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Abstract: In this paper, our contributions address the issues of sub-metric positioning and navigation of a mobile ad hoc network, by using widely available low cost resources. In this framework, we propose to assess the positioning performance of a loosely coupled INS/DGPS integration system improved with radio-based ranging collaborative algorithms. We propose two collaborative solutions: Linear Matrix Inequalities (LMI) based method and Bounding Box method. These methods consist in estimating a node’s position by finding the regions in which the node has a high probability to stay in, either by intersecting convex regions (LMI) or rectangular regions (Bounding Box). The latter is less accurate but simpler in term of complexity and hence easier to embed. The absolute positioning accuracy of each loosely coupled INS/DGPS-aided Collaborative approach will be compared to a loosely coupled INS/DGPS single differencing solution using simulated datasets. The obtained improvements of the proposed methods will be discussed.

BIOGRAPHIES

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Tanguy Pérennou received his Ph.D from the National Polytechnic Institute of Toulouse (INPT) in January 1997. He has then been working as a computer science engineer for various aeronautics-, space-and research-related companies and institutes between 1997 and 2001, including Astrium-Space and the French Air Traffic Management Research Center. In April 2001, he joined the ISAE (the French Aerospace Engineering School) as an associate professor. He is also a member of the SARA group at LAAS-CNRS. His research interests now focus on collaborative positioning, in particular in indoor environments, and on emulation platforms dedicated to the test of heterogeneous networking.

Vincent Calmettes received a Ph.D. degree in signal processing from ISAE, Toulouse, France. He is a training and research scientist at the signal, communications, antenna and navigation laboratory of ISAE. His research interests include Bayesian filtering and multi-sensors data fusion, with applications to tracking and navigation in harsh environments. He is also working on new Galileo signal processing and GNSS receiver architectures and is involved in several projects devoted to the development of embedded navigation system based on DSP and programmable logic devices.
1 INTRODUCTION

In many applications, like unmanned vehicle formation flight, the mobile vehicle must coordinate and move in a cooperative way with respect to the other vehicles in the fleet. Consequently, this raises the issues of keeping an accurate relative distance between vehicles. Thus collaborative positioning plays a central, if not critical, role for such applications. In this paper, our main concern deals with the problem of collaborative sub-metric positioning in mobile ad hoc networks. To this end, we propose a novel approach to enhance the performance of a loosely coupled INS/DGPS. This approach consists in exploiting simple information received from the other mobile vehicles, to derive a smaller error bound compared to the INS/DGPS system. In particular the new approach exploits distance between vehicles measurements. We propose two algorithms to process this external information.

Traditionally, Global Navigation Satellite Systems (GNSS), such as GPS, have become widely used for localization purposes. However, standard GPS systems suffer from many different errors and bias that limit its accuracy to about 10 meters 95% of the time. Several techniques have been developed to improve the performance of GPS systems. Augmented GPS systems, such as Differential GPS (DGPS) or approaches using carrier phase measurements, known as Real-Time Kinematics (RTK), offer higher precision through the use of fixed reference stations at well-known positions. The main difference between these two techniques is that DGPS sends the calculated pseudorange correction to a mobile device while RTK sends a carrier-phase correction. One of the main drawbacks of these techniques is that the device's receiver has to be within a certain range of a reference station in order to receive corrections. Moreover, these solutions often require cumbersome equipment, which are not always embeddable on lightweight mobile devices.

Other approaches, which may be complementary to the above cited techniques, combine GPS measurements with information provided by additional systems. The integration of GPS with an inertial navigation system (INS), for instance, has been exploited extensively (Groves, 2008). Different techniques have been proposed for coupling these two systems. In the conventional loosely coupled architecture, a GPS/INS estimator fuses the GPS navigation solution and inertial measurements in a Bayesian approach to obtain an improved navigation solution. GPS estimates of position and velocity form the measurement inputs of the estimation filter. There are two main advantages of a loosely coupled approach which lies in its simplicity of implementation and its redundancy. On the other hand, at least four satellites, which provide reliable measurements, are required to determine the GPS solution otherwise the system relies on the INS standalone solution.

