried^∞ X¬married) \rightarrow ⊥\]

- same complexity as ATL
No uncertainty, strategic actions: STIT

- **Language of Seeing-To-It-That Logic STIT**
  
  \[ \text{Stit}_J \varphi \quad = \quad \text{“by following their current strategy the agents in } J \text{ guarantee that } \varphi \text{ is true, whatever the other agents do”} \]

  \[ \Diamond \varphi \quad = \quad \text{“it is historically possible that } \varphi \text{”} \]

  \[ \mathcal{F} \varphi \quad = \quad \text{“...” (temporal operators)} \]

- **In focus: reasoning about causality (‘agency’)**

  \[ \text{Cond} \rightarrow \text{Stit}_{\{i_1, \ldots, i_n\}} \text{ Fact} \]

- **Reasoning about strategic ability à la ATL:**

  \[ \langle J \rangle X \psi = \Diamond \text{Stit}_J X \psi \]

- **Satisfiability undecidable**

  [H & Schwarzerentruber, AiML 2008]
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Individual knowledge, no actions

- language of modal logic S5:
  \[ K_i \varphi = \text{“agent } i \text{ knows that } \varphi \text{ is true”} \]

- principles
  - \[ K_i \top \] (omniscience)
  - \[ (K_i \varphi \land K_i (\varphi \rightarrow \psi)) \rightarrow K_i \psi \] (omniscience)
  - \[ K_i \varphi \rightarrow \varphi \] (knowledge implies truth)
  - \[ K_i \varphi \rightarrow K_i K_i \varphi \] (positive introspection)
  - \[ \neg K_i \varphi \rightarrow K_i \neg K_i \varphi \] (negative introspection)

- “the” logic of knowledge?
  - generally adopted in AI
  - but. . .
Individual knowledge, no actions

- Language of modal logic S5:
  \[ K_i \phi = \text{“agent } i \text{ knows that } \phi \text{ is true”} \]
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- “The” logic of knowledge?
  - Generally adopted in AI
  - But...
Individual knowledge 🤗, no actions

- negative introspection axiom $\neg K_i \varphi \rightarrow K_i \neg K_i \varphi$ too strong

1. suppose $B_i K_i p$
   - $i$ strongly believes to know $p$
   - should not imply $K_i p$

2. suppose $\neg p$

3. then $\neg K_i p$ (knowledge implies truth)

4. then $K_i \neg K_i p$ (neg. introspection)

5. then $B_i \neg K_i p$ (knowledge implies belief)

6. $\bot$ (belief consistent)

$\Rightarrow (B_i K_i p \land \neg p) \rightarrow \bot$ ?!

- logic of knowledge should rather be S4.2

$\Rightarrow$ dynamic epistemic logics get more involved...
Individual knowledge 🤖, no actions

- negative introspection axiom $\neg K_i \varphi \rightarrow K_i \neg K_i \varphi$ too strong
  
  [Lenzen 1978, Voorbraak 1993]

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Individul knowledge ⬇️, no actions

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Individual knowledge $\square_i$, nonstrategic actions:

**PAL**

- Public Announcement Logic PAL
  \[\langle \psi! \rangle \varphi = \text{“the truthful public announcement of } \psi \text{ can be made and } \varphi \text{ will be true afterwards”}\]

- reduction axioms (aka regression):
  \[\langle \psi! \rangle p \leftrightarrow \psi \land p\] facts don’t change (epistemic change only)
  \[\langle \psi! \rangle K_i \varphi \leftrightarrow \psi \land K_i[\psi!] \varphi\]

- complexity of satisfiability:
  - same as underlying epistemic logic
  - but more succinct

[Lutz, AAMAS 2006]
[French et al., IJCAI 2011]
Individual knowledge $\Box$, nonstrategic actions: the problem of closure under updates in PAL

- most papers choose S5 as the logic of knowledge
  - others adopt K for generality
- S5-based PAL ‘works’ because the set of S5 models is closed under updates by announcements
  - holds also in modal logic K
- fails in logic of belief KD45 and in logic of knowledge S4.2
  
  \[\text{[Balbiani, van Ditmarsch & H, AiML 2012]}\]

- reason: confluence node may be eliminated by update

\[
\begin{align*}
\{p\} & \xrightarrow{R} \emptyset \\
\{p\} & \xrightarrow{R} \{p\} \\
\{p\} & \xrightarrow{R} \{p\}
\end{align*}
\]

\[
\begin{align*}
p! & \quad \implies \quad \{p\} \\
\{p\} & \xrightarrow{R} \{p\}
\end{align*}
\]

- similar problem with other modal logics
Individual knowledge, nonstrategic actions: variants of PAL

