Influence of acoustic streaming on thermo-diffusion in a binary mixture under microgravity

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ABSTRACT

An analytical and numerical study of the influence of acoustic streaming on species separation of a binary mixture under microgravity is presented. A rectangular cell filled with binary fluid is submitted to an ultrasonic propagating wave along a portion of one of its small walls while the opposite wall is perfectly absorbing. A temperature gradient is applied between the two other walls. The unicellular flow induced by the Eckart streaming may lead to significant species separation. In a first part, the hypothesis of parallel flow is used to determine the analytical solution which describes the unicellular flow and the separation is calculated analytically based on the acoustic streaming parameter, \( A \), the acoustic beam width, \( e \), and the Schmidt number, \( Sc \). Theses analytical results are corroborated by direct numerical simulations.

In a second part, a linear stability analysis of the unicellular flow is performed. The eigenvalue problem resulting from the temporal stability analysis is solved by the Galerkin method, a spectral Tau–Chebychev method and by a finite element method. The thresholds for the stationary and oscillatory instability depend on the normalized acoustic beam width.

1. Introduction

The propagation of ultrasonic waves in a fluid may induce a stationary flow at large scale: this phenomenon is called “acoustic streaming”, Lighthill [1], Eckart [2].

A review of most of the pertinent information on the acoustic streaming can be found in the following recent papers. Lei et al. [3], performed 3D numerical experiments in order to investigate the convection in an enclosure subjected to a horizontal temperature gradient and a longitudinal sound field. The authors’ contribution concerns Rayleigh streaming created along the walls by a standing wave in a three-dimensional cavity. The contribution of Dridi et al. [4] concerns Eckart streaming in a three-dimensional cavity. The structure of the flow induced by acoustic streaming and its stability were determined numerically. The authors found that the thresholds increased when the acoustic streaming effect was enhanced. Destabilization effects were also observed for a specific parameter range. Dridi et al. [5] also studied Eckart acoustic streaming in their most recent papers. The authors considered an incompressible liquid layer, of thickness \( H \), confined between two infinite horizontal walls. This layer was subjected to a temperature gradient and to a radiation pressure caused by a transducer. The ultrasound beam, which was applied inside the layer, had a normalized width \( e = \frac{H_p}{H} \) in the \( z \) direction and was uniform in the transverse direction \( x \).

The velocity profiles of the basic flows were determined analytically when the beam was centered on the side of the cavity and in the case where the beam was not centered but was positioned at a given location \( z = z_b \).

The authors studied the linear stability of Eckart streaming flows in an isothermal or laterally heated mono-constituent fluid layer in the gravity field. The critical value of the acoustic streaming parameter, \( A_c \), leading to a Hopf bifurcation, was determined in the case of an isothermal flow for an infinite horizontal fluid layer. For a centered beam, \( A_c \) is minimum for an acoustic beam width \( e = 0.32 \) and increases when either \( e \) decreases or increases.

In the present paper, the coupling between Eckart streaming and Soret effect in a binary fluid, in weightlessness, was investigated. In accordance with the contribution [5], the Rayleigh streaming influence was not introduced in our model. Dridi et al. [5] showed that the streaming flow induced by a traveling wave, in a channel with a typical height of a few centimeters is predominantly generated by Eckart streaming. The Rayleigh streaming contribution is negligible. Unlike the works developed in [4,5] where the acoustic source was centered or not, in the current study, the beam was always placed in contact with the wall \( z = H \) to obtain a unicellular flow. This is a necessary condition to obtain
the separation of species as previously obtained in thermogravitational columns. The ultrasonic waves lead to important species separation for particular values of the acoustic streaming parameter, A, the acoustic beam width, ε, and the Schmidt number, Sc.