Collaborative positioning allows the localization of a set of mobile devices by exploiting not only local information on the devices themselves but also external information received from neighbor devices. This approach, initially studied for wireless sensor networks (Amundson et al., 2009), has many advantages beyond the mere localization solution, when devices have limited resources in term of battery power and/or processing capacities, or when dealing with harsh environments where GPS signals may be degraded or lost. Collaborative positioning methods can be either deterministic or probabilistic. The first approach applies least square optimization, multidimensional positioning, multilateration or other optimization techniques. The second approach exploits Bayesian estimation to find the most probable positions. For the purpose of embeddability, in this paper, we are inspired by two simple, yet efficient deterministic methods: Bounding Box (Savvides et al, 2002) and LMI (Doherty et al., 2001). Traditionally, these approaches consider only external information. We propose two improvements of these algorithms, which consist in taking into account also the internal information, provided by a loosely coupled INS/DGPS single differencing process.

In the present study, we consider a network of mobile devices. Each node of the network is equipped with a GPS receiver, a MEMS based inertial module unit (IMU) and Wi-Fi transmitters. At each node, an Extended Kalman Filter (EKF) is applied to process the measurements from the IMU and the GPS receiver, by using a DGPS/INS loosely-coupled integration, locally updating the node's internal state. Concerning the GPS measurements, a master node performs a Precise Point Positioning (PPP) solution and is used as a DGPS mobile reference station for the other nodes. The latter use the GPS measurements, i.e. pseudorange and Doppler frequency shift, broadcasted by the master node to perform single differencing. At this stage, a first position estimate is available, as well as a covariance matrix that gives an indication on the positioning error.
In a second stage, nodes estimate the maximum distance between them, using more or less precise measurements such as time of arrival (TOA), time difference of arrival (TDOA) or Received Strength Signal (RSS). These maximum distances are then used to refine the position estimation in a collaborative way. Considering a Wi-Fi communication channel, in our collaborative positioning method, nodes first estimate the maximum distances to their neighbor nodes by using RSS, which is the measurement of the power of the received Wi-Fi signal. This measure is used for the estimation of the maximum distance to the neighbor in a robust way, even for propagation channels with difficult fading conditions. Though highly dependent on the environment and propagation model, this maximum distance estimation technique can be easily and rapidly implemented on any Wi-Fi receiver.

Nodes also receive their neighbors’ estimated positions broadcasted via Wi-Fi connections, RSS being measured during the broadcast. We consider two collaborative solutions: Linear Matrix Inequalities (LMI) based method and Bounding Box method. These methods consist in estimating a node’s position by finding the regions in which the node has a high probability to stay in, either by intersecting convex regions (LMI) or rectangular regions (Bounding Box). The latter is less accurate but simpler in term of complexity and hence easier to embed.

The expected outcome of this study is to reach sub-metric positioning accuracy of each mobile device in the network. To do so, we compare the 3D positioning performance of the two loosely coupled INS/DGPS-aided collaborative approaches with the loosely coupled INS/DGPS single differencing solution using simulated datasets. The improvements of the proposed methods will be discussed.

The rest of this paper is structured as follows. In Section 2, we introduce the loosely coupled INS/DGPS single differencing system. In Section 3, we introduce our collaborative approach. Simulation results are discussed in Section 4.

2 LOOSELY COUPLED INS/DGPS SINGLE DIFFERENCING

This section describes the loosely coupled INS/DGPS single differencing integration using an Extended Kalman Filter (EKF) algorithm. The general architecture of the system implemented on each slave is illustrated in Figure 1. The EKF algorithm is one of the most popular estimators when it comes to INS/GNSS and multi-sensor integration (Grewall, 2001), (Groves, 2008). It is an iterative process and is a non-linear version of the standard Kalman Filter (Kalman, 1960). In this study, we concern ourselves with pseudorange and Doppler frequency shift measurements.
3.1 GPS measurements

A GPS can measure a range, or distance, between a satellite \( k \) and a receiver \( u \). In reality, the true range is corrupted with series of errors produced at the satellite and receiver levels and by atmospheric effects such as ray bending and propagation delays. Accounting all these errors, the measured range is actually defined as a pseudorange. Its mathematical model can be expressed as:

\[
\rho_u^{(k)} = R_u^{(k)} + c(\delta t_u - \delta t^{(k)}) + l_u^{(k)} + T_u^{(k)} + \epsilon_{pu}
\]  

(1)

In equation (1), \( \delta t^{(k)} \) and \( \delta t_u \) are the errors introduced respectively by the satellite clock errors and the receiver clock bias, \( l_u^{(k)} \) and \( T_u^{(k)} \) reflect the delays associated with the transmission of the signal through the ionosphere and the troposphere respectively, and \( \epsilon_{pu} \) account for all non-modeled errors including multipath and receiver noise. In the above equation, \( \rho_u^{(k)} \) is the measured pseudorange in meters and \( R_u^{(k)} = \| \mathbf{p}^{(k)} - \mathbf{p}_u \| \) is the true receiver-satellite geometric range in meters with \( \mathbf{p}^{(k)} \) denoting the known satellite position, which can be obtained from the navigation message, and \( \mathbf{p}_u \) is the receiver antenna position.