- **DEL = Dynamic Epistemic Logic**  
  - agents perceive events only incompletely  
  - event models  
  [Baltag & Moss, Synthese 2004]

- **GAL = PAL plus Group announcements**  
  - $\langle J \rangle \varphi = \"J can achieve $\varphi$ by announcing some known formulas\"$  
  - cf. ATL, CL  
  [Ågotnes et al. 2010]

- **APAL = PAL plus Arbitrary announcements**  
  - $\langle ! \rangle \varphi = \"there is a $\psi$ such that $\langle \psi! \rangle \varphi\"$  
  [Balbiani et al., RSL 2008]
Individual knowledge 🤔, nonstrategic actions: the problem of uniform choices in APAL

- You don’t see B’s and C’s cards, and they only see their cards.
- Among the ace of spades and the ace of clubs, B has one and C has one, but You don’t know who has which.
- You want agent B to know both Spades and Clubs, but not C.
- Is there a public announcement doing the job?
Individual knowledge \( K \), nonstrategic actions: the problem of uniform choices in APAL

\[
Init = K_Y \text{Spades} \land K_Y \text{Clubs} \land K_Y (K_B \text{Spades} \land \neg K_C \text{Spades}) \lor (K_B \text{Clubs} \land \neg K_C \text{Clubs})
\]

\[Goal = K_B (\text{Spades} \land \text{Clubs}) \land \neg K_C (\text{Spades} \land \text{Clubs})\]

• provable in PAL:

\[
(K_B \text{Spades} \land \neg K_C \text{Spades}) \rightarrow (\text{Spades} \rightarrow \text{Clubs!}\)Goal

\[
(K_B \text{Clubs} \land \neg K_C \text{Clubs}) \rightarrow (\text{Clubs} \rightarrow \text{Spades!}\)Goal
\]

• so \( Init \rightarrow K_Y \langle \exists! \rangle Goal \), … but you don’t know what to say!

• in Group Announcement Logic GAL:

\[K_Y \langle \{Y\} \rangle \varphi \text{ vs. } \langle \{Y\} \rangle K_Y \varphi\]
Individual knowledge $\diamondsuit$, nonstrategic actions: the problem of uniform choices in APAL

- in S5:

  \[ \text{Init} = K_Y \text{Spades} \land K_Y \text{Clubs} \land K_Y \left( (K_B \text{Spades} \land \neg K_C \text{Spades}) \lor (K_B \text{Clubs} \land \neg K_C \text{Clubs}) \right) \]

  \[ \text{Goal} = K_B (\text{Spades} \land \text{Clubs}) \land \neg K_C (\text{Spades} \land \text{Clubs}) \]

- provable in PAL:

  \[ (K_B \text{Spades} \land \neg K_C \text{Spades}) \rightarrow \langle \text{Spades} \rightarrow \text{Clubs}! \rangle \text{Goal} \]

  \[ (K_B \text{Clubs} \land \neg K_C \text{Clubs}) \rightarrow \langle \text{Clubs} \rightarrow \text{Spades}! \rangle \text{Goal} \]

- so \( \text{Init} \rightarrow K_Y \langle \exists! \rangle \text{Goal} \), … but you don’t know what to say!

- in Group Announcement Logic GAL:

  \[ K_Y \langle \{ Y \} \rangle \varphi \text{ vs. } \langle \{ Y \} \rangle K_Y \varphi \]
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Individual knowledge ♦, strategic actions

- Alternating-time Temporal Epistemic Logic ATEL
  - \([\text{van der Hoek \\ Wooldridge, Studia Logica 2003}]
  - \(\ll J \gg \varphi = \text{"coalition } J \text{ can achieve } \varphi \text{ (whatever opponents do)"} \)
  - \(K_i \varphi = \text{"agent } i \in \text{Agt} \text{ knows that } \varphi \text{"} \)

- problem of uniform strategies [Schobbens, ENTCS 2004]
  - same as problem of uniform choice for APAL, v.s.
  - \(K_i \ll i \gg X^{\text{safeOpen}} \)

- solution in ATELEA = ATEL with Explicit Actions
  - \(K_i \ll i \gg i: \text{dial}_{1234} X^{\text{safeOpen}} \)
Individual knowledge 人工智能, strategic actions

- Alternating-time Temporal Epistemic Logic ATEL
  
  \[ \langle\langle J\rangle\rangle \varphi = \text{“coalition } J \text{ can achieve } \varphi \text{ (whatever opponents do)"} \]
  
  \[ K_i \varphi = \text{“agent } i \in \text{Ag} \text{t knows that } \varphi \" ]

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  - same as problem of uniform choice for APAL, v.s.

\[ K_i \langle\langle i\rangle\rangle X \text{safeOpen} \]

