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Micro-scale modeling of carbon-fiber reinforced thermoplastic materials

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Abstract
Thin-walled textile-reinforced composite parts possess excellent properties, including lightweight, high specific strength, internal torque and moment resistance which offer opportunities for applications in mass transit and ground transportation. In particular the composite material is widely used in aerospace and aircraft structure. In order to estimate accurately the parameters of the constitutive law of woven fabric composite, it is recommended to canvass multi-scale modeling approaches: meso, micro and macro. In the present investigation, based on the experimental results established by carrying out observations by Scanning electron microscope (SEM), we developed a micro-scale FEM model of carbon-fiber reinforced thermoplastic using a commercial software ABAQUS. From the SEM cartography, one identified two types of representative volume elementary (RVE): periodic and random distribution of micro-fibers in the yarn. Referring to homogenization method and by applying the limits conditions to the RVE, we have extracted the coefficients of the rigidity matrix of the studied composites. In the last part of this work, we compare the results obtained by random and periodic RVE model of carbon/PPS and we compute the relative error assuming that random model gives the right value.

Introduction
The determination of the mechanical performance of woven fabric composites materials is based on the study of the behavior of the texture and the composite under different solicitations. Currently, the multi scale modeling of composites (figure 1) is one of the most used methods and it was adapted by several researchers. In fact, using this approach, F. Costanzo and L. Gray [1] have implanted a survey on periodicity and boundary conditions; P. Boisse [2] has raised the constructive equations of the mechanical behavior of the composites woven during the forming; Gilles Hivet [3] has elaborated a mathematical approach to identify the trajectory and the different sections of the yarn in texture, the profiles of the contacts’ curves and the contact’s sections according to the conic equations; L. Orgéas [4] has studied in meso-scale, the permeability of the reinforcements woven of stratified composites by surveying the velocity in such composites; J. Wang [5] has studied the predictive mechanical behavior modeling in woven composite structure, by analyzing 3D finites elements ; P. Badel and P. Boisse [6,7] have determined fibers orientations, in reinforcements woven during and after composites’ formation.

In order to identify the behavior of the studied composite using multi-scale approach, we have developed in this paper a simulation of the reinforcement’s woven fabric composite (figure 1).

We have started by using an experimental characterization of the texture to prepare a geometrical description of fibers’ diameters and distributions in the Polyphenylene sulfide (PPS) matrix. Then, we
identified two types of RVE (periodic and random one) in order to estimate the errors’ values in the results. Then, basing on the homogenization method and after applying the boundary conditions to the RVE, one has extracted the coefficients of the rigidity matrix and the parameters of the yarn composites. Finely, we have identified the able RVE to characterize accurately the yarn of our woven fabric composite.

I. Carbon-fiber reinforced thermoplastic materials

The composite texture consists of a carbon fibers and a PPS matrix and the volumetric fraction of the fibers in the composites is $V_f = 0.5$. The characteristics of the materials forming the composite are summarized in the following table:

<table>
<thead>
<tr>
<th>Material</th>
<th>Filament diameter ($\mu$m)</th>
<th>Volumetric Mass $\rho$ (kg/m$^3$)</th>
<th>Longitudinal Elasticity module $E$ (Mpa)</th>
<th>Module of Shearing $G$ (Mpa)</th>
<th>Poisson Coefficient $v$</th>
<th>Constrained of rupture (traction) MPa</th>
<th>Elongation to The rupture (%)</th>
<th>Thermal dilation coefficient $C_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Fiber</td>
<td>6.24</td>
<td>1800</td>
<td>390 000</td>
<td>20 000</td>
<td>0.35</td>
<td>2500</td>
<td>0.6</td>
<td>0.08 $10^{-3}$</td>
</tr>
<tr>
<td>PPS</td>
<td>1300</td>
<td>4000</td>
<td></td>
<td></td>
<td></td>
<td>65</td>
<td>100</td>
<td>5 $10^{-5}$</td>
</tr>
</tbody>
</table>

Table 1: the characteristic of the materials forming the composite

The characterization of the texture of the composite has been carried out through two main steps. In the first one, we determine the texture’s character, the trajectory, and the sections of the yarns (Texture of the composite: satin 4x1 in three layers). Then, in a second step, we find out the micrographic arrangement of the fibers in a yarn (figure 3).