For each satellite, the Doppler frequency shift provides a direct way to compute precise line-of-sight (LOS) velocity and helps to improve the quality of positioning. The frequency shift experienced by the receiver by a satellite \( k \) can be modelled as:

\[
f_d^{(k)} = \left( f_u^{(k)} - f_{L1} \right) = -\frac{f_{L1}}{c} \cdot \left( \dot{R}_u^{(k)} + c(\delta \dot{t}_u - \delta \dot{t}^{(k)}) + \epsilon_{pu} \right)
\]  

(2)

where \( c \) is the speed of light, \( f_{L1} \) is the L1 carrier frequency (1575.42 MHz), \( \delta \dot{t}_u \) and \( \delta \dot{t}^{(k)} \) are respectively the receiver and satellite clock drift errors, \( \epsilon_{pu} \) denotes all non-modeled errors and \( \dot{R}_u^{(k)} \) is the relative radial velocity between the satellite and the receiver in the line-of-sight (LOS) direction. The relationship between the measured Doppler frequency shift and a receiver's velocity can be formulated the following way:

\[
f_d^{(k)} = -\frac{f_{L1}}{c} \cdot \left( \mathbf{e}^{(k)^T} \cdot (\mathbf{v}^{(k)} - \mathbf{v}_u) + c(\delta \dot{t}_u - \delta \dot{t}^{(k)}) + \epsilon_{pu} \right)
\]  

(3)

where \( \mathbf{v}^{(k)} \) and \( \mathbf{v}_u \) are the velocities of the satellite and the receiver respectively and \( \mathbf{e}^{(k)} \) is the LOS unit vector defined as:

\[
\mathbf{e}^{(k)} = \frac{\mathbf{p}^{(k)} - \mathbf{p}_u}{\| \mathbf{p}^{(k)} - \mathbf{p}_u \|}
\]  

(4)

3.2 DGPS single differencing process

Concerning GPS measurements, in the case of this ad hoc network, a master node performs a Precise Point Positioning (PPP) solution and is used as a mobile reference station for the other nodes. The GPS measurements, i.e. pseudorange and Doppler frequency shift, are broadcasted by the master to the slave nodes. The transmitted measurements are then differenced with the slave’s GPS measurements in order to estimate the relative state vector of each slave, \( \Delta \mathbf{x}_{IM} \), which is defined by:

\[
\Delta \mathbf{x}_{IM} = \left[ \Delta \mathbf{x}_{IM}^{pos}, \Delta \mathbf{x}_{IM}^{vel} \right]^T
\]  

(5)

in which \( \Delta \mathbf{x}_{IM}^{pos} = [r_{IM} \, \Delta b_h]^T \) and \( \Delta \mathbf{x}_{IM}^{vel} = [v_{IM} \, \Delta d_b]^T \). \( r_{IM} \) and \( v_{IM} \) are the baseline and relative velocity between the slave \( i \) and master mobiles, and \( \Delta b_h \) and \( \Delta d_b \) are the receiver clock bias and clock drift differences. This technique is referred to single differencing and is often used in applications where the user position must be known accurately with respect to a reference station. This relative position is known as a baseline. Other differencing methods, known as double and triple
3.3 Relative position and velocity using single differencing

To compute the relative position and velocity, we will consider least-squares as a means of estimating the unknown relative state vector defined previously.