The combination of two phenomena, convection and pure thermomodiffusion, is called thermogravitational diffusion. The coupling of these two phenomena leads, in some circumstance, to large species separation. Clusius and Dickey [6] successfully carried out the separation of gas mixtures in a vertical cavity heated from the side, namely a thermogravitational column (TGC). The optimum separation, between the two ends of the cell is obtained for an appropriate choice of the convective velocity and the mass diffusion time. This optimum is obtained for a very small thickness of the vertical cell. In a vertical thermogravitational column the thermal gradient applied on the wall induces simultaneously a pure diffusion phenomenon and a natural convection flow. Furry, Jones and Onsager [7] developed a fundamental theory to interpret the experimental processes of isotope separation. Lorenz and Emery [8] introduced a porous medium in the TGC columns to increase the separation. More recently, many works were carried out with the aim to increase the separation and to study the linear stability of Soret-driven convection in different configurations. Platten et al. [9] used a tilted thermogravitational column. Experiments were performed with water–ethanol mixtures. Bou-Ali et al. [10] were interested in mixtures with negative Soret coefficient in the TGC. Elhajjar et al. [11] developed a linear stability analysis of the unicellular flow which appears when the separation ratio is higher than a certain positive value, in a Rayleigh–Bénard configuration. Zebib and Bou-Ali [12] performed a linear stability analysis of a binary mixture buoyant return flow in a tilted differentially heated infinite layer using asymptotic long-wave analysis and pseudo-spectral Chebyshev numerical solutions. Elhajjar et al. [13] presented a theoretical and numerical study of species separation in an inclined porous cavity.

In the present configuration, in order to obtain an analytical solution of the unicellular flow, a shallow cavity along the x direction is considered (B = L/H >> 1, B being the aspect ratio of the rectangular cavity). A portion of one of its small sides along the z axis is submitted to an ultrasonic propagating wave while the opposite wall is a perfectly absorbing wall. The sides along the x axis are maintained at uniform temperature T1 and T2, respectively. The normalized width : z of the acoustic beam varies between 0 and 1, and the beam is applied from the top of the cell (z = H). Acoustic streaming describes a steady flow generated by an ultrasound wave propagating in a fluid. This effect was first observed in 1831 by Faraday [14]. It is well known that it is a non linear phenomenon whose origin is due to Reynolds stress and the dissipation of the acoustic energy flux. Nyborg [15] showed that a constant radiation pressure associated to an ultrasonic traveling wave and generated in a given direction (x direction in our configuration) is associated to a body force oriented along the x axis. Its intensity is given by: \( F = \rho \Delta U_T^2 e^{-\alpha z} \), where \( \alpha \) is the amplitude attenuation coefficient for an ultrasound wave, and \( U_T \) is the amplitude of the acoustic velocity oscillation. This expression of the force \( F \) is valid under the assumptions of plane wave and negligible divergence of the beam. Under the assumption that the attenuation of the wave is sufficiently weak, the body force is considered as a constant \( F = \rho x U_T^2 \) inside the beam of height \( H_b \) and equal to zero outside the beam. This body force can be introduced in the Navier–Stokes equations [1].

The species separation per unit of length, m, was calculated as a function of the acoustic streaming parameter A, Sc and ε. These analytical results were corroborated by direct 2D numerical simulations. The linear stability analysis of the unicellular flow, performed for an infinite layer, showed that the unicellular flow loses its stability via Hopf bifurcation for all the values of the acoustic beam width, ε, except for ε = 0.5 for which the transition is a stationary one.

2. Mathematical formulation

We consider a rectangular cavity of large aspect ratio \( B = L/H \), where \( H \) is the height of the cavity along the z-axis and \( L \) is the length along the x-axis. The cavity is filled with a binary fluid mixture of density \( \rho \) and dynamic viscosity \( \mu \). The two walls \( x = 0 \) and \( x = L \) are adiabatic and impermeable. The two other walls \( z = 0 \) and \( z = H \) are kept at uniform temperature \( T_1 \) for \( z = 0 \) and \( T_2 \) for \( z = H \). The mathematical formulation is presented taking into account the gravity field in order to obtain the general formulation: \( \ddot{g} = -\gamma \), (Fig. 1). The Boussinesq approximation is assumed valid, thus, the thermo-physical properties of the binary fluid are constant, except the density in the buoyancy term which varies linearly with the local temperature \( T \) and the mass fraction C of the considered component:

\[
\rho = \rho_0 (1 - \beta_T (T - T_{\text{ref}}) - \beta_C (C - C_{\text{ref}}))
\]

where \( \beta_T \) and \( \beta_C \) are respectively the thermal and mass expansion coefficients of the binary fluid.

The dimensionless mathematical formulation of the problem is given by:
\[
\n\n\frac{\partial}{\partial t} \vec{V} = 0 \\
\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\nabla p + \mathbf{A} \delta \vec{e}_z + G_r(T + \psi C)\vec{e}_z + \nabla^2 \vec{V}
\]

(2)

where the parameter \( \delta \), in the system (2), is defined by: \( \delta = 1 \) for \( z \in [1 - \varepsilon, 1] \) and \( \delta = 0 \) for \( z \in [0, 1 - \varepsilon] \).

