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Adjoint-based sensitivity and feedback control of noise emission

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Plan

1. Introduction and testcase
2. Sensitivity analysis
3. Direct feedback output control
4. Application and analysis
5. Summary and perspectives
Active flow control strategies

- In addition to the passive flow control and the shape optimization, it is a necessity to control flows to enhance performances in transportation vehicles: drag, lift, noise emission, flow instabilities, separation, ...

- My challenge: to propose a methodology based on theoretical and numerical approaches for actuation law design dealing with large system (DoF $\geq 10^6$)

1. **Open loop control**:
   - optimization problem with full PDE (DNS, LES, ...)
   - expensive and time consuming
   - low robustness

2. **Feedback control**: more efficient, used in real flow and systems, robustness can be a parameter or an issue, first step towards adaptive control.
Feedback control: to manage the huge size of fluid flow configurations

- Need a Reduced Order Model

1. ROM based on global stability modes: laminar flow $\rightarrow$ turbulent flow

2. ROM based on POD modes: laminar flow $\rightarrow$ moderate turbulent flow.
   Rowley (Balanced POD, 2012), Airiau & Cordier (2013), ...

3. POD analysis + heuristic feedback law based on physical considerations
   Pastoor et al (2008), ...

- Usual feedback loop (sensors $\rightarrow$ state estimate $\rightarrow$ actuation)
  Done with ROM

- Present work: with DNS, direct output feedback law
  sensor outputs $\rightarrow$ actuation law
**Introduction and testcase**

**General methodology: control of a ROM**

- Application to any flow control as soon as a POD is relevant
- 8 steps: large developments and programming
- DNS: large DOF (\(> 10^6\))
- ROM/LINEAR CONTROL: very low DOF (\(< 10\))
- Computational cost: 2 DNS + ROM
- Many parameters, options and choices...
- What else?

**Issues for an efficient feedback law:**

- Optimal position and type of actuators (controlability)
- Optimal position and type of sensors (observability)
- Well capturing the response of the flow (computed by DNS) to any actuation: actuation mode(s)

**POD:** Proper Orthogonal Decomposition
**1a - Testcase : 2D compressible cavity flow**

- $M_\infty = 0.6$, $R_\theta = 688$, $Re_L = 2981$, noise control is a good (severe) testcase

- $\text{SPL in dB}$
  - $n=101$, $x/h=3.33$, $y/h = 0.82$ and wall
  - $O_{1}$ and $O_{3}$ : center point of the observation domain for sensitivity
  - $O_{2}$ : probes for spectral analysis (SPL)

- Self-sustained instability due to a feedback effects with the impingement of the shear layer on the downstream cavity corner

- instantaneous pressure, acoustic wave directivity Rowley’s case

- Rossiter 2 : $St_L = 0.74$

- Tescase $L/H = 2$
  - (Rowley, JFM 2002)
  - $St_L = \frac{n - 0.25}{M_\infty + 1.754}$
  - $St_L = 0.703$
Actuator position and type provided by the sensitivity analysis

1. Observed quantity (functional)

\[
J(q, f) = \int_\Omega \int_0^T j_{\text{observed}} d\Omega dt, \quad j_{\text{observed}} = \frac{1}{2}(p - \bar{p})^2
\]

\(q\) : state vector, \(f\) forcing/actuation vector

2. Variational problem and Fréchet derivative : sensitivity \(S(x, t)\)

\[
\delta J = \left\langle \frac{\partial j_{\text{observed}}}{\partial q_k}, \delta q_k \right\rangle_\Omega = \left\langle S_{f_i}, \delta f_i \right\rangle_\Omega
\]

3. Adjoint Navier-Stokes equations : sensitivity is related to the adjoint state : \(S_{f_i} = q_i^*\)

4. true for wall localized forcing and global volume forcing
Sensitivity analysis

0 - Actuators settings

- 2D Fourier modes, adjoint state (case 10) from adjoint DNS
  - Stationary mode sensitivity: steady actuation
    - 2nd Rossiter mode sensitivity: unsteady actuation
      - \( \hat{p}^* \) and \( \hat{m}_x^* \)