A single differencing measurement is obtained by subtracting the GPS measurements of two receivers referred to the same satellite. Assuming there are two vehicles, one specified as the slave \( i \), the other the master \( M \), and one satellite denoted \( k \), as shown in Figure 2, the pseudorange and Doppler-based single differencing measurements can be expressed by:

\[
\Delta \rho_{iM}^{(k)} = \rho_{i}^{(k)} - \rho_{M}^{(k)} \tag{6}
\]

\[
\Delta f_{d_{iM}}^{(k)} = f_{d_{i}}^{(k)} - f_{d_{M}}^{(k)} \tag{7}
\]

For practical reasons, it is easier to work with velocity measurements rather than Doppler shift measurements. We thus define the velocity-based single differencing measurement as a function of the Doppler-based single differencing measurement by the following relation:

\[
\Delta v_{d_{iM}}^{(k)} \equiv -\frac{c}{f_{L1}} \Delta f_{d_{iM}}^{(k)} \tag{8}
\]

**Relative positioning**

By substituting equation (1) in (6), the single difference between pseudoranges can be modeled as:

\[
\Delta \rho_{iM}^{(k)} = \left( R_{i}^{(k)} - R_{M}^{(k)} \right) + c(\delta t_{i} - \delta t_{M}) + \left( f_{i}^{(k)} - f_{M}^{(k)} \right) + \left( T_{i}^{(k)} - T_{M}^{(k)} \right) + (\epsilon_{p_{i}} - \epsilon_{p_{M}}) \tag{9}
\]
When applying the single difference, and under the assumption of close proximity of both receivers, the ionospheric and the tropospheric effects are nearly the same for both receivers and thus become negligible compared to the errors due to multipath and receiver noise. Moreover, the satellite clock bias $\delta t^{(k)}$, which is assumed to be common for both receivers, is cancelled out as well. Thus, for a relatively short distance between two receivers, equation (9) is simplified to:

$$\Delta \rho_{IM}^{(k)} = \Delta R_{IM}^{(k)} + \Delta b_h + \Delta \varepsilon_\rho$$

(10)

In the above equation, $\Delta b_h$ is the relative clock bias between the two receivers and $\Delta R_{IM}^{(k)}$ is the difference between the true ranges from both receivers to the satellite. Considering that the true ranges from the satellite to the receivers are much larger than the distance between receivers, it can be assumed that the LOS vector, $e_i^{(k)}$, from the slave to the satellite is parallel to the LOS vector, $e_M^{(k)}$, from the master to this same satellite. Therefore, $\Delta R_{IM}^{(k)}$ can be approximated by the scalar product between the LOS unit vector $e^{(k)}$ and the baseline vector $r_{IM}$. This relation is given by:

$$\Delta R_{IM}^{(k)} \approx e^{(k)T} \cdot r_{IM}$$

(11)

The baseline vector $r_{IM} = p_i - p_M$ is expressed in the ENU local coordinate frame. Equation (10) can be re-written as:

$$\Delta \rho_{IM}^{(k)} = e^{(k)T} \cdot r_{IM} + \Delta b_h + \Delta \varepsilon_\rho$$

(12)

When $N_s$ satellites are available at each time epoch, the above relation can be written in the following state-space representation:

$$\Delta \rho_{IM} = H \Delta x_{IM}^{pos} + \Delta \varepsilon_\rho$$

(13)

where:

$$H = \begin{bmatrix} e^{(1)} & e^{(2)} & \ldots & e^{(N_s)} \\ 1 & 1 & \ldots & 1 \end{bmatrix}^T$$

(14)

Relationship (13) constitutes the measurement model to be considered for determining the relative position. For $N_s \geq 4$ it can be solved by the linear least-squares estimator:

$$\Delta x_{IM}^{pos} = (H^T H)^{-1} H \Delta \rho_{IM}$$

(15)

Relative velocity

The relative motion between the slave and the master results in changes in the observed Doppler frequency shift. This can be modeled by the following expression, which is derived by (3) and (7):

$$\Delta f_{d_{IM}}^{(k)} = -\frac{f_{d_{IM}}}{c} \cdot \left( e^{(k)T} \cdot (v_i - v_M) + \Delta d_h + \Delta \varepsilon_\rho \right)$$

(16)

where $\Delta d_h$ is the relative clock drift between the two receivers, $v_i$ and $v_M$ are the velocities of slave $i$ and the master respectively. Regarding the velocity-based single differencing measurement defined by (8), equation (16) can be rewritten as:

$$\Delta v_{d_{IM}}^{(k)} = e^{(k)T} \cdot v_{IM} + \Delta d_h + \Delta \varepsilon_\rho$$