The corresponding dimensionless boundary conditions are:

\[
\begin{align*}
\theta &= 0 \quad & & \quad \text{at } x = 0, x = B \ ; \ \vec{V} = 0 \quad & & \quad \text{at } z = 1
\end{align*}
\]

(3)

Note: If we choose \( \Delta C = -\Delta \theta (\beta_1 / \beta_2) \) rather than \( \Delta C = -\Delta \theta (1 - C_0) / D_T / D_r / \theta / \theta_0 \) as reference scale for the mass fraction, the separation ratio, \( \psi \), will appear in the right-hand-side term of the species equation and in the boundary conditions but it will not be present in the Navier-Stokes equation.

3. Analytical solution in the case of a shallow cavity: uncellular flow in microgravity (\( G_r = 0 \))

3.1. Closed-form analytical solution

In the case of a shallow cavity \( B \gg 1 \), in microgravity (i.e. \( G_r = 0 \)), the parallel flow approximation, used by many previous authors [11], is considered. The solution corresponding to the uncellular flow is given as follows:

\[
\begin{align*}
\bar{U}_0 &= U_0(z) \vec{e}_z & & \quad \text{at } z = 1 - \varepsilon \\
\bar{T}_0 &= 1 - z & & \quad \text{at } C_0(x, z) = m x + f(x) \quad (4)
\end{align*}
\]

where \( m \) is the mass fraction gradient along the \( x \) axis.

The traveling wave is supposed not to interact with the wall at \( z = 1 \).

With these assumptions and for the steady state, the system of Eq. (2) with the boundary conditions (3) is reduced to a set of the following equations solved using Maple software:

\[
\begin{align*}
\frac{d^2 \vec{e}_z}{d z^2} + A \vec{e}_z &= 0 \\
\frac{d \theta}{d z} &= 0 \\
mU_0Sc &= \frac{d \theta}{d z}
\end{align*}
\]

(5)

The two first equations obtained in the system (5) were considered by Dridi et al. [5]. To solve the system (5), the following assumptions are considered:

- Continuity of the velocity, the stress constraint, the mass fraction and temperature at the interface of the acoustic beam for \( z = 1 - \varepsilon \).
- The mass flow rate through any cross section perpendicular to the \( x \)-axis is equal to zero.
- The mass flow rate of the component of mass fraction \( C \) on all the cell is equal to zero.

Thus the expression of the velocity, the mass fraction and the temperature field are given by the following expressions:

\[
\begin{align*}
U_0(z) &= \frac{1}{2} A (\varepsilon - 1)^2 (z - 1) / 2 (z + 1) - 1 \quad \text{for } 1 - \varepsilon \leq z \leq 1 \\
C_0(x, z) &= mx + f_1(x, A, B, m, Sc, \varepsilon) \quad \text{for } 0 \leq z \leq 1 - \varepsilon \\
C_0(x, z) &= mx + f_1(x, A, B, m, Sc, \varepsilon) - \frac{1}{24} \bar{A} \bar{Sc} m (\varepsilon + 1 - z)^4 \\
& \quad \text{for } 1 - \varepsilon \leq z \leq 1
\end{align*}
\]

(6)

where \( f_1 = \frac{1}{24 \bar{A} \bar{Sc} m (\varepsilon + 1 - z)^4} \)

The flow velocity component \( U_0(z) \) is thus proportional to the intensity of the acoustic parameter \( A \). The expressions (6) give the velocity a particular case of those obtained by Dridi et al. [5] for a non-centered beam. Our case corresponds to a particular condition \( 2z + \bar{H}_0 = 1/2 \), where \( z_0 \) is the coordinate of the center of the beam with respect to the centre of the layer and \( \bar{H}_0 = \varepsilon \) in the nomenclature of the paper [5]. For a given value of \( A \), the maximum velocity \( U_{\text{max}} \) is obtained for:

\[
\varepsilon_1 = \frac{1}{2} (1 + \sqrt{13}) / 6 \simeq 0.434 \text{ at } z_1 = 1 + \sqrt{13} / 6
\]

\[
\varepsilon_2 = \frac{7 - \sqrt{13}}{6} \simeq 0.566 \text{ at } z_2 = 5 - \sqrt{13} / 6 \simeq 0.232 \text{ with } U_{\text{max}} = \pm A (46 + 13 \sqrt{13}) / 54 \simeq 0.0162 A.
\]

These two sets of values \( (\varepsilon_1, z_1) \) and \( (\varepsilon_2, z_2) \) are symmetrical with respect to the point \( (\varepsilon = 1/2, z = 1/2) \) and verify the two following relations:

\[
\varepsilon_1 + \varepsilon_2 = 1 = z_1 + z_2 = 1.
\]

In Section 4 an interpretation of this result is provided.