The yarn is composed by thousands of small fibers whose diameter in the order of 6.24 $\mu$m (figure 2). This value is established by calculating the average of 150 fibers’ diameter. The disorganized arrangement of the fibers in the yarn presented in figure3 will produce a variation in the local properties influenced by the distance between these fibers. Then it is necessary to start by characterizing the fibers’ arrangement in order to determine the minimal size of the representative volume elementary (RVE) of the yarn. To do so, we can characterize the distribution of the fibers by analyzing the yarn’s picture and using the covariance concept adapted already by [8]:

$$C(x, x + h) = P\{x \in d, x + h \in d\}$$

(2)

The covariance is defined as the probability of adherence of two points” x “and “x+h” in the same phase d, and it can be valued by carrying out the Fourier’s transformed of the figure 2. According to the works of [7] the periodicity of the microstructure is presented by the periodicity of the covariance.

II. The micro scale modeling

1. The geometric model of RVE

The choice of the RVE which is a cubic shape was based on several researches works [10, 11, 12 and 13]. This RVE it should have the smallest size which makes it representative of the yarn material. We opted for this step of the simulation for two cubic cells shapes and we considered the fiber has a
cylindrical form. The first cell (figure 3-a) is periodic and the second is random (figure 3-b). The volumetric fraction of the reinforcement is calculated by the report between the volume of the fibers and the total volume of the basic cell:

\[ V_f = \frac{V_{fibers}}{V_{total}} = n \frac{\pi d^2}{4a^2} \]  (3)

Where: \( d \) is the diameter of the fiber, \( a \) is the side of the basis cell, and \( n \) is the number of fibers by cell

Figure 3: The periodic representative elementary volume of the yarn, number of fiber \( N = 2 \) fibers (a) and \( v_f = 0.505 \). The random representative elementary volume of the yarn, number of fiber \( N= 14 \) fibers and \( v_f = 0.475 \) (b)

2. The elastic constructive equations of the yarn’s homogenization

The elastic properties, are calculated by a periodic homogenization via a finite element method developed using ABAQUS software. It will give us the opportunity to study the elastic behavior of the yarn and to calculate the elastic coefficients of the composite material. For 3D RVE (cubic shape), submitted to a volumetric load, its elastic behavior can be presented as follow:

\[ \varepsilon = \Phi \sigma \]  (4)

Where: \( \varepsilon \) is the strain tensor, \( \sigma \) is the stress tensor, and \( \Phi \): the suppleness Matrix

Then, the stress distribution in the elementary volume can be written as follow:

\[ \sigma = C \varepsilon \]  (5)

Where \( \Phi = C^{-1} \)

The mechanical behavior of the yarn is equivalent and it depends on the mechanical and geometric properties of the different constituent: the fiber geometry, behavior, and distribution in the matrix, the matrix behavior and the characteristic of the fiber-matrix interface. The process of homogenization consists in assimilating a material characterized by an important heterogeneity by a homogeneous one. This process was applied to the RVE.

The main step of the homogenization consists in the determination of the stress and displacement fields within the RVE.

The average of the microscopic stress of this RVE can be expressed as follow:

\[ \langle \sigma \rangle = \frac{1}{V} \int_{\Omega} \sigma \, dv = \Sigma \]  (6)

In the same way, the average of the microscopic strain is give by:

\[ \langle \varepsilon \rangle = \frac{1}{V} \int_{\Omega} \varepsilon \, dv = E \]  (7)

Where \( E \) is the macroscopic strain and \( \Sigma \) is the macroscopic stress

From equation (6) and (7), one can write the Hooke criteria:

\[ \langle \sigma: \varepsilon \rangle = \langle \sigma \rangle: \langle \varepsilon \rangle = \Sigma: E \]  (8)

\( \Rightarrow \) The macroscopic stress \( (\Sigma = <\sigma>) \) is a linear function of the macroscopic strain \( (E = <\varepsilon>) \)

\[ \Sigma = C^{hom} E \]  (9)
Where $C^{\text{hom}}$ represents the macroscopic tensor obtained by the homogenization method.