(17)
Considering \( N_s \) shared satellites, the state-space form of the above equation is:

\[
\Delta v_{dlM} = H \cdot \Delta x_{IM}^{vel} + \Delta \xi
\]  

(18)

By analogy with (15), the least squares relative velocity solution is then:

\[
\Delta x_{IM}^{vel} = (H^T H)^{-1} H \cdot \Delta v_{dlM}
\]  

(19)

### 3.4 State representation and measurement model

Regarding a slave \( i \) in the ad hoc network, we define the absolute state vector \( x_i \) to be estimated as follows:

\[
x_i = [p_i \quad v_i \quad q_i \quad b_{ai} \quad b_{gi}]^T
\]  

(20)

where \( p_i = [x \quad y \quad z]^T \), \( v_i = [vx \quad vy \quad vz]^T \) and \( q_i = [qw \quad qx \quad qy \quad qz]^T \) represent respectively the position, the velocity and the attitude in quaternion of a given slave node in the East-North-Up frame (ENU-frame). \( b_{ai} \) contains the IMU acceleration biases and \( b_{gi} \) contains the IMU gyro biases.

The dynamic model which describes the propagation of the state vector can be formulated as:

\[
\dot{x}_i(t) = f(x_i(t), u_i(t)) + w_i(t)
\]  

(21)

where \( f(\cdot) \) is a non-linear relation describing the INS, \( u_i = [f_{ib} \quad \omega_{ib}]^T \) denotes the vector of the IMU outputs, in which \( f_{ib} \) and \( \omega_{ib} \) are the accelerations and angular rate provided by the accelerometers and gyroscopes, and \( w_i(t) \) is the system noise vector, modeled as white noise.

In the loosely coupled mode, the DGPS single differencing process generates absolute position and velocity. These values are used as measurements by the EKF algorithm and are represented by the measurement vector \( z_i \), which is expressed by the following formulation:

\[
z_i = \begin{bmatrix} r_{IM} + p_M \\ v_{rIM} + v_M \end{bmatrix}
\]  

(22)

where \( x_M = [p_M \quad v_M]^T \) is the absolute state of the master node. Finally, the measurement equation is given by:

\[
\Delta z_i = z_i - h(x_i)
\]  

(23)

where \( h(x_i) = [p_i \quad v_i]^T \).

### 3 LOOSELY COUPLED INS/DGPS-AIDED COLLABORATIVE APPROACH

This Section describes how our collaborative algorithm is integrated to the loosely coupled INS/DGPS process to use positioning information from the neighboring nodes (master and/or slaves). Figure 3 illustrates the general architecture of this collaborative approach. At each node (master or slave) an iterative process is performed. Each iteration consists of the following main steps:

1. The node broadcasts part of its state vector to the other nodes using the radio, e.g. Wi-Fi;
2. The node receives all information broadcasted by the other nodes, and measures the radio received signal strength (RSS) to infer the maximum sender distance;
3. The node uses all the information received from the other nodes to refine its own position estimation and error.
The collaborative algorithms used in this paper determine a volume in which a receiver node has a high probability to reside. This volume is the intersection of volumes comprising each sender node as well as the receiver node.

4.1 Broadcasted information

The information that each node broadcasts includes:
- the estimated position $\hat{\mathbf{p}}_i$ extracted from the estimated state vector,
- standard deviations on $\hat{\mathbf{p}}_i$ extracted from the covariance matrix, $\mathbf{P}_i$, and denoted $\mathbf{\sigma}_i \equiv [\sigma_{xx} \ \sigma_{yy} \ \sigma_{zz}]^T$.

This information allows the computation of a volume where node $i$ is very likely to be located. According to the collaborative algorithm variant used, the volume can be either a box or an ellipsoid centered on the EKF position estimate. The dimensions of this bounding volume are proportional to the standard deviation of $\hat{\mathbf{p}}_i$. The exact dimensions are computed so that the actual node position is inside the bounding volume with a given confidence, e.g. with a 99% confidence. For a target confidence $\alpha_p$ (i.e. $\alpha_p$ for each dimension), the volume half-dimensions should be: $\lambda \cdot \mathbf{\sigma}_i$, with $\lambda$ such that:

$$\Pr(|\mathbf{p}_i - \hat{\mathbf{p}}_i| \leq \lambda \cdot \mathbf{\sigma}_i) = \alpha_p^3 \quad (24)$$

For a 99% confidence, $\lambda = F^{-1}(0.99) = 2.3263$, where $F^{-1}$ is the normal cumulative distribution function (CDF). Figure 4 illustrates bounding volumes.
The obtained bounding volume is denoted $\hat{b}_i$ such that:

$$\hat{b}_i = \hat{p}_i \pm \lambda \cdot \sigma_i$$  \hspace{1cm} (25)$$

In this paper, the shapes used are either boxes for the Bounding Box-based variant of the collaborative algorithm, or ellipsoids for the LMI-based variant. Both variants will be presented later on.