The choice of \( \varepsilon_1 \) or \( \varepsilon_2 \) leading to the maximum velocity affects the time required for the species separation, \( S \) (§ 3.2).

The velocity profile is presented, in Fig. 2, for \( \varepsilon = 0.2, 0.5 \) and \( \varepsilon = 0.8 \) for \( A = 0.45 \), value of the acoustic parameter for which the species separation is maximum for \( Sc = 700 \) (see § 3.2). It can be observed that the velocity profile is symmetrical with respect the point \( z = 1/2, U_0 = 0 \) only for \( \varepsilon = 0.5 \). Thus, the velocity profiles for \( \varepsilon = 0.2 \) and \( \varepsilon = 0.8 \) for \( A = 0.45 \) are symmetrical to one another with respect to the point \( (U_0 = 0, z = 0.5) \). Similar profiles, corresponding to beams along the boundaries (width = 0.3), were obtained by Dridi et al. [5].

As \( \varepsilon \) tends to 0 or 1, the velocity profile generated across the cell by the ultrasonic wave is similar to the one obtained when a constant tangential velocity \( U_t \) is applied in \( z = 1 \) or 0. Indeed, at the boundaries \( z = 0 \) or 1 the velocity is equal to zero in the presence of acoustic streaming and equal to \( U_t \) when a constant tangential velocity is applied to one of the horizontal walls. (Fig. 3). Note that when \( \varepsilon \) tends to 0 or 1, the velocity profiles at constant \( A \) tend towards zero amplitude profiles.

Note: In all the following sections, the Schmidt number is \( Sc = 700 \) corresponding to a water–ethanol binary fluid where the values of Lewis number and Prandtl number are around 100 and 7 respectively.
3.2. Study of the species separation, $S$

The species separation in the binary mixture is studied in this section. The separation $S$ is defined as the difference in mass fraction of the considered species between the two ends of the cell, $x = 0$ and $x = B$, and its expression is $S = mB$, with $m$ defined as:

$$m = -\left(\frac{105}{2}\right)^{-1} \frac{A \text{Sc} \varepsilon^2 (1 - \varepsilon)^7}{A^4 \text{Sc} \varepsilon^2 (\varepsilon^2 - 3)(1 - \varepsilon)^4 - 1260}$$  (7)

As indicated in Eq. (7), the separation $S = mB$ is function of $A$, $\text{Sc}$, $\varepsilon$ and $B$ (concentration gradient $m$ does not depend on $B$). The expressions giving the mass fraction $G_0$ (Eq. (6)) and the mass fraction gradient $m$ (Eq. (7)) depend on $A$ and on $\text{Sc}$ but only via the product $A \text{Sc}$ which can be written $A \delta = \pi B^2 H^3/\rho D$ as and called modified acoustic streaming parameter. From the Eq. (7), the mass fraction gradient, $m$, is equal to $0$ when $\varepsilon = 0$ or $1$ or when the modified acoustic streaming parameter $A \delta = 0$. Indeed, for these particular values of $A \delta$ and $\varepsilon$, the binary fluid velocity is equal to zero inside the cell.

The maximum mass-fraction gradient $m_{\text{max}}$ with respect to $\varepsilon$ is obtained analytically using Eq. (7) for $\varepsilon = 0.5$ when $A \delta \leq \frac{120 \cdot 455}{5} \approx 315.04$. In this range, there is an increase of $m_{\text{max}}$ with $A \delta$ from $0$ to $m_{\text{max}} = \frac{120 \cdot 455}{5} = 0.41$. For $A \delta > \frac{120 \cdot 455}{5} \approx 315.04$, $m_{\text{max}}$ is no more obtained for $\varepsilon = 0.5$ but for two values of $\varepsilon$, which are symmetric with respect to $\varepsilon = 0.5$ and which evolve with $A \delta$. In this second range, $m_{\text{max}}$ increases slowly and reaches $m \approx 0.427$ when $\varepsilon$ tends to $0$ or $1$.