The calculation of the $C^{\text{hom}}_{ijkl}$ coefficients takes place while calculating the stress field that corresponds to an imposed macroscopic displacement. Supposing that the yarn represents a composite with orthotropic characteristic, the macroscopic elasticity relation is expressed as follow:

$$\begin{bmatrix}
\Sigma_{11} \\
\Sigma_{22} \\
\Sigma_{33} \\
\Sigma_{23} \\
\Sigma_{13} \\
\Sigma_{12}
\end{bmatrix} = \begin{bmatrix}
C^{\text{hom}}_{1111} & C^{\text{hom}}_{2211} & C^{\text{hom}}_{3311} & 0 & 0 & 0 \\
C^{\text{hom}}_{1222} & C^{\text{hom}}_{2222} & C^{\text{hom}}_{3322} & 0 & 0 & 0 \\
C^{\text{hom}}_{1333} & C^{\text{hom}}_{2333} & C^{\text{hom}}_{3333} & 0 & 0 & 0 \\
0 & 0 & 0 & C^{\text{hom}}_{2323} & 0 & 0 \\
0 & 0 & 0 & 0 & C^{\text{hom}}_{1313} & 0 \\
0 & 0 & 0 & 0 & 0 & C^{\text{hom}}_{1212}
\end{bmatrix} \begin{bmatrix}
E_{11} \\
E_{22} \\
E_{33} \\
E_{23} \\
E_{13} \\
E_{12}
\end{bmatrix}
$$

For $i = j = k = l$, $i, j, k, l \in \{1, 2, 3\}$, the $C^{\text{hom}}_{ijkl}$ coefficients, have been determined by imposing a shear loading whose main directions correspond with the symmetry’s axes of the cell; that’s means:

$$F = E_{11} \varepsilon_1 \otimes \varepsilon_1 + E_{22} \varepsilon_2 \otimes \varepsilon_2 + E_{33} \varepsilon_3 \otimes \varepsilon_3$$ \hspace{1cm} (10)

For $i = k$ and $j = l$, $i, k \in \{1, 2\}$ and $j, l \in \{2, 3\}$, the coefficients $C^{\text{hom}}_{ijkl}$, have been determined by imposing to the basic cell a macroscopic displacement of type "simple shear" which can be expressed as follow:

$$F = E_{ij} / 2 (\varepsilon_i \otimes \varepsilon_j + \varepsilon_j \otimes \varepsilon_i)$$ \hspace{1cm} (11)

In the order to have a periodic applied displacement’s filed, it is necessary that every cell satisfies the following conditions [10]:

1. The continuity of the vector $\sigma. n$
2. The compatibility of the strain fields $\varepsilon$; therefore the neighboring should not be separated or superposed.

The periodicity of the passage from a cell to its neighbor is equivalent to pass a face from one face of the cell to the opposite face. The condition (1) becomes: $\sigma. n$ must be on the first opposite to that in the other face. The stress field $\sigma$ is called periodic on the cell while the field $\sigma. n$ is anti-periodic on its contour.

## III. Homogenization of the yarn based on micro scale finite element model

### 1. The micro scale constructive finite elements models

The adapted method consists in applying three simple traction loads following the three main axes (1, 2 and 3) and three simple shear loads in the directions 2-3, 1-2 and 2-3 (figure 4). In order to applying this method we should be imposed a displacements loading and putting a specific boundary conditions for each load, this method has been adapted by several authors [10, 14].

![Figure 4: The six different cases to be solved in order to calculate the homogenized elastic properties of the RVE.](image)

The calculation of $\Sigma_{ij}$ is approximated by the summation of all the volumetric elements of structure already calculated by elementariness integrations throw every finite element. Then we have the following equation:

$$\Sigma_{ij} = \langle \sigma \rangle = \frac{1}{V} \int_{\Omega} \sigma \, dV \equiv \frac{\sum_{k=1}^{n} \psi_k \sigma_{ij}(\alpha_{ij})_{k}}{\sum_{k=1}^{n} \psi_k}$$ \hspace{1cm} (12)
Where: \( V_k \) is the volume of the \( k \)th element and \( \sigma_{ij} \) is the composing \( ij \) of the microscopic constraint of the \( k \)th element.