### 4.2 Inter-node maximum distance estimation

When node $i$ receives the information broadcasted by node $j$, the radio device measures the received signal strength (RSS). Various propagation models allow transforming RSS into an estimation of the $d_{ij}$ relative distance between nodes. Accurate distance estimation using radio signal can only be performed in free space, i.e. when no signal obstruction or reflection occurs. In the general case, for instance within a fleet of drones, in an urban environment, a number of impairments lead to large random variations of RSS for a given distance. It is then only possible to estimate a maximum distance $D_{ij}$ with a given confidence. Under these assumptions, for a given RSS value $\beta$, maximum distance estimation with confidence $\alpha_d$ is such that:

$$\Pr(d_{ij} \leq D_{ij} | RSS_{ij} = \beta) = \alpha_d$$  \hspace{1cm} (26)$$

In this paper, for the sake of simplicity, we use a very simple assumption that the measured distance $\tilde{d}$ follows a Gaussian law centered on the real distance $d$ with standard deviation $\sigma_d$:

$$\tilde{d} \sim N(d, \sigma_d)$$  \hspace{1cm} (27)$$

Under this assumption, an estimation of $D_{ij}$ is given by:

$$\tilde{D}_{ij} = \tilde{d}_{ij} + \sigma_d \cdot F^{-1}(\alpha_d)$$  \hspace{1cm} (28)$$

This estimated maximum distance $\tilde{D}_{ij}$ is then used to enlarge $\hat{b}_j$ (bounding volume for node $j$, centered on $\hat{p}_j$) so that the resulting bounding volume $\tilde{b}_{ij}$ (also centered on $\tilde{p}_j$) is very likely to also enclose node $i$:

$$\tilde{b}_{ij} = \tilde{p}_j \pm h_{ij} \text{ where } h_{ij} = \begin{bmatrix} \lambda \cdot \sigma_j + [\tilde{D}_{ij} \tilde{D}_{ij} \tilde{D}_{ij}] \end{bmatrix}$$  \hspace{1cm} (29)$$
4.3 Bounding volumes intersection

By construction, receiver node $i$ has a high probability to be located within the intersection of the $\mathbf{b}_{ij}$ bounding volumes of all senders $j$. It also resides within the bounding volume returned by the INS/DGPS single difference process, $\mathbf{b}_i$. Both variants of the collaborative algorithm use the intersection $\mathbf{b}_i^*$ of these volumes:

$$\mathbf{b}_i^* = \mathbf{b}_i \cap \bigcap_{j \neq i} \mathbf{b}_{ij}$$  \hspace{1cm} (30)

Both variants compute a box enclosing $\mathbf{b}_i^*$ and return the opposite min and max corners of this box, named $\mathbf{b}_i^{\text{min}}$ and $\mathbf{b}_i^{\text{max}}$. The new estimate $\hat{\mathbf{p}}_i^*$ of receiver node $i$ position is chosen as the center of this box:

$$\hat{\mathbf{p}}_i^* = \frac{\mathbf{b}_i^{\text{min}} + \mathbf{b}_i^{\text{max}}}{2}$$  \hspace{1cm} (31)

If receiver node $i$ receives information from $N$ senders, the maximum error on position $\mathbf{p}_i$ with confidence $\alpha_p^3$ on the INS/DGPS position error and confidence $\alpha_d$ in the maximum distance error is then:

$$\Pr \left( \|\hat{\mathbf{p}}_i^* - \mathbf{p}_i\| \leq \left\| \mathbf{b}_i^{\text{max}} - \mathbf{b}_i^{\text{min}} \right\| / 2 \right) = \alpha_p^3 \alpha_d^N$$  \hspace{1cm} (32)

The remainder of this section describes how the intersection $\mathbf{b}_i^*$ is computed for each variant of the collaborative algorithm, and how the opposite corners $\mathbf{b}_i^{\text{min}}$ and $\mathbf{b}_i^{\text{max}}$ of the box enclosing $\mathbf{b}_i^*$ are computed.