These values are close to the one obtained for the thermo-gravitational columns (TGC). For a horizontal porous cell saturated by binary fluid, the optimum value of $m$ is $0.45$ [13]. To highlight this result, the curves of the mass fraction gradient, $m$, versus $\varepsilon$ are plotted for four values of $A \delta (230, 315.04, 1428$ and $1428 \times 10^2)$ in Fig. 4. A symmetry of the curves with respect to $\varepsilon = 0.5$ can be observed in Fig. 4. This result is to be expected as the experiment takes place in microgravity.

In Table 1, we present the different values of the mass fraction gradient obtained analytically for $A \delta \in [0, 14285]$ and in Fig. 5 the evolution of the mass fraction gradient versus $A \delta$ for $\varepsilon = 0.5$.

### 4. Numerical simulations

The system of Eq. (2) (with $Gr = 0$) associated to the boundary conditions (3) is solved numerically using a finite element code.
(Comsol industrial code) with a rectangular grid, better suited to the rectangular configuration. For the computations, an aspect ratio $B = 10$ is considered. The quadrangle spatial resolution is $120 \times 20$ or $150 \times 30$ for high values of the modified acoustic streaming parameter $A_d$.

In order to eliminate the boundary effect, to determine the separation, the curve $C = g(x)$ is plotted at a given value of $z$ ($z = 0.5$ for instance) and the slope of the curve, which is a straight line in the central part of the cell, is calculated.

As mentioned in § 3, in Fig. 4, the numerical results obtained for $m$ versus $\varepsilon$ are reported on the curve determined analytically for $A_d = 315.04$. A very good agreement is observed between analytical and numerical results. A similar result is also observed in Figs. 2 and 5.

The mass fraction field, obtained for $A_d = 315.04$ is presented for four values of $\varepsilon$: $\varepsilon = 0.1, 0.5, 0.8, 0.9$, and $\varepsilon = 1$, in Fig. 6. The black lines represent the iso-concentrations. As expected the mass fraction gradient values $m$, obtained for $\varepsilon = 0.1$ and 0.9 are equal. Indeed for a given value of the acoustic streaming parameter $A$, the unicellular flow induced by an acoustic beam of thickness $\varepsilon$ is opposite to the flow generated by an acoustic beam of thickness $(1-\varepsilon)$ positioned on the top of the opposite side. This unicellular flow is also identical to the flow generated by an acoustic beam of thickness $(1-\varepsilon)$ positioned on the bottom of the opposite side of the cell. Therefore, this study can be limited to $\varepsilon$ varying from 0 to 0.5 or $\varepsilon$ varying from 0.5 to 1.

The evolution of the mass fraction field is presented, in Fig. 7, for $A_d = \frac{100}{\varepsilon}$ ($0.6, 22, 50, 100, 200, 600, 1000$) and for $\varepsilon = 0.5$.

For very small values of $A_d$, the concentration field stratification is horizontal and for values of $A_d$ exceeding $A_{dopt} = 315.04$ there is a significant deformation of the mass fraction field leading to a small separation.

5. Linear stability analysis of the unicellular flow

In order to analyze the stability of this unicellular flow described in § 3, the perturbations of velocity $\vec{v} = (u, v, w)$, of temperature $\theta$, of mass fraction $c$ and pressure $p$ are first introduced. The perturbations ($\vec{v}, \theta, c, p$) are assumed to be of small amplitude, then, after simplification the following linearized equations are obtained:

\[
\begin{align*}
\frac{\partial^2}{\partial t^2} - m \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} \right) &= 0 \\
\frac{\partial}{\partial t} - m \frac{\partial}{\partial z} + U_0 \frac{\partial}{\partial x} - \frac{\partial}{\partial x} (U_0) - \nabla^2 w - Gr \frac{\partial}{\partial t} (\theta + \psi c) &= 0 \\
\frac{\partial}{\partial z} - m \frac{\partial}{\partial x} + U_0 \frac{\partial}{\partial y} - m \frac{\partial}{\partial y} (U_0) - \frac{\partial}{\partial x} (\nabla^2 (c - \theta)) &= 0
\end{align*}
\]

(8)

The associated boundary conditions are:

\[w = \theta = 0, \text{ and } \frac{\partial \theta}{\partial z} = \frac{\partial c}{\partial z} \text{ for } z = 0 \text{ and } z = 1 \ \forall x, y, t\]  

(9)

In microgravity the Grashof number, $Gr$, is equal to zero. The second and the third equations of the system (8) are decoupled from the first one which involves only the perturbation velocity component $w$.