- solution in ATELEA = ATEL with Explicit Actions

\[ K_i \langle\langle i\rangle\rangle_{i:dial_{1234}} X \text{safeOpen} \]
Individual knowledge 🧑, strategic actions

- Alternating-time Temporal Epistemic Logic ATEL
  
  [van der Hoek & Wooldridge, Studia Logica 2003]

  $\langle J \rangle \varphi = \text{“coalition } J \text{ can achieve } \varphi \text{ (whatever opponents do)”}$

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Group knowledge, no actions

- $S5^C = S5$ plus Common knowledge
  - $\text{CK}_J\varphi =$ “it is common knowledge in $J \subseteq \text{Agt}$ that $\varphi$”
  - $= \text{EK}_J\varphi \land \text{EK}_J\text{EK}_J\varphi \land \text{EK}_J\text{EK}_J\text{EK}_J\varphi \land \cdots$

- fixpoint axiom:
  - $\text{CK}_J\varphi \leftrightarrow \text{EK}_J(\varphi \land \text{CK}_J\varphi)$

- induction axiom:
  - $\left(\varphi \land \text{CK}_J(\varphi \rightarrow \text{EK}_J\varphi)\right) \rightarrow \text{CK}_J\varphi$

$\Rightarrow$ will be criticized in the next section
Group knowledge 🗝️, no actions

- $S_5^C = \text{S5 plus Common knowledge}$
  
  $\text{CK}_J \varphi = \text{“it is common knowledge in } J \subseteq \text{Agt} \text{ that } \varphi”$
  
  $= \text{EK}_J \varphi \land \text{EK}_J \text{EK}_J \varphi \land \text{EK}_J \text{EK}_J \text{EK}_J \varphi \land \cdots$

- fixpoint axiom:

  $\text{CK}_J \varphi \leftrightarrow \text{EK}_J (\varphi \land \text{CK}_J \varphi)$

- induction axiom:

  $\left( \varphi \land \text{CK}_J (\varphi \rightarrow \text{EK}_J \varphi) \right) \rightarrow \text{CK}_J \varphi$

  $\Rightarrow$ will be criticized in the next section
Group knowledge ☕, no actions

- **S5^C =** S5 plus Common knowledge
  
  CK_Jφ = “it is common knowledge in J ⊆ Agt that φ”
  
  = EK_Jφ ∧ EK_J EK_Jφ ∧ EK_J EK_J EK_Jφ ∧ ⋯

- **fixpoint axiom:**
  
  CK_Jφ ↔ EK_J (φ ∧ CK_Jφ)

- **induction axiom:**
  
  \((φ ∧ CK_J(φ → EK_Jφ)) → CK_Jφ\)

⇒ will be criticized in the next section
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Group knowledge 🍄, nonstrategic actions

- PAL$^C = \text{PAL plus Common knowledge}$
- semantics: same as PAL
- accessibility relation for $CK_J = \text{greatest fixpoint of } EK_J$ relation
  - ⇒ ‘rebuilt’ after each update
  - ⇒ no reduction axioms for $CK_J$:
    \[
    \models CK_J[\psi!]\varphi \rightarrow [\psi!]CK_J\varphi
    \]
    \[
    \not\models [\psi!]CK_J\varphi \rightarrow (\neg\psi \lor CK_J[\psi!]\varphi)
    \]
  - ⇒ common knowledge may ‘pop up’ in an unforeseeable way!
Group knowledge 🍔, nonstrategic actions: the ignorant compatriots

Agents $B$ and $C$ are both Italian and don’t know each other. They meet during the coffee break and start to talk in English.

\[
\text{Init} = K_B IT_B \land CK_{\{B,C\}}(IT_B \rightarrow K_B IT_B) \land (\neg IT_B \rightarrow K_B \neg IT_B) \land
\]
\[
K_C IT_C \land CK_{\{B,C\}}(IT_C \rightarrow K_C IT_C) \land (\neg IT_C \rightarrow K_C \neg IT_C)
\]

1. first scenario:
a third agent truthfully says: “Hey, you are both Italian!”

\[
\text{Init} \rightarrow \langle IT_B \land IT_C \rangle \ CK_{\{B,C\}}(IT_B \land IT_C)
\]

2. second scenario:
a third agent truthfully says: “Hey, you are compatriots!”

\[
\text{Init} \rightarrow \langle IT_B \leftrightarrow IT_C \rangle \ CK_{\{B,C\}}(IT_B \land IT_C)
\]

[Lorini & H, 2013]
Group knowledge 🧐, nonstrategic actions: the ignorant compatriots

Agents B and C are both Italian and don’t know each other. They meet during the coffee break and start to talk in English.