**Bounding Box algorithm variant**

In the Bounding Box variant of the collaborative algorithm, all bounding volumes used by receiver node $i$ are boxes. This variant derives from the work of (Savvides et al, 2002) in the context of wireless sensor networks.

Bounding volume $\mathbf{b}_i$ is a box centered on $\hat{\mathbf{p}}_i$ with half-dimensions $\lambda \cdot \sigma_i$ as defined in equation (25) and bounding volumes $\mathbf{b}_{ij}$ are boxes centered on $\hat{\mathbf{p}}_j$ with half-dimensions $\mathbf{h}_{ij}$ as defined in equation (29). Box $\mathbf{b}_i^*$ is the intersection of all $\mathbf{b}_{ij}$ boxes and by construction is a box which corners $\mathbf{b}_i^{\text{min}}$ and $\mathbf{b}_i^{\text{max}}$ are defined as:

$$\begin{cases} 
\mathbf{b}_i^{\text{min}} = \max \left( \mathbf{\hat{p}}_i - \lambda \cdot \sigma_i, \max_{j \neq i} (\mathbf{\hat{p}}_j - \mathbf{h}_{ij}) \right)^T \\
\mathbf{b}_i^{\text{max}} = \min \left( \mathbf{\hat{p}}_i + \lambda \cdot \sigma_i, \min_{j \neq i} (\mathbf{\hat{p}}_j + \mathbf{h}_{ij}) \right)^T
\end{cases}$$  \hspace{1cm} (33)

**LMI algorithm variant**

In the Linear Matrix Inequalities (LMI) variant of the collaborative algorithm, the highly probable inclusion of receiver node $i$ in bounding volumes $\mathbf{b}_i$ and $\mathbf{b}_{ij}$ is expressed as a set of inequalities with respect to each dimension $x$, $y$ and $z$:

$$\forall i, \forall j \neq i \left\{ \begin{array}{l} |\mathbf{p}_i - \mathbf{\hat{p}}_i| \leq \lambda \cdot \sigma_i \\
|\mathbf{p}_i - \mathbf{\hat{p}}_j| \leq \mathbf{h}_{ij} \end{array} \right. \hspace{1cm} (34)$$
This variant is derived from the work of (Doherty et al., 2001) in the context of wireless sensor networks. Intuitively, bounding volumes \(\bar{b}_i\) and \(\bar{b}_{ij}\) are ellipsoids centered on \(\bar{p}_i\) or \(\bar{p}_{ij}\) with half-dimensions \(\lambda \cdot \sigma_i\) or \(h_{ij}\). This set of inequalities constitutes an LMI problem that can be solved with e.g. the Matlab Robust Control toolbox. The mincx variant of Matlab's LMI solver computes a box enclosing the intersection \(b_i^j\) of the enclosing ellipsoids and its opposite corners \(b_{min}^i\) and \(b_{max}^i\).

4 SIMULATION RESULTS

We performed intensive simulations to show the performances of the previously introduced INS/DGPS-aided collaborative approach. We choose the absolute positioning error as performance metric. Let us consider a network of 9 mobile nodes moving together in a 3D space. We choose a simple square formation in which the master is located at the center and the slaves are located at the corners and the edges’ midpoints. In this work, we developed a 3D mobility model that incorporates realistic accelerations and turns (we omit the details of this model as they are beyond the scope of this paper). We consider the case in which the formation's orientation stays constant during the entire trajectory.

We implemented our simulator in MATLAB, including the INS/DGPS-aided collaborative positioning system and the mobility model. In this study, a low-cost IMU is considered whose specifications are given by the following noise parameters: \(Q_a = 10^{-3} \, \text{m/s}^2/\sqrt{\text{Hz}}\), \(Q_g = 10^{-4} \, \text{rad/s}/\sqrt{\text{Hz}}\), \(K_a = 10^{-4} \, \text{m/s}^2/\sqrt{\text{Hz}}\) and \(K_g = 10^{-5} \, \text{rad/s} \cdot \sqrt{\text{Hz}}\).

