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A bypass transition in the Lamb-Oseen vortex

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Outline

**Observation of high energy transient amplifications** in the Lamb-Oseen vortex by means of the Linear Optimal Perturbation approach

a) Nonlinear Optimal Perturbation analysis?

Axisymmetrization process in the Lamb-Oseen monopole: a generic process?

**Emergence of tripole vortex observation in numerical simulations**

b) Nonlinear Optimal Perturbation: a potential path to a nonlinear bypass transition?

Rossi et al. 1997
The Lamb-Oseen vortex

Reynolds number \(Re = \frac{\Omega_0 R_0}{\nu}\)
Axial vorticity \(Z(r, t) = 2 \exp(-r^2/(1 + 4t/Re))/(1 + 4t/Re)\)
Angular velocity \(\Omega(r, t) = [1 - \exp(-r^2/(1 + 4t/Re))] / r^2\)

Linear stability analysis: one-signed vorticity gradient distribution is **linearly stable** at large times solution.

Shear-diffusion mechanism drives the axysimmetrization process (on the \(Re^{1/3}\) time scale for large Reynolds number flows - Bernoff and Lingevitch, 1994).
Transient energy growth

Why?

Classical stability analysis:

- **small perturbations** in the base flow
  \[ \phi(r, \theta, z, t) = \Phi(r, \theta, z) + \phi'(r, \theta, z, t) \quad \phi' / \Phi \ll 1; \]
- **linearization** around the base state + modal decomposition
  \[ \phi'(r, \theta, z, t) = \hat{\phi}(r) \exp \{i(kz + m\theta - \omega t)\}; \]
- eigenvalue analysis.

If all eigenvalues are in the stable complex half-plane,

The flow is linearly stable.

**But..** transient energy amplifications are possible if the governing system is not normal.

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Time exponential damping

Trefethen et al. 1993
Transient energy growth
Why?

Classical stability analysis:

- **small perturbations** in the base flow
  \[ \phi(r, \theta, z, t) = \Phi(r, \theta, z) + \phi'(r, \theta, z, t) \quad \phi' / \Phi << 1; \]
- **linearization** around the base state + modal decomposition
  \[ \phi'(r, \theta, z, t) = \hat{\phi}(r) \exp \{ i (kz + m\theta - \omega t) \}; \]
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If all eigenvalues are in the stable complex half-plane,

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Classical stability analysis:

- **small perturbations** in the base flow
  \[ \phi(r, \theta, z, t) = \Phi(r, \theta, z) + \phi'(r, \theta, z, t) \quad \phi'/\Phi \ll 1; \]
- **linearization** around the base state + modal decomposition
  \[ \phi'(r, \theta, z, t) = \hat{\phi}(r) \exp \{ i(kz + m\theta - \omega t) \}; \]
- eigenvalue analysis.

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Trefethen et al. 1993
The Optimal Perturbation approach

Analytical background

The “optimal perturbation” maximizes the gain at a given time (Farrell 1988).

Lagrangian multiplier technique to find Nonlinear Optimal Perturbation (Pringle & Kerswell 2010).

\[ \mathcal{L} = E(\tau) - \langle \mathcal{F}(u), a \rangle - \lambda(E(0) - E_0) \]

\[ \frac{\partial \mathcal{L}}{\partial \epsilon} \delta \epsilon = 0 \quad \Rightarrow \]

- Navier Stokes equations + b.c.
- Adjoint equations + b.c.
- Compatibility conditions

... and a pseudo-spectral code!
Transient energy growth
2D Linear Optimal Perturbation

\[ Re = 5000; k = 0 \]
Transient energy growth
2D Linear Optimal Perturbation

\[ Re = 5000; k = 0 \]
Transient energy growth
2D Nonlinear Optimal Perturbation
Transient energy growth
2D Nonlinear Optimal Perturbation

$t = 0$  $t = 15$  $t = 30$  $t = 60$

$E_0 = 0.0001$

$E_0 = 0.001$

$E_0 = 0.01$

$t = 120$  $t = 200$

$t = 0$  $t = 15$  $t = 30$  $t = 60$  $t = 120$  $t = 200$
Conclusions

- Nonlinear optimal perturbations: remarkable differences with respect to the linear case;

- Axisymmetrization is a systematic process only in the linear approach;

- High-energy tripole generation as a nonlinear bypass transition induced by a nonlinear transient growth mechanism revealed by a nonlinear optimal perturbation analysis;

- $\overline{E}_0$ 'threshold' as $(Re, \tau)$ function (in Rossi & al, 1997 and Barba & Leonard, 2007 but differences...);

- Kinematic energy gain: the most effective objective function to induce transition?