Modes localized in space associated to the controllability property

Figure 3 - Modes at \( St_1 \) for \( \hat{m}_x^*/U_\infty \) (blue: 0.48, red: 7.22) and \( \hat{p}^*/U_\infty^2 \) (blue: 0.23, red: 3.71)
1b - DNS response to a generic actuation

- Need to define actuation mode in the ROM from actuated DNS
- Wall normal velocity forcing
  - Distributed actuation centered at $x_f = 2.72X/D$
    \[ f_w(x, t) = \gamma(t) \exp[-r^2/\sigma^2], \quad r^2 = ||x - x_{for}||^2, \sigma = 50\Delta y \]
  - Large frequency bandwidth actuation $A_1(t)$:
    \[ \gamma(t) = A_1 \sin(2\pi St_1 t) \times \sin(2\pi St_2 t - A_2 \sin(2\pi St_3 t)) \]

To excite and therefore later to control all possible physical unstable perturbations

$St_{cavity} \approx 0.7$
Proper Orthogonal Decomposition of unactuated flow field \( \rightarrow \phi_i^u(x) \)

POD mesh size \(<\) DNS mesh \(\Rightarrow\) gain in CPU time and accuracy
Optimal size?

\[
q^a(x, t) = \bar{q}^a(x) + \sum_{i=1}^{N} a_i^a(t) \phi_i^u(x) + \gamma(t) \psi(x)
\]  

- \((\phi_{i=1,N}, \psi)\) orthogonal basis, truncation to \(N\) modes
- Assumption: \(\bar{q}^a(x) \approx \bar{q}^u(x)\)
- \(q = (\zeta = 1/\rho, u, v, p)\)
3 - Actuation mode, $A_1 = 0.001$

Sine forcing, $u$ velocity, Vigo’s and isentropic models

Distributed actuation along the wall and in the shear layer

Chirp, $u$ velocity, Vigo’s model

Actuation close to the noise source location
Direct feedback output control

5, 6a, 6b - ROM-Galerkin projection

- **Projection** of NSE (formulation with $\zeta = 1/\rho$, Vigo-98, Bourguet-09) on $(\phi^u_{i=1,N})$:
  Nonlinear forced dynamical system of low order

  \[
  \dot{a} = C + La + a^t Qa + \gamma \hat{L}a + \gamma \hat{C} + \gamma^2 \hat{Q}
  \]  

- **Calibration** of ROM (find $C$ and $L$ for $a(t)_{POD} = a(t)_{ROM}$)

- **Equilibrium (steady) state** (many states can exist):

  Physical domain:
  \[
  q^e(x) = \bar{q}(x) + \sum_{i=1}^{N} a^e_i \phi_i(x)
  \]

  Equilibrium state of the NS eq.

- **Linearization** with $\tilde{a} = a - a^e$:

  \[
  \dot{\tilde{a}} = L\tilde{a} + \tilde{a}^t Qa^e + (a^e)^t Q\tilde{a} + (\hat{L}a^e + \hat{C})\gamma
  \]

  \[
  \dot{\tilde{a}} = \tilde{A}\tilde{a} + \tilde{B} \gamma \quad \text{State equation}
  \]
4 - Use of sensors: output identification model

Required for the feedback control law design

- Unsteady pressure sensors ($\tilde{y}_i = \tilde{p}_i$):
  
  \[ \tilde{y} = \tilde{C}\tilde{a} + \tilde{D}\gamma, \quad \tilde{y} = y - \bar{y} - \tilde{C}a^e \]  

- Sensor $y_i$ is located on the POD mesh at $x_k$:
  
  $\tilde{C}_{ij} = \phi_j^u(x_k)$ and $\tilde{D}_i = \psi_k = \psi(x_k)$.

$N_s = 6 \ (1 \rightarrow 6)$ sensors are used to build the actuation law.