The stability of the basic flow is then investigated, using only the first equation of system (8), by a temporal linear analysis. The perturbation quantities are chosen as follows:

\[w = \tilde{w}(x) \exp(i k x + i h y + \sigma t)\]

(10)

where $k$ and $h$ are the wavenumbers respectively in the $x$ and $y$ directions, $\sigma = -1$, and $\sigma = \sigma_0 + i \omega_0$ is a complex eigenvalue. The real part of $\sigma$, $\sigma_0$, represents an amplification rate and its imaginary part, $\omega_0$, the Hopf pulsation.

Fig. 5. Analytical curve of the mass fraction gradient $m$ versus the acoustic parameter $A_d$ for $\varepsilon = 0.5$. Solid circles give the mass fraction gradient $m$ versus $A_d$ obtained in a two dimensional cavity for $\varepsilon = 0.5$ and $B = 10$.

Fig. 6. Mass fraction fields and iso-concentrations for different values of $\varepsilon$ and $A_{dopt}$.
Of interest are the instabilities leading to convective rolls with axis perpendicular to the direction of the base flow due only to ultrasonic waves in microgravity, therefore, the wavenumber $h$ is equal to zero. For $Gr = 0$, the first equation of system (8), after substituting $w$ by its expression given by Eq. (10), leads to a fourth order equation depending only on the velocity component $\tilde{w}(z)$:

$$
\zeta^2(\tilde{w}) - (\sigma + IkU_0)\zeta(\tilde{w}) + Ik\tilde{w}\frac{d^2}{dz^2}(U_0) = 0
$$

(11)

where the derivative operators $\zeta = \frac{d}{dz} - k^2$ and $\zeta^2 = \frac{d^2}{dz^2} - 2k^2 \frac{d}{dz} + k^4$.

The same stability problem was treated by Dridi et al. [5] in the case of a non-centered ultra sound beam. However, in the current work, the focus is different as only specific beams along the boundaries are considered. The critical values of the acoustic parameter, $A_c$, are first determined using the Galerkin method for stationary and Hopf bifurcation: the disturbance $\tilde{w}$ is developed as a polynomial function which verifies all the boundary conditions and the different polynomials obtained when $i$ varies from 1 to $N$ form a complete basis of the studied problem:

$$
\tilde{w}(z) = \sum_{i=1}^{N} a_i(z - 1)^{i-1} z^2
$$

(12)

The convergence for the resolution using the Galerkin method is obtained for $N \geq 12$.

Two other methods have been used to solve the eigenvalue problem resulting from the temporal stability analysis for the determination of the critical parameters corresponding to either stationary or Hopf bifurcation: a spectral Tau–Chebychev method with more than 200 collocation points and a finite element method with quadrangle spatial resolution $150 \times 30$. The results obtained by these three methods are in good agreement for both stationary and Hopf bifurcation.

The neutral stability curves describing the evolution of the critical acoustic streaming parameter $A_c$, Hopf pulsation $\omega_c$ and wave length $k_c$ as a function of $\varepsilon$ are presented respectively in Figs. 8(a)–8(c). It can be observed that the bifurcation is a stationary one ($\omega = 0$) only for $\varepsilon = 0.5$.

The neutral curve which corresponds to the onset of oscillatory instabilities presents a minimum value for $\varepsilon = 0.5$, associated to a stationary bifurcation characterized by the critical parameters: $A_c = 3019$ and $k_c = 3.33$. The critical value $A_c$ increases and $k_c$ decreases for smaller and higher values of $\varepsilon$ around $\varepsilon = 0.5$ while the absolute value of the critical angular frequency increases. The increase of $A_c$ for large or small values of $\varepsilon$, symmetrical with