2. Periodic representative elementary volume:

During the simulation, it is necessary to apply the loads as imposed displacements and to impose boundary conditions to the limits for every load. At first, we have supposed that the material is orthotropic. Then, the numeric simulation and the calculations by periodic homogenization gave the rigidity matrix of the yarn:

\[
C = \begin{bmatrix}
198953,521 & 2916,098 & 2915,550 & 0 & 0 & 0 \\
2916,099 & 10427,265 & 1874,303 & 0 & 0 & 0 \\
2915,550 & 1874,303 & 10426,265 & 0 & 0 & 0 \\
0 & 0 & 0 & 5820,813 & 0 & 0 \\
0 & 0 & 0 & 0 & 5960,149 & 0 \\
0 & 0 & 0 & 0 & 0 & 5960,646
\end{bmatrix}
\]

The calculation of the inverse rigidity matrix, will give the values of the suppleness matrix \( \Phi \), so we can determinate the material parameters. These parameters are summarized, in the following table:

<table>
<thead>
<tr>
<th>The Young Modules (MPa)</th>
<th>Poisson Coefficients</th>
<th>Shear Modules (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 ) = 197570,919</td>
<td>( v_{12} = 0,176 )</td>
<td>( G_{23} = 5820 )</td>
</tr>
<tr>
<td>( E_2 ) = 10061,284</td>
<td>( v_{13} = 0,237 )</td>
<td>( G_{13} = 5960 )</td>
</tr>
<tr>
<td>( E_3 ) = 10060,343</td>
<td>( v_{22} = 0,237 )</td>
<td>|</td>
</tr>
</tbody>
</table>

| \( E_3 \) = 10060,343 | \( v_{13} = 0,237 \) | \| |

Table 2: The periodic RVE elastic parameters

The yarn’s material is unidirectional and the results of the simulation of the periodic RVE using Von Mises constraint are provided in figure 9.

The Von Mises constraint in the RVE structure is

\[
\bar{\sigma} = \sqrt{\frac{3}{2} \sum_{ij} \sigma_{ij}^2} = \sqrt{\frac{3}{2} \sum_{ij} \sigma_{ij} \sigma_{ij} - \frac{1}{2} \left( \sum_k \sigma_{kk} \right)^2}
\]  

(13)

Figure 5: Results of simulation of the RVE, Von Mises constraint in the different loads (plan y z),

3. Random representative elementary volume

By one applying the same boundary conditions and the same loads on the random cell, we can determine the constants of the rigidity matrix \( C \) of the yarn and the suppleness matrix \( \Phi \), and consequently we can determinate the material random parameters which are shown in the following table:

<table>
<thead>
<tr>
<th>The Young Modules (MPa)</th>
<th>Poisson Coefficients</th>
<th>Shear Modules (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 ) = 183019,394</td>
<td>( v_{22} = 0,093 )</td>
<td>( G_{23} = 4498 )</td>
</tr>
<tr>
<td>( E_2 ) = 11588,458</td>
<td>( v_{13} = 0,222 )</td>
<td>( G_{13} = 5554 )</td>
</tr>
<tr>
<td>( E_3 ) = 9951,280</td>
<td>( v_{12} = 0,243 )</td>
<td>( G_{12} = 5369 )</td>
</tr>
</tbody>
</table>

Table 3: The random RVE elastic parameters
The results of the simulation of the random RVE using Von Mises constraint are provided in figure 11.

Figure 6: The results of simulation of the RVE, Von Mises constraint in the different loads (plan y z).