The INS, GPS and collaborative positioning system measures are taken at the rate of 100Hz, 1Hz and 1Hz respectively. The standard deviation of the distance estimation is set to \(\sigma_d = 0.05\) meters. To compare the collaborative positioning algorithms, each one is run on the same data generated by the underlying INS/DGPS system and on the same trajectories. We then repeat the experiments 100 times and measure each time the absolute positioning error of the INS/DGPS single differencing process and the INS/DGPS-aided collaborative solutions (both Bounding box and LMI variants) along each node trajectory. Table 1 summarizes our simulation settings.

<table>
<thead>
<tr>
<th>Table 1. Simulation settings</th>
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</thead>
<tbody>
<tr>
<td>Number of nodes</td>
</tr>
<tr>
<td>Formation</td>
</tr>
<tr>
<td>Duration</td>
</tr>
<tr>
<td>DGPS</td>
</tr>
<tr>
<td>INS frequency</td>
</tr>
<tr>
<td>Collaborative frequency</td>
</tr>
<tr>
<td>(\sigma_d)</td>
</tr>
<tr>
<td>Number of simulations</td>
</tr>
</tbody>
</table>
Figure 5 show the absolute error ranges of all the algorithms during the trajectory, obtained as the average over all the simulation runs. The boxplot shows the average (black dots), the standard deviation (gray boxes) and the 95% confidence interval (error bars) of this measure. It is clear that, with only simple additional information on the distance and the positions of the other nodes, our collaborative solutions enhance the performance of the loosely coupled INS/DGPS.

For each simulation, we compute the absolute positioning accuracy gain along the trajectory. We then take the average over all the simulations and finally compute the quadratic mean over all the gains obtained along the trajectory. As a consequence, this quantity measures the average absolute positioning accuracy gain and its variation along the trajectory. The values are shown in Table 2. We can see that all the slaves (i.e., nodes 2 to 9 in the table) benefit from the collaborative algorithm, while the master’s (i.e., node 1 in the table) gain is 0. This can be explained by the fact that its position is determined by a PPP solution, which is already highly precise, so the collaborative algorithms fail to find a smaller bounding volume.

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bounding Box</td>
<td>0</td>
<td>2.7</td>
<td>2.1</td>
<td>3.3</td>
<td>2.5</td>
<td>2.5</td>
<td>2.2</td>
<td>3.5</td>
<td>2.0</td>
</tr>
<tr>
<td>LMI</td>
<td>0</td>
<td>2.8</td>
<td>2.1</td>
<td>3.1</td>
<td>2.7</td>
<td>2.6</td>
<td>2.3</td>
<td>3.2</td>
<td>2.0</td>
</tr>
</tbody>
</table>

We plot the cumulative probability distribution of the absolute errors of these 3 solutions in Figure 6. It is interesting to see that the collaborative approach increases the probability of getting sub-metric performance by 10% over the INS/DGPS non collaborative solution. The figure also shows that the probability of obtaining an error less than 1m is higher than 0.9 for our collaborative algorithms. This means that our collaborative approach can ensure a sub-metric performance over 90% of time, which is a very promising result.
Finally, to evaluate the impact of the ranging measure on the performance of the collaborative approach, we perform the same experiment with different values of $\sigma_d$. Figure 7 shows that the absolute positioning accuracy gain (for node 2 in this figure) decreases as the distance measure’s error increases, with respect to both Bounding Box and LMI. This result shows that collaborative algorithms are quite sensitive to distance measure’s error. Thus choosing the appropriate ranging technique is crucial to obtain the best performance.
5 CONCLUSION

We have presented in this paper a new approach to improve the performance of loosely coupled INS/DGPS systems in mobile ad hoc networks. This simple approach consists of exploiting the information received from the other mobile nodes, i.e., their estimated positions and the estimated distances to them, to derive a bounding volume smaller than the one obtain with the INS/DGPS. We have proposed two algorithms to compute this bounding volume. Via simulations, we showed that this approach is capable of decreasing the INS/DGPS’s absolute positioning error. Moreover, it can ensure a sub-metric performance for over 90% of time. Although this is a promising result, we are convinced that there is still room for improvement. We have pointed out that this approach is quite sensitive to distance measurement errors. One solution to deal with this issue is choosing ranging techniques that allow highly precise measures, such as the Ultra Wide Band ranging technique. Besides, optimizing the proposed algorithms is also our ongoing work. This can be done by, for example, taking into account the estimate history.

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REFERENCES