Optimal positions: physical considerations, observability
direct output feedback control law design

- linear state space model: \( \dot{\tilde{a}} = \tilde{A}\tilde{a} + \tilde{B}\gamma \)
- feedback control law: \( \gamma = -K_c \tilde{a} \)
- minimization of \( J = \int_0^T (\tilde{a}^T \tilde{a} + \ell^2 \gamma^2) \, dt \).
- Ricatti equation: \( K_c = \frac{1}{\ell^2} \tilde{B}^T X \cdot (\tilde{A}^T X + X\tilde{A} - \frac{1}{\ell^2} X\tilde{B}\tilde{B}^T X + I d = 0) \).
- outputs: \( \tilde{y} = \tilde{C}\tilde{a} + \tilde{D}\gamma \)
- direct feedback output control: \( \gamma(t) = \alpha(y(t) - \bar{y}) + \beta \) \hspace{1cm} (5)

- \( N_{POD} = N_{Sensors} \) : \( \gamma(t) = -K_c(\tilde{C} - \tilde{D}K_c)^{-1}(y - \bar{y} - \tilde{C}a^e) \)
- \( \beta \) imposes the mean actuation velocity, \( \alpha \) imposes the damping of the time variation of the actuation
- implementation in DNS code
Application and analysis

application: efficiency & robustness

- Efficient and robust feedback control law: decay of pressure fluctuation levels

\[ \gamma(t) = \alpha(y(t) - \bar{y}) + \beta, \]

- Tuning \( \beta \) to improve the efficiency
  - A) \( \beta \approx 5 \text{ m/s} \),
  - B) \( \beta \approx 10 \text{ m/s} \),
  - C) \( \beta \approx 17 \text{ m/s} \)

- Robustness, feedback law
  - Time window: \( \text{it}=25000 \)
  - Spatial window: red box

- Near and far field noise damping

- Subharmonic and harmonics: weakly nonlinear effects
Noise reduction

Case C: maximum of -10 dB

- Global noise reduction
- Few areas with increase, but SPL remains low
- Wavy SPL contours: weakly nonlinear effect
- Actuation modifies the mean pressure: nonlinearity
Deeper analysis: RIC content & POD eigen values

- Actuation drastically modifies the flow dynamic:
  \[ q(x, t) = \bar{q}(x) + \sum_{i=1}^{N} a_i(t)\phi_i(x) \]

**Fig. 1:** POD eigenvalues

**Fig. 2:** Mode relevance

**Fig. 3:** \( a_i(t) \), no actuation

**Fig. 4:** \( a_i(t) \), with control

Relative Information Content (RIC):
- more relevant modes are required

Eigen values distribution:
- new spectra
- time coefficients \( a_i(t) \to \text{constant for } t \to \infty \), time damping
Phase portrait: convergence towards a steady (equilibrium) state?

- POD decompositions:
  1) with unsteady actuation
  \[ q^a(x, t) = \bar{q}^a(x) + \sum_{i=1}^{N} a_i^a(t) \phi_i^u(x) + \gamma(t) \psi(x) \]
  2) with stabilization \((t \to \infty)\):
  \[ q^\infty(x) = \bar{q}^{ac}(x) + \sum_{i=1}^{N} a_i^\infty \phi_i^{ac}(x) \]

- Next step: determine the final state \(q^\infty(x)\)
Summary and perspectives

- Some concluding remarks
  1. Feedback output control law implemented in DNS (2 DNS + ROM \(\rightarrow\) low cost)
  2. Efficient and quite robust (when time \(\rightarrow\) \(\infty\))
  3. Several dB of noise reduction (up to -10 dB)
  4. Possible to tune the feedback law to improve efficiency, towards nonlinearity
  5. Independent on the DNS code: \(\rightarrow\) LES?

- Current works
  1. Linear Quadratic Gaussian control with ROM with state estimate
  2. Feedback law: \[ \gamma(t) = \int_{t-t_c}^{t} \sum_{i=1,N_s} G_i(t-\tau)y_i(\tau) \, d\tau \]

- Some improvements and perspectives
  1. Sensitivity analysis to many parameters included in the approach: \(N_{POD}, N_s\), actuation location and type, option of the ROM or POD, ...
  2. To test other flow decomposition to better take into account of actuation (with \(\dot{\gamma}\))
  3. To increase physical parameters (Re, Ma) and to test other flows
  4. Nonlinear feedback analysis and robustness analysis (\(H_2, H_\infty\))
  5. Validation/comparisons with experiments on low Reynolds number reference flows
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