4. Comparisons between periodic and random model

The results gotten for the periodic and random model are reported in table 4. We can identify a fluctuation in the Young modules and the Poisson coefficients among the two models: the relative error for $E_2$ reaches 13% and, for the Poisson coefficients $\nu_{12}$ and $\nu_{13}$ it is respectively 2.47% and 6.76%. These results converge with the 2D studies in simple traction following the (OY) axis achieved by D.Trias [13], where the Young module present a differentiation of 12% and 6% for the Poisson coefficient.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Young Module</th>
<th>The share Modeling</th>
<th>The poisons coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_1$</td>
<td>$E_2$</td>
<td>$E_3$</td>
</tr>
<tr>
<td>Periodic RVE</td>
<td>197570.919</td>
<td>10061.284</td>
<td>10060.343</td>
</tr>
<tr>
<td>Random RVE</td>
<td>183019.394</td>
<td>11588.548</td>
<td>9951.280</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>7.95</td>
<td>13.18</td>
<td>1.096</td>
</tr>
</tbody>
</table>

Table 4: Computation of effective properties for the periodic and random model of the yarn

Our survey in 3D simulation will give some results more advanced than [13]. The difference between the random and the periodic RVE in Shear Modules $G_{13}$ and $G_{12}$ is roughly 11% and 29.39% for $G_{23}$. Concerning the Poisson coefficient $\nu_{23}$ the relative error between the two models is around 89%.

In the numerical results, for the periodic REV, we observe a like value of YOUNG modules $E_2$ and $E_3$ ($E_2=10061,284$ MPa and $E_3=10060,343$ MPa) and a regular behavior in the tow transverse directions. But for the random model, a small difference between the value of the two YOUNG modules ($E_2=11588,548$ MPa and $E_3=9951,280$ MPa), this difference is generally due to the proposed arrangement of fibers and the irregular distances inter-fibers in the REV (see figure 3a-b). Also a variation of the value of $E_2$ of 13% and the value of $G_{23}$ of 29.39 % has been observed in the two cases random and periodic REV. This deference is due to the closeness between fibers in the random REV who will give a more resistance.

The results of the distribution using Von Mises constraint in the matrix and the fibers (figure 5 and 6) present a huge difference between the two types of RVE. Indeed, the random model gives a more real response than the periodic model.

5. Analytic results

In order to validate the numerical results presented in the previous section, a simplest theoretical approaches. These theoretical models are able to predict the composites parameters'. The Voigt model and the Reuss model are expressed by [16 and 17]:

Reuss model (transverse model):

$$E_t = \left[ V_f/E_f + V_m/E_m \right]^{-1}$$ (14)
The Voigt model (longitudinal model):

\[ E_l = E_f V_f + E_m V_m \]  \hspace{1cm} (15)

Shear Module and poisons coefficient are calculated by the mixtures law as follow:

\[ \nu_{ll} = \nu_f V_f + \nu_m V_m \]  \hspace{1cm} (16)
\[ G_{ll} = \left[ \frac{V_f}{G_f} + \frac{V_m}{G_m} \right]^{-1} \]  \hspace{1cm} (17)

The analytical results bases in Reuss and Voigt approaches are:

\[ E_1 = 197000 \, \text{MPa} \; ; \; E_2 = E_3 = 7918.718 \times 10^3 \, \text{D} \nu_{23} \]

Particularly, the analytic results prove the numerical prediction in periodic model of \( E_1 \), poisons coefficient \( \nu_{23} \), but we observe a small variation of the values of \( E_2, E_3, \nu_{12} \) and \( \nu_{13} \), this difference is due to the no into account the morphology of the composite material in the used theoretical models.

**Conclusions:**

The micro scale modeling adopted in this work has permitted to extract the elastic features of the composite yarn and the simulation of the periodic and random RVE gave that the yarn material is unidirectional. According to the works of D.Trias [13] where two types of 2D representative models (random and periodic) were compared, we can conclude that the periodic models could be used in some cases when the observed error is considered like negligible and no assessment for the material’s security. But this type of model cannot be adopted to calculate accurately the material properties. The uses of periodic models could cause misjudge estimation (crack in the matrix and initiation of the damages), contrarily to the random models which can provide useful information for reliability analysis not achieved with periodic models. We have confirmed the numerical simulation by classical analytic models (Reuss and Voigt) but it is recommended to develop an appropriate law for our composite yarn. The results gotten using the random RVE will be implanted shortly in the meso-scale modeling of our woven fabric composite. This study is promoter and it requires an advance model in damage and the rupture problems and to define the constitutive law of the yarn.

**References:**