\[
\text{Init} = K_B IT_B \land CK_{\{B,C\}}(IT_B \rightarrow K_B IT_B) \land (\neg IT_B \rightarrow K_B \neg IT_B) \land \\
K_C IT_C \land CK_{\{B,C\}}(IT_C \rightarrow K_C IT_C) \land (\neg IT_C \rightarrow K_C \neg IT_C)
\]

1 first scenario:
a third agent truthfully says: “Hey, you are both Italian!”

\[
\text{Init} \rightarrow \langle IT_B \land IT_C! \rangle CK_{\{B,C\}}(IT_B \land IT_C)
\]

2 second scenario:
a third agent truthfully says: “Hey, you are compatriots!”

\[
\text{Init} \rightarrow \langle IT_B \leftrightarrow IT_C! \rangle CK_{\{B,C\}}(IT_B \land IT_C)
\]
Group knowledge 🎓, nonstrategic actions: the ignorant compatriots

Agents $B$ and $C$ are both Italian and don’t know each other. They meet during the coffee break and start to talk in English.

$$\text{Init} = \mathsf{K}_B \mathsf{IT}_B \land \mathsf{CK}_{\{B,C\}}(\mathsf{IT}_B \rightarrow \mathsf{K}_B \mathsf{IT}_B) \land (\neg \mathsf{IT}_B \rightarrow \mathsf{K}_B \neg \mathsf{IT}_B) \land \mathsf{K}_C \mathsf{IT}_C \land \mathsf{CK}_{\{B,C\}}(\mathsf{IT}_C \rightarrow \mathsf{K}_C \mathsf{IT}_C) \land (\neg \mathsf{IT}_C \rightarrow \mathsf{K}_C \neg \mathsf{IT}_C)$$

**1** first scenario:
a third agent truthfully says: “Hey, you are both Italian!”

$$\text{Init} \rightarrow \langle \mathsf{IT}_B \land \mathsf{IT}_C ! \rangle \mathsf{CK}_{\{B,C\}}(\mathsf{IT}_B \land \mathsf{IT}_C)$$

**2** second scenario:
a third agent truthfully says: “Hey, you are compatriots!”

$$\text{Init} \rightarrow \langle \mathsf{IT}_B \leftrightarrow \mathsf{IT}_C ! \rangle \mathsf{CK}_{\{B,C\}}(\mathsf{IT}_B \land \mathsf{IT}_C)$$

[Orini & H, 2013]
Group knowledge ☰, nonstrategic actions: the ignorant compatriots, ctd.

- After the announcement of $IT_B \leftrightarrow IT_C$, is it part of the common ground of the conversation that $IT_B \land IT_C$???

- implicit vs. explicit common knowledge

\[ Init \rightarrow \langle IT_B \leftrightarrow IT_C! \rangle \left( ICK_{\{A,B\}}(IT_B \land IT_C) \land \neg ECK_{\{A,B\}}(IT_B \land IT_C) \right) \]

- implicit common knowledge = $\text{PAL}^C$ common knowledge
  - induction axiom: OK
  - reduction axiom: KO

- explicit common knowledge: accessibility relation for $ECK_J$ is some fixpoint, but not necessarily the greatest
  - induction axiom: KO
  - reduction axiom: OK

\[ [\psi!]ECK_J \varphi \leftrightarrow (\psi \rightarrow ECK_J[\psi!] \varphi) \]
Group knowledge ♻, nonstrategic actions: the ignorant compatriots, ctd.

- After the announcement of $IT_B \leftrightarrow IT_C$, is it part of the common ground of the conversation that $IT_B \land IT_C$??

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- explicit common knowledge: accessibility relation for $ECK_J$ is some fixpoint, but not necessarily the greatest
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  $$[\psi!]ECK_J\phi \leftrightarrow (\psi \rightarrow ECK_J[\psi!]\phi)$$
Group knowledge 🎉, strategic actions

- $\text{ATEL}^C = \text{ATEL plus common knowledge}$
- problem: which form of group knowledge required for (uniform) group strategies?
  - sometimes distributed knowledge $\text{DK}_{J\varphi}$
  - sometimes shared knowledge $\text{EK}_{J\varphi}$
  - sometimes common knowledge $\text{CK}_{J\varphi}$
## Conclusion

|------------------|--------------|------------------|------------------|--------------|----------------|----------------|--------------|

### Table

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- S5: inadequate as a logic of knowledge
- S5\(^C\): questionable as *the* logic of common knowledge
- APAL and ATEL: can’t talk about uniform strategies
- ATL: commitment to strategies missing
## Conclusion

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<tr>
<th><strong>Conclusion</strong></th>
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- revisited logics for MAS and their problems
  - S5: inadequate as a logic of knowledge
  - S5^C: questionable as *the* logic of common knowledge
  - APAL and ATEL: can’t talk about uniform strategies
  - ATL: commitment to strategies missing
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