Fig. 7. Mass fraction fields and iso-concentrations for different values of $A_d$: $\varepsilon = 0.5$. 
Some particular beams along the boundaries were considered by Dridi et al. [5], namely those corresponding to \( z_0 = 0.035 \) with \( \varepsilon = 0.3 \). The corresponding results can be found in their Fig. 6, and it can be seen that such beams with \( \varepsilon = 0.3 \) correspond to instabilities with negative values of the angular frequency \( \omega_c \). We show that for \( \varepsilon = 0.3 \) and \( \varepsilon = 0.7 \), \( \omega_c(\varepsilon) = -\omega_c(1-\varepsilon) \) whereas \( A_1(\varepsilon) = A_1(1-\varepsilon) \) and \( k_1(\varepsilon) = k_1(1-\varepsilon) \). The critical values obtained for \( \varepsilon = 0.5 \) and 0.3 for different orders of spectral approximation are also presented in Table 2. The values obtained for \( A_1 \), \( \omega_c \) and \( k_1 \) are in good agreement with those extrapolated from Fig. 6 presented by Dridi et al. [5], for \( \varepsilon = 0.3 \) and \( z_0 = 0.035 \).

### 6. Conclusion

In this paper, the influence of the acoustic streaming on species separation in a rectangular cavity, filled with a binary fluid in weightlessness, was presented. A new experimental configuration was considered in order to obtain species separation in a binary mixture. To the authors’ knowledge, no work has yet been presented on this topic.

In a first part an analytical solution of the unicellular flow induced by the ultrasound traveling wave was determined using the assumption of parallel flow observed for \( B \gg 1 \). The velocity profiles were plotted for \( \varepsilon = 0.5 \) and 0.2. As expected the velocity profile is symmetrical with respect to \( z = 0.5 \) only for \( \varepsilon = 0.5 \). Furthermore the velocity profiles \( U_d(\varepsilon) \) obtained for \( \varepsilon = 0.2 \) and 0.8 are symmetrical to one another with respect to the point \( (U_d(\varepsilon) = 0, z = 0.5) \).

Thus, the mass-fraction gradient \( m \) was calculated as function of the acoustic streaming parameter, \( A_1 \), the acoustic beam width, \( \varepsilon \), the Schmidt number and its variation was presented as a function of the modified acoustic streaming parameter \( A_d \) and \( \varepsilon \). The maximum separation was obtained for \( \varepsilon = 0.5 \) and for \( A_{dopt} = \frac{A_d}{5\pi R^2} \approx 315.04 \).

The analytical results were corroborated by direct numerical simulations using a finite element code (Comsol). The mass-fraction gradients versus \( \varepsilon \) were plotted for different values of \( A_d \). The symmetry of the curves \( m = f(\varepsilon) \) with respect to \( \varepsilon = 0.5 \) is thus illustrated. The mass fraction fields were also plotted for different values of \( \varepsilon \) for \( A_{dopt} \) and for different values of \( A_d \) for \( \varepsilon = 0.5 \). The evolution of the iso-concentrations when \( A_d \) increases was also presented.

In a second part a linear stability analysis of the unicellular flow was performed using the Galerkin method, a spectral Tau–Chebychev method and a finite element method. The results obtained by these three methods were in good agreement. This

### Table 2

Values of the critical parameters for different orders of spectral approximation for \( \varepsilon = 0.3 \) and 0.5.

<table>
<thead>
<tr>
<th>Spectral approximation orders</th>
<th>( A_1(\varepsilon = 0.3) )</th>
<th>( k_1(\varepsilon = 0.3) )</th>
<th>( \omega_c(\varepsilon = 0.3) )</th>
<th>( A_1(\varepsilon = 0.5) )</th>
<th>( k_1(\varepsilon = 0.5) )</th>
</tr>
</thead>
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<td>3.12</td>
<td>-38.77</td>
<td>3136</td>
<td>3.25</td>
</tr>
<tr>
<td>6</td>
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<td>3.11</td>
<td>-44.60</td>
<td>3025</td>
<td>3.31</td>
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<tr>
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Dridi et al. [5]

for \( \varepsilon = 0.3 \)
study showed that the unicellular flow loses its stability via a Hopf bifurcation for all values of $\varepsilon$, except for $\varepsilon = 0.5$, for which the transition is a stationary one. The values of the critical acoustic parameter $A_c$ obtained for different values of $\varepsilon$ are much larger than the optimum value $A_{opt}$ leading to the maximum of separation.

Acknowledgment

We wish to acknowledge the valuable comments made by the anonymous referee and specially his helpful suggestion concerning the reference scales used to obtain the present nondimensional formulation